Extended Abstract: Finite Verification of Infinite Families of Diagram Equations

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This work has already been submitted to QPL, and a preprit is available on the arXiv at:

https://arxiv.org/abs/1904.00706

The work was written with a focus on the direct application of the results, especially in relation to computer verification of theorems. This was done at the expense of focussing on the categorical underpinnings that led to these results. It was submitted with the following abstract:

The ZX, ZW and ZH calculi are all graphical calculi for reasoning about pure state qubit quantum mechanics. All of these languages use certain diagrammatic decorations, called !-boxes and phase variables, to indicate not just one diagram but an infinite family of diagrams. These decorations are powerful enough to allow complete rulesets for these calculi to be expressed in around ten rules. On the other hand reasoning involving decorated diagrams can be significantly more complicated. We present here a method for constructively reducing these infinite families of equations into finite verifying subsets. The only requirement for this construction is a property of our !-box structure that we call separability. This allows both researchers and proof assistants to reduce infinite families of problems down to undecorated, case-by-case verification, in a way not previously possible. In particular we note the removal of the need to reason directly with !-boxes in verification tasks as something entirely new, as well as extending a previously known result about removal of phase variables in verification tasks. This forms part of larger work in automated verification of quantum circuitry, and diagrammatic languages in general. The methods described here extend to any diagrammatic languages that meet certain simple conditions.

The work is relevant to ACT not just because it deals with graphical languages for quantum computing, but primarily because the results in the paper rely on a novel way of functorially interpreting these families of diagrams. Although the functorial nature of this interpretation is not emphasised in the paper itself, it would be the focus of a presentation given to an audience like ACT's. To give a shorter, but more categorical, summary:

> We have found a novel way of interpreting families of diagrams used in languages like ZX, in particular a functorial interpretation into $\mathbb{C}[X_1,\ldots,X_n]-$Mod. From there we use properties of $\mathbb{C}[X_1,\ldots,X_n]-$Mod in such a way that we can infer verification procedures for families of diagrams.

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