Graphical Linear Algebra

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Pawel Sobocinski
joint work with Filippo Bonchi and Fabio Zanasi

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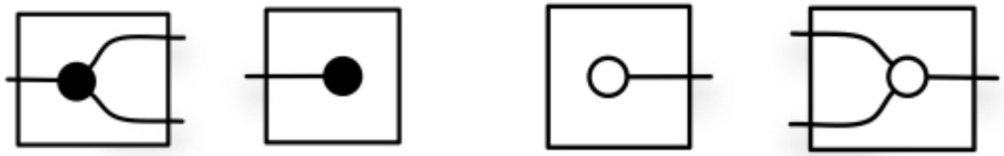
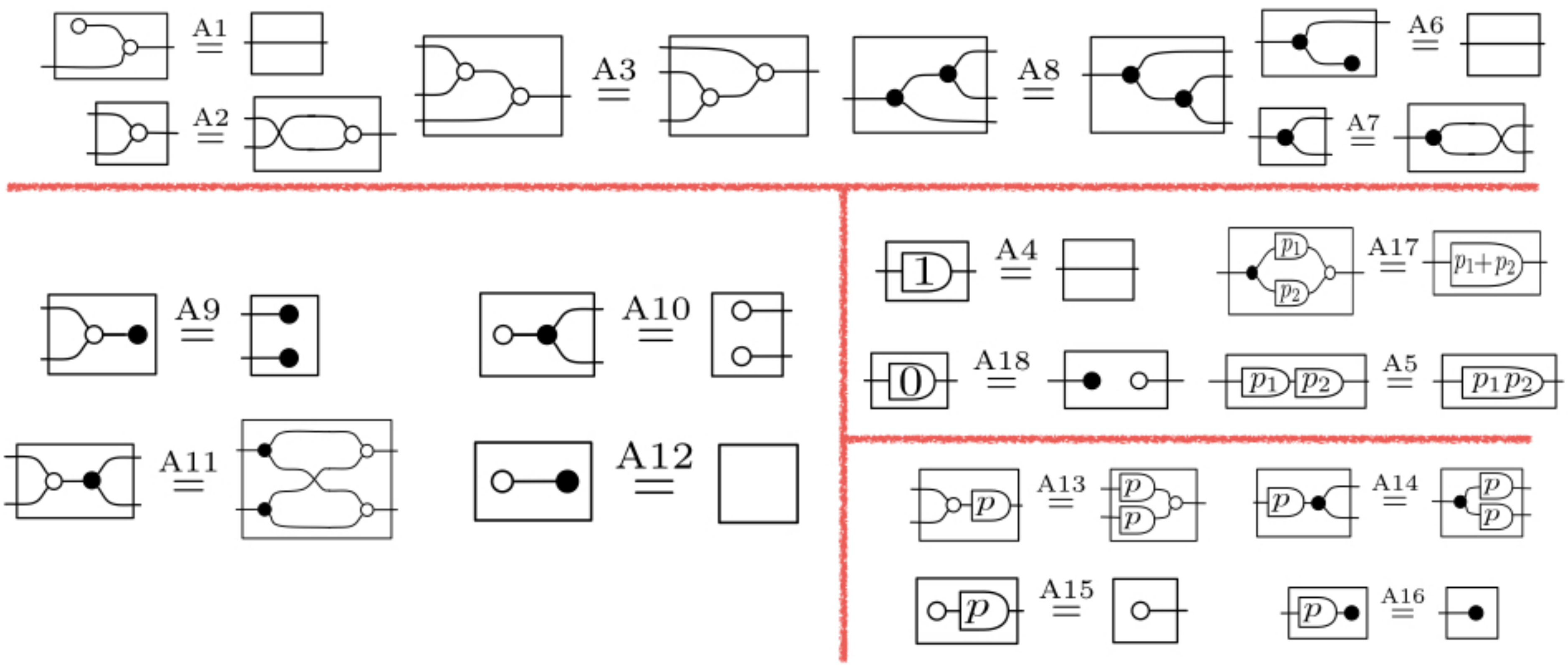
Recent work

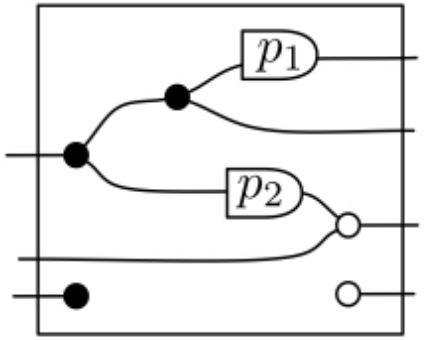
- string diagrams as “denotational semantics” for signal flow graphs
    - Bonchi Sobocinski Zanasi, *A categorical semantics of Signal Flow Graphs*, CONCUR`14
    - Bonchi Sobocinski Zanasi, *Full abstraction for Signal Flow Graphs*, PoPL`15

Hopf Algebra = graphical theory of linear transformations





Example
String diagrams = matrices



$$\begin{pmatrix} p_1 & 0 & 0 \\ 1 & 0 & 0 \\ p_2 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

There is an **isomorphism of PROPs** between the free symmetric monoidal theory HA (Hopf Algebra) and Mat R, the PROP where arrows from m to n are mxm R-matrices

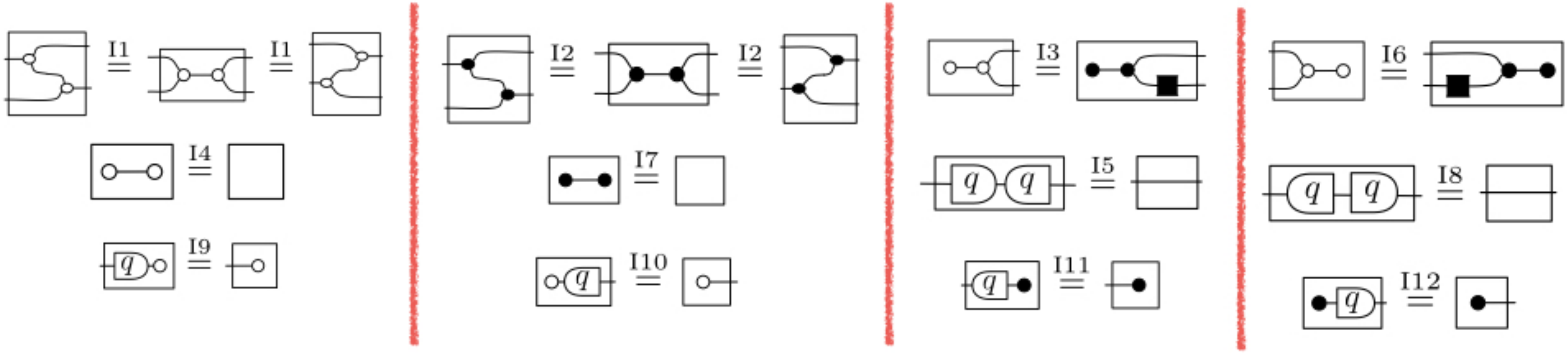
tl;dr this is the graphical theory of matrices

Interacting Hopf Algebras = graphical theory of (linear) spaces

Bonchi, Sobocinski, Zanasi. *Interacting Hopf Algebras*, Arxiv, March 2014.

+ Hopf algebra

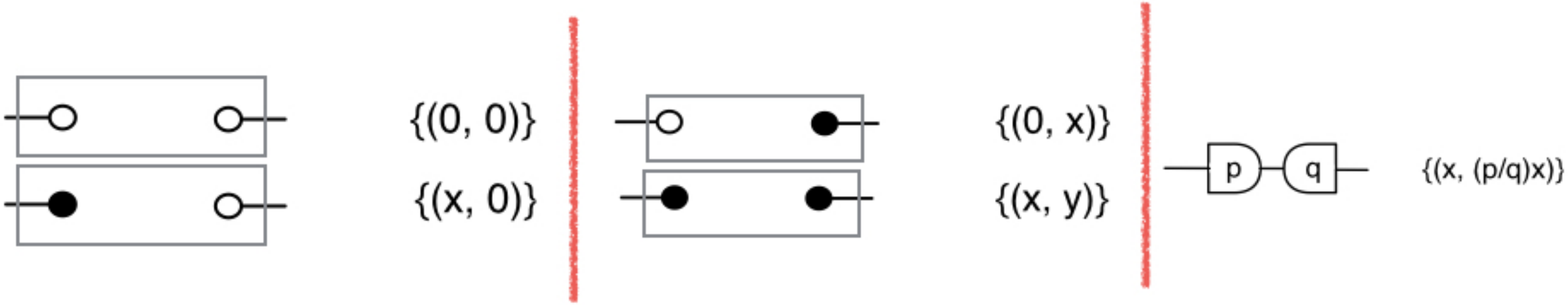
+ Hopf algebra op



for q not 0

Example
string diagrams = spaces

- The combinatorics of subspaces of $\mathbf{Q}^2$



There is an **isomorphism of PROPs** between the free symmetric monoidal theory IH (Interacting Hopf Algebra) and LinRel k, where k is the field of fractions of R. LinRel k is the PROP where arrows from m to n are subspaces of $k^m \times k^n$ and composition is relational.

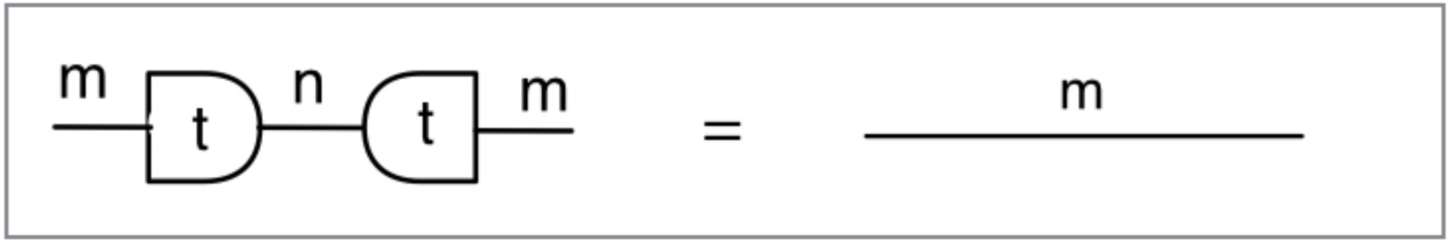
tl;dr this is the graphical theory of linear algebra

Lets redo linear algebra!

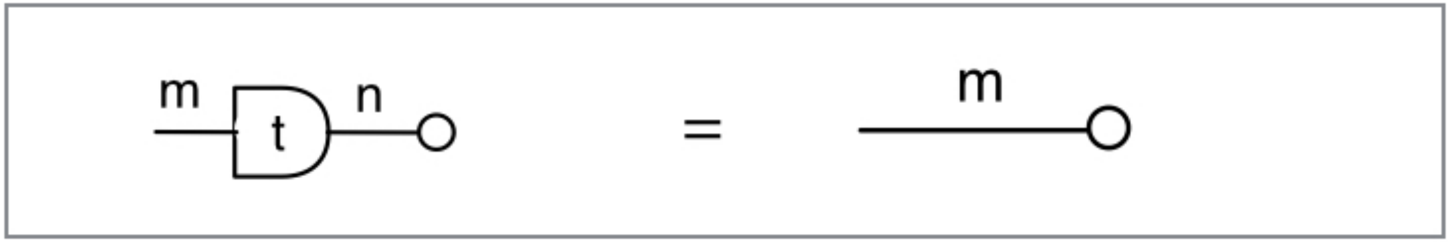
- A linear transformation t is injective iff ker t = 0.
- **Textbook proof:**
  - Assume ker t = 0. If tf = tg then t(f-g)=0, so by assumption f-g = 0, hence f=g.
  - Assume t injective and tx = 0. Since always t0=0, tx = t0 and so x=0.

Translating the statement

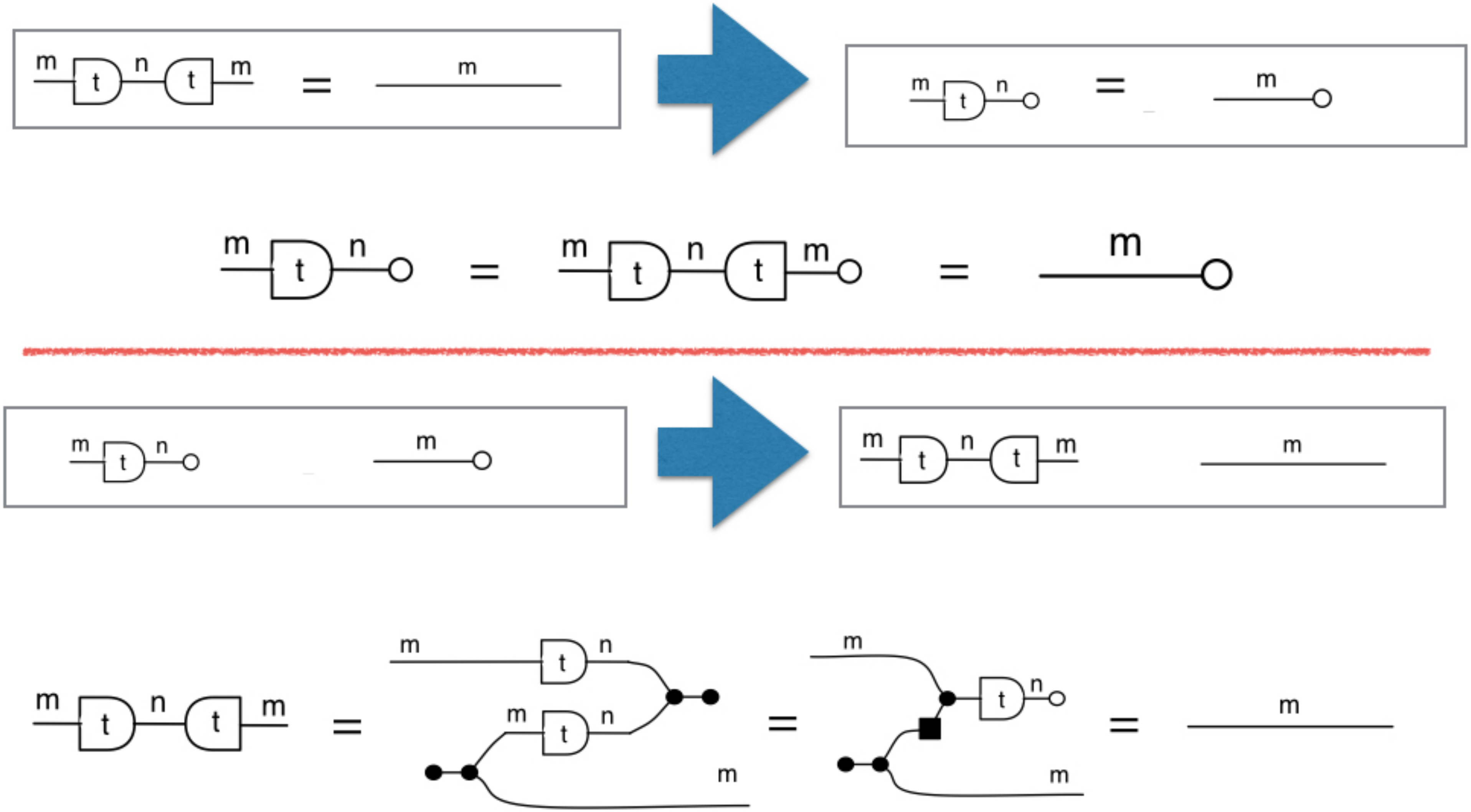
- t mono



- ker t = 0



Graphical proof



Conclusion

- The compositional language of string diagrams as the language of linear algebra
  - no set theory — a space is now a string diagram not a “set of vectors” closed under blah blah. No basis vectors etc.
  - no sudden change of language moving from linear transformations to subspaces — the classical language is a hack! String diagrams talk about both matrices and spaces.
  - previously hidden symmetries become apparent
  - closer to applications (e.g. signal flow graphs, electrical circuits, etc)