Categorical methods in linguistics

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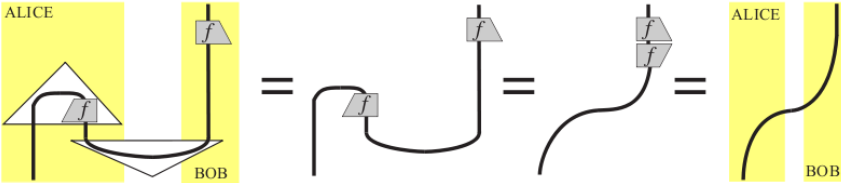
2014-10-16

What does



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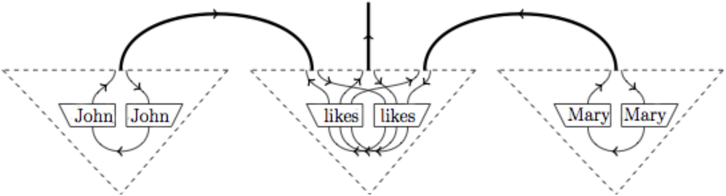
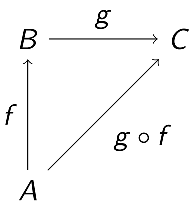


?

Categories capture composition



Reconciling syntax and semantics

Formal, symbolic, logical

"*I reject the contention that an important theoretical difference exists between formal and natural languages.*"

- English as a Formal Language, Montague

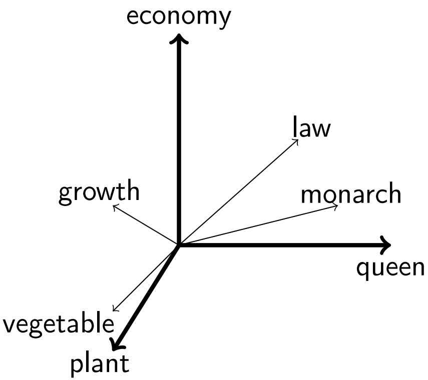
$$\dfrac{\vdash \text{who} : N \circ W \qquad \dfrac{\text{x} : W \vdash \text{x} : W \qquad \dfrac{\vdash \text{did} : (S\backslash W)/S \qquad \dfrac{\vdash \text{John} : N \qquad \dfrac{\text{y} : N \vdash \text{y} : N \qquad \vdash \text{see} : (S\backslash N)/N}{\text{y} : N \vdash \text{see y} : S\backslash N}[/E]}{\text{y} : N \vdash \text{John see y} : S}[\backslash E]}{\text{y} : N \vdash \text{did John see y} : S\backslash W}[/E]}{\text{x} : W, \text{y} : N \vdash \text{x did John see y} : S}[\backslash E]}{\vdash \text{who did John see} : S}[\circ E]$$

Compositional but incomplete model of meaning.

Distributional

"*You shall know the word by the company it keeps*"

- J.R. Firth



Good model of meaning of individual words but not obviously compositional

Compositional distributional model

- With categorical methods we wish to provide a general and abstract **interface** between these two approaches.
- A functor

$$\text{Syntax} \xrightarrow{\quad F \quad} \text{Semantics}$$

- What structure do we need to express basic syntactic and semantic information? Category in which
  - objects are grammatical types;
  - morphisms are ???.

What structure?

- Monoidal: type of a sentence is the tensor of the types of the words

$$\text{My fake plants died} : Pr \otimes Adj \otimes N \otimes V^i$$

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$$NP \otimes (NP \Rightarrow S) \xrightarrow{Eval_{NP,S}} S$$

* satisfying the appropriate universal properties.

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$$(S \Leftarrow NP) \otimes NP \xrightarrow{Lave_{NP,S}} S$$

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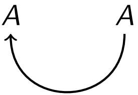
A more convenient framework: compact closed categories

We get a diagrammatic calculus for free!

$\eta^l_A =$


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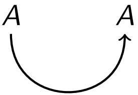

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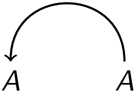

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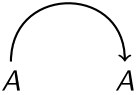

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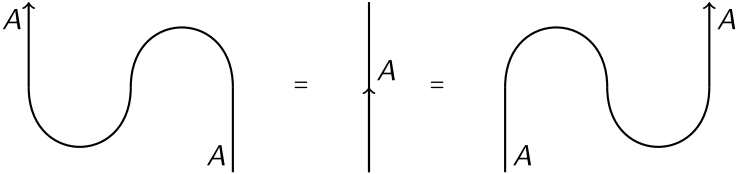
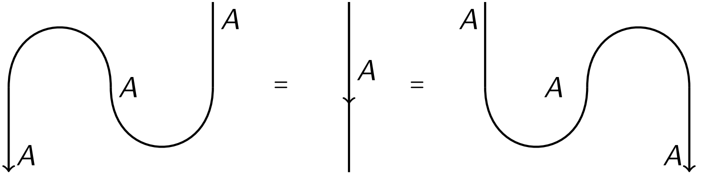


and



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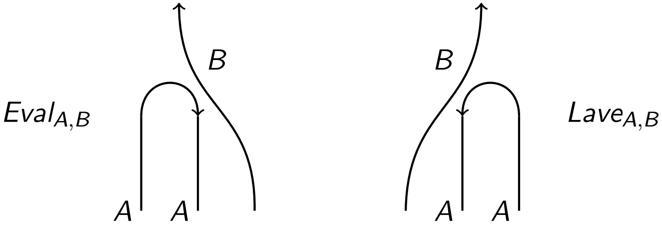
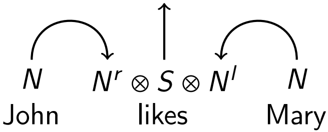
$$A \otimes (A^r \otimes B) \xrightarrow{\epsilon^r_A \otimes 1_B} B \qquad (B \otimes A^l) \otimes A \xrightarrow{1_B \otimes \epsilon^l_A} A$$



Reductions

- Morphisms in the internal language of a monoidal closed category.
- A reduction looks like



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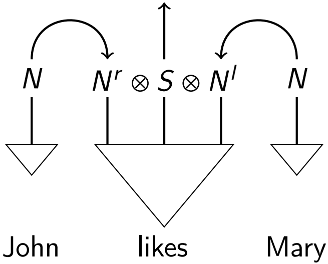
- Dagger: involutive, identity on object contravariant functor*. To compare the proximity of meaning:

$$I \xrightarrow{love} N^r \otimes S \otimes N^l \xrightarrow{like^\dagger} I$$

*satisfying coherence conditions with the compact closed structure.

Functorial interpretation

Strict monoidal functor from our syntactic category to our semantic category.



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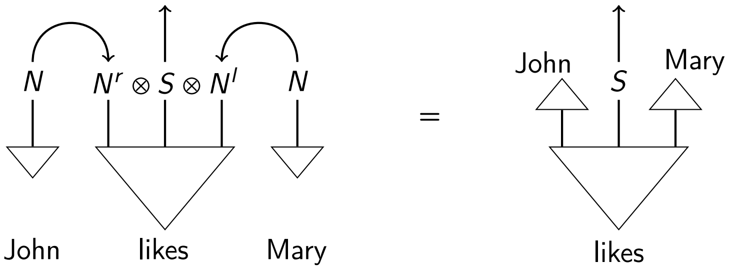

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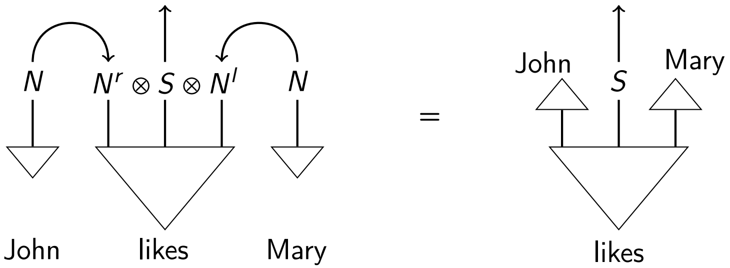

Examples:

$$Gr \xrightarrow{F} \text{FdHilb}, \qquad \sum_{ijk} \langle John | v_i \rangle s_j \langle v_k | Mary \rangle$$

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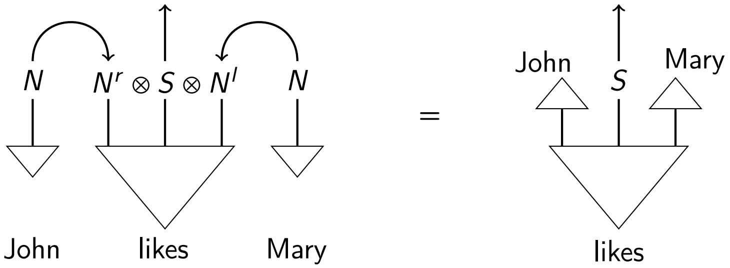

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$$Gr \xrightarrow{F} CPM(\text{FdHilb})$$

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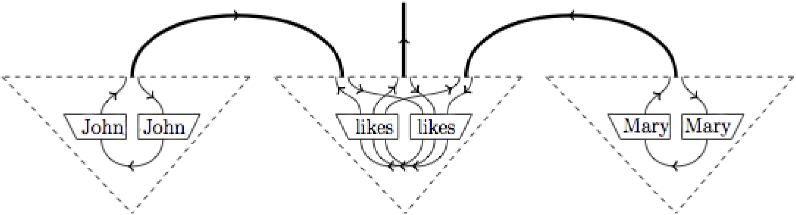


$$\ulcorner \mathrm{Tr}_{N,N}(\rho(\mathit{like}) \circ (\rho(\mathit{John}) \otimes 1_S \otimes \rho(\mathit{Mary}))) \urcorner$$

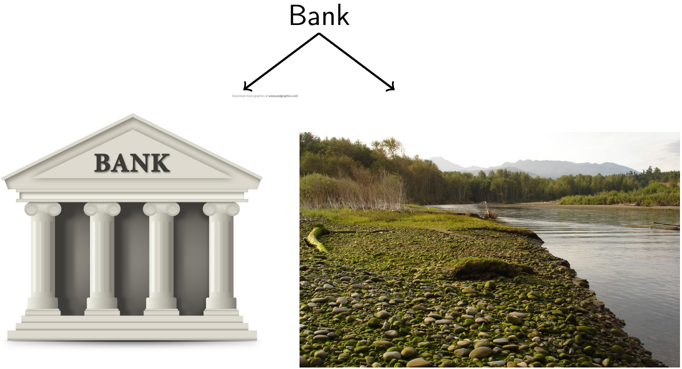
Disambiguation

Queen

Context determines meaning:

- "the queen overruled the decision of the prime minister"
- "Queen captures the rook in a1"

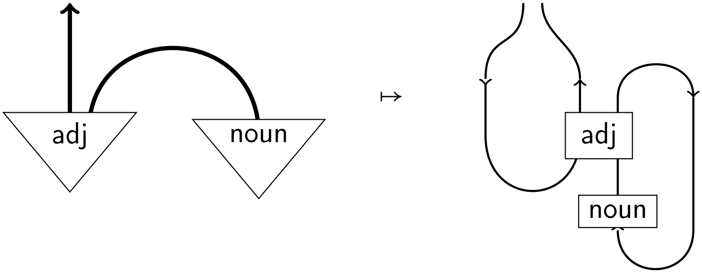
Toy example



Context determines meaning:

"river bank"

Flow of ambiguity



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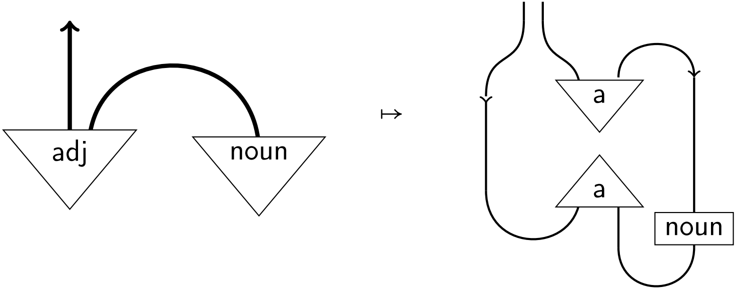

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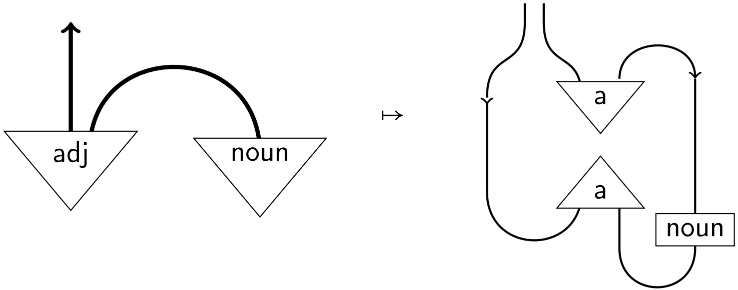

More refined process: CP^*

Moral of the story

Contextual features of natural language can be built in the wires.