Reality of the Quantum State: A Stronger ψ-ontology Theorem

Shane Mansfield

UNIVERSITY OF OXFORD
DEPARTMENT OF COMPUTER SCIENCE

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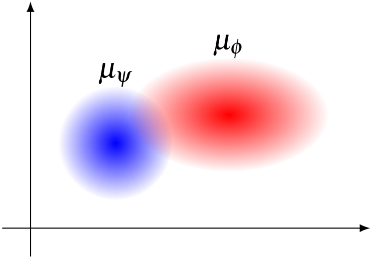
The Quantum State ψ — Real or Phenomenal?

- Assume some space Λ of *ontic states*
- Preparation of *quantum states* $\psi, \phi \in \mathscr{H}$ induce probability distributions μ_ψ, μ_ϕ over Λ, etc.



- If distributions can overlap $\rightarrow$ ψ-epistemic
- If distributions never overlap $\rightarrow$
  Each $\lambda \in \Lambda$ encodes a *unique* quantum state, so ψ-ontic

Harrigan & Spekkens, arXiv:0706.2661 [quant-ph]

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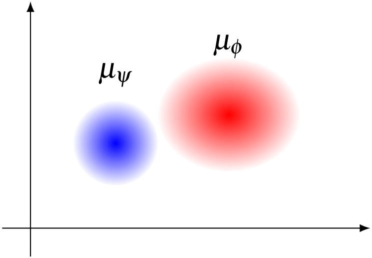

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The PBR Theorem*

The following assumptions

1. systems have an objective physical state
2. quantum predictions are correct
3. *preparation independence*

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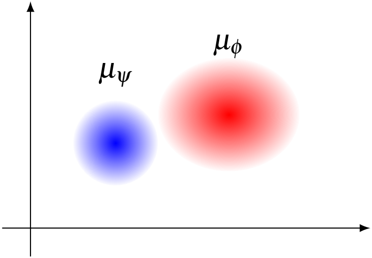

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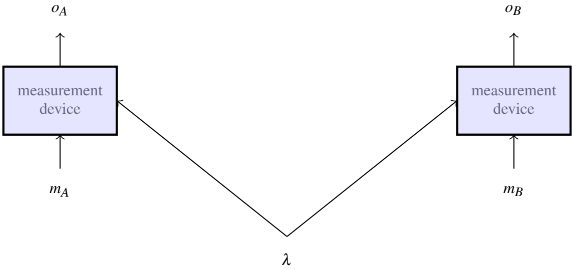
Preparation Independence

The only reasonable option?

$$\mu(\lambda_A, \lambda_B \mid p_A, p_B) = \mu(\lambda_A \mid p_A) \times \mu(\lambda_B \mid p_B)$$

Comparison with Bell Locality

An intuitive notion in measurement scenarios



$$p(o_A, o_B \mid m_A, m_B, \lambda) = p(o_A \mid m_A, \lambda) \times p(o_A \mid m_B, \lambda)$$

(Ruled out by Bell's Theorem)

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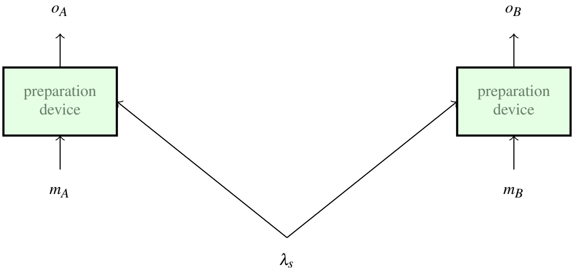
Weakening Preparation Independence

An intuitive notion of independence
(from the analogy with Bell Locality)



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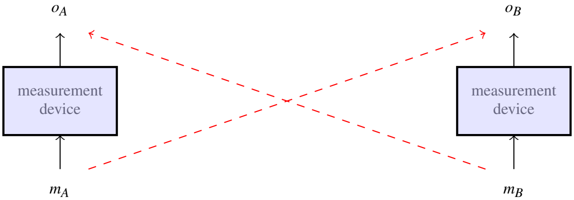
No-signalling



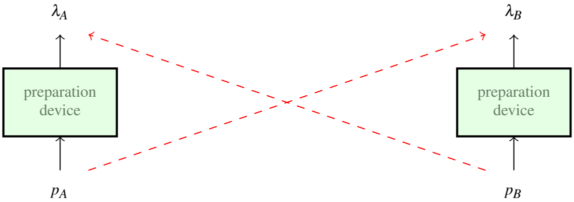
$$p(o_A \mid m_A, m_B) = p(o_A \mid m_A)$$

$$p(o_B \mid m_A, m_B) = p(o_B \mid m_B)$$

- Allows good notion of subsystem
- Consistent with SR

An Alternative to Preparation Independence

Idea: make the minimum assumption that will allow a reasonable notion of subsystem



$$\mu(\lambda_A \mid p_A, p_B) = \mu(\lambda_A \mid p_A)$$

$$\mu(\lambda_B \mid p_A, p_B) = \mu(\lambda_B \mid p_B)$$

Escaping PBR's Conclusion

A ψ-epistemic model realising PBR statistics:

Define $\mu_{00}, \mu_{0+}, \mu_{+0}, \mu_{++}$ by the table below and measurement response functions as on the right

		System 2			
		$\|0\rangle$		$\|+\rangle$	
		λ_δ	λ_0	λ_δ	λ_+
System 1: $\|0\rangle$	λ_δ	0	1/4	0	1/4
	λ_0	1/4	1/2	1/4	1/2
$\|+\rangle$	λ_δ	0	1/4	0	1/4
	λ_+	1/4	1/2	1/4	1/2

$$\xi_{\neg(\psi,\psi)}(\boldsymbol{\lambda}) := \begin{cases} 1 & \text{if } \boldsymbol{\lambda} = (\lambda_{\phi-\psi}, \lambda_{\phi-\psi}) \\ 1/2 & \text{if } \boldsymbol{\lambda} \in \{(\lambda_{\psi\wedge\phi}, \lambda_{\phi-\psi}), (\lambda_{\phi-\psi}, \lambda_{\psi\wedge\phi})\} \\ 0 & \text{otherwise} \end{cases}$$

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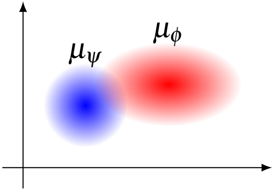
The Crux of the Matter (Outline)

One step of the PBR argument:

- Suppose μ_ψ and μ_ϕ have overlapping supports, such that a device that can prepare $p \in \{\psi, \phi\}$ results in an ontic state in the overlap region with probability at least q
- Then with two such preparation devices, there is probability at least q^2 that *both* ontic states lie in the overlap region



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A Stronger ψ-ontology Theorem

- Uniformly sample n preparations from m preparation devices

- Even allowing for the loophole in PBR, we prove a bound on the probability of being in the overlap region

$$q \leq \frac{1}{\left\lceil \frac{n}{m-1} \right\rceil}$$

- $q \to 0$ as $m \to \infty$

Conclusion

- Take a reasonable weaker notion of independence
- ψ can be interpreted statistically

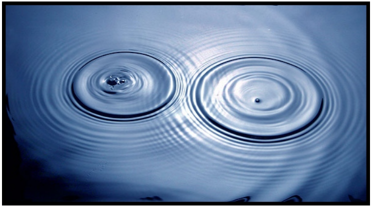
BUT!!!

- The degree to which systems may be composed limits the degree to which ψ may be statistical
- Fine for simple toy theories, but not for fully fledged physical theories
- ψ is still real!

Appendix: The Quantum State ψ — Real or Phenomenal?

ψ-ontic:

- A real physical wave (on configuration space?)
- Easiest way to think about interference
- PBR theorem



ψ-epistemic:

- ψ gives probabilistic information
- Collapse $\rightarrow$ Bayesian updating

- Can't reliably distinguish non-orthogonal ψ, ϕ
- ψ is exponential in the number of systems
- Can't be cloned
- Can be teleported