# Complexity Resources in Physical Computation

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# Abstracts of Talks

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## The Computational Power of Correlations

**Janet Anders**, University College London

In measurement-based quantum computation (MBQC) the computation proceeds by adaptive single-qubit measurements on a multi-qubit entangled state, such as the cluster state. The adaptive measurements require a classical control computer which processes the previous measurement outcomes to determine the correct bases for the following measurement. Feeding the control computer with the correlated bits extracted by the measurements thus raises its computational power considerably, in the case of measurements on the cluster state, it reaches quantum universality.

We define a general framework which allows us to study the intrinsic computational power of correlations and establish a notion of resource states for measurement-based classical computation [1,2]. Surprisingly, the Greenberger-Horne-Zeilinger and Clauser-Horne-Shimony-Holt problems emerge naturally as optimal examples. Our work exposes an intriguing relationship between the violation of local realistic models and the computational power of entangled resource states. Finally, we show that the correlations obtained by measurements on separable states can not produce any deterministic computation beyond the power of the control computer [3].

[1] J. Anders and D. E. Browne, Phys Rev Lett PRL 102, 050502 (2009)
[2] D. E. Browne, J. Anders, Lecture Notes in Computer Science 5028:94 (2008)
[3] J. Anders and D. E. Browne, "Correlations, adaptiveness and classical control in measurement-based computation", in preparation.

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## Hybrid Quantum Simulators

**Ville Bergholm**, Harvard University

Most current quantum simulation algorithms are based on the quantum circuit model and, because of the Trotter decomposition used in implementing the Hamiltonian, require the coherent execution of millions of quantum gates for each simulation run. Scientifically useful quantum simulations are thus well out of reach for present-day quantum hardware.

We have developed a model for a computationally efficient, exact (within a given basis and to a required numerical precision) and error-resistant adiabatic quantum simulator with a far more feasible control scheme. The simulator consists of an adiabatically evolving simulation register with tunable two-local interactions, and an individually addressable readout register which can be controlled and measured nonadiabatically. The model requires somewhat complicated effectively k-local interactions between the qubits, which we propose to implement using perturbative Hamiltonian gadgets.

# About Combining Special Relativity and Computational Complexity Theory

**Jake Biamonte**, University of Oxford

The Lorentzian length of a timelike curve connecting both endpoints of a computation in Minkowski spacetime is smaller than the Lorentzian length of the corresponding geodesic. In this talk, I will point out some properties of spacetime that allow an inertial classical computer to outperform a quantum one, at the completion of a long journey. We will focus on a comparison between the optimal quadratic Grover speed up from quantum computing and an $n = 2$ speedup using classical computers and relativistic effects. These results are not practical as a new model of computation, but allow us to probe the ultimate limits physics places on computers.

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# What is a Resource?

**Ed Blakey**, University of Oxford

We discuss the notion of 'computational resource' in a general, computation-model-independent context, in particular considering several interpretations of resource: as *commodities* consumed during computation; as operations permitted by the *paradigm* of computation, or by enveloping *physical laws*; as a system's *manufacturing* (as opposed to running) *costs*; etc. We argue that qualifying our notion of resource using only something like Blum's axioms still allows undesirable complexity behaviour, and suggest suitable further restriction.

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# Computational Correlations and Multi-Partite Bell Inequalities

**Dan Browne**, University College London

Bell's inequality has been a cornerstone of our understanding of the fundamental differences between quantum and classical physics. In recent years, a computational interpretation of the Bell-CHSH inequality has been proposed [1], which has led to a number of new insights, including, most recently, a proposed derivation [2] of the quantum upper bounds of the Bell-CHSH inequality from information theoretic principles. Recently, [3] we have shown a link between the CHSH inequality, the GHZ paradox and the intrinsic computational power which correlations can possess (see [3] and Janet Anders' talk).

In my talk I will explore the extent to which such approaches can be generalised to the multi-party case. I will show that a broader class of Bell inequalities have a similar computational interpretation, and that this leads naturally to new multipartite generalisations of the Bell inequalities and the GHZ paradox [4]. This work illustrates a surprising connection between the Bell inequalities and measurement-based quantum computation.

[1] Wim Van Dam, DPhil Thesis (Oxford, 1999)
[2] M. Pawlowski, et al., A new physical principle: Information Causality, arXiv:0905.2292
[3] J. Anders and D. E. Browne, The computational power of correlations, Phys. Rev. Lett. 102, 050502 (2009)
[4] M. Hoban, E. T. Campbell, K. Loukopoulos and D. E. Browne, "Computational Bell Inequalities", in preparation.

## Why Is Quantum Non-Locality Limited? Perspectives from Non-Locality Distillation

**Nicolas Brunner**, University of Bristol

Quantum mechanics is a non-local theory, however not a maximally non-local one according to relativity. More precisely, there exist alternative theories containing more non-locality than quantum mechanics that still respect the no-signaling principle. Why these theories are unlikely to exist in nature, and what physical principle limits quantum non-locality is still not known today, despite an intensive research effort.

Here I will start by briefly reviewing general non-signaling theories, in particular focusing on non-local boxes. Then I will show how non-locality can be distilled out of non-local boxes, which will have important implications for information processing in general non-signalling theories.

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## Bound States for Magic State Distillation

**Earl Campbell**, University College London
(joint work with Dan Browne, University College London)

Magic state distillation is an important primitive in fault-tolerant quantum computation. The magic states are pure non-stabilizer states which can be distilled from certain mixed non-stabilizer states via Clifford group operations alone. Hence, any distillable non-stabilizer state is a resource for fault-tolerant quantum computation. Due to the Gottesman-Knill theorem, convex mixtures of Pauli eigenstates are not expected to be magic state distillable, but it has been an open question whether all mixed states outside this octahedral set may be distilled. I will outline our recent result showing that, when protocols are of finite size, non-distillable states exist outside the stabilizer octahedron. In analogy with the bound entangled states, which arise in entanglement theory, we call such states bound states for magic state distillation.

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## Contextuality as a Resource

**Bob Coecke**, University of Oxford

We study the connections between recent work by Anders and Browne on non-locality as a resource in measurement-based classical computing and by Coecke, Edwards and Spekkens on the group-theoretic passage from local 'almost quantum models' to genuine non-local ones. Key is the contextual reliance of GHZ-non-locality on the AND-gate, and its representation as the four-element cyclic group.

## Computable Scientists, Uncomputable World

**José Félix Costa**, Instituto Superior Técnico, Universidade Técnica de Lisboa
(joint work with Edwin Beggs and John V. Tucker, both Swansea University)

In this talk we adopt the perspective of scientists modelled as Turing machines. Such an idea has been developed by John Case in the recursive analysis of learning theory inspired by problems in artificial intelligence. Our motivation is physics: we inspect the physical world by using experimental apparatus and observations as an oracle to a Turing machine.

The physical world might be computable or rather uncomputable. We investigate the complexity classes induced by some classes of physical experiments and the structural relations between them.

Moreover, the results so far obtained contradict some aspects of classical measurement theory (a chapter of logic and the philosophy of science started by, e.g., Campbell, Nagel, Hempel, Carnap, and Suppes, in the first half of the 20th century). We show that, by changing the basic axioms of measurement, we obtain a new logic compatible with computational complexity.

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## Non-Classical Resources for Interactive Proofs

**Joseph Fitzsimons**, University of Oxford

Quantum mechanics allows for a more general class of correlations than are possible with purely classical states. These non-classical states fundamentally distinguish quantum information processing from classical computation, and are a prerequisite for many quantum communications protocols including quantum key distribution and teleportation. In this talk I will discuss the use of non-classical quantum states as a resource for interactive proofs. In particular, I will show that non-classical aspects of quantum states allow for the interactive proof of any problem within BQP given a classical verifier and a quantum prover. This problem is believed to be impossible with only classical communication and resources.

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## Fault Tolerant Quantum Computation—<br>The Curse of the Open System

**Amit Hagar**, Indiana University

Quantum computer scientists would like us to believe that (1) skepticism about the computational superiority of quantum computers is tantamount to skepticism about the universal applicability of quantum mechanics, and that (2) the reason we still have no scalable quantum computer is only technological. Support for this view comes from the well known threshold theorems, according to which, under a certain level of noise, an arbitrary long quantum computation can be executed with only moderate (at most polynomial) overhead in computational costs. In this talk I shall question the physical content of the threshold theorems, and raise doubt with respect to the validity of (1) and (2).

## Information Flow in Quantum and Classical Computation

**Clare Horsman**, Hewlett-Packard

I address the question of unifying quantum and classical resources in measurement-based computing. I introduce a new graphical calculus for information flow in such computation, which concentrates on the physical and causal structure of the flow, in distinction to the logical flow models already developed (Coecke, Abramsky, Kashefi…). I show how quantum and classical processes are formally integrated in this model, and give an analysis of the joint resources used in a typical measurement-based computation. This leads to a new understanding of the roles played by the cluster state resource and the classical measurement channels in measurement-based quantum computing.

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## Permutational Quantum Computing

**Stephen Jordan**, California Institute of Technology

In topological quantum computation the geometric details of a particle trajectory are irrelevant; only the topology matters. Taking this one step further, we consider a model of computation that disregards even the topology of the particle trajectory, and computes by permuting particles. Whereas topological quantum computation requires anyons, permutational quantum computation can be performed with ordinary spin-½ particles, using a variant of the spin-network scheme of Marzuoli and Rasetti. I do not know whether permutational computation is universal. It may represent a new complexity class within BQP. Nevertheless, permutational quantum computers can in polynomial time approximate matrix elements of certain irreducible representations of the symmetric group and evaluate certain spin foams in the Ponzano-Regge model. No polynomial time classical algorithms for these problems are known.

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## Quantum Simulation and Analogue Computation

**Viv Kendon**, University of Leeds

The quantum version of analogue computation—usually known as continuous variable quantum computing (CVQC)—is relatively unexplored compared to digital quantum computation. We know that universal quantum computation is possible in an analogue setting [Lloyd + Braunstein PRL 82 1784 1999], with the same caveats as classical analogue computation where the resources scale unfavourably with precision due to the lack of binary encoding of the data. Little else is known about the theoretical underpinning and practical application. Simulation of quantum systems also does not binary encode the data [Brown et al, PRL 97 050504 2006]. In this talk I will explore the commonalities between analogue computation and quantum simulation, and the implications this has for the development of both.

## Complexity Resources and Sinks in Noisy Quantum Computation

**Marco Lanzagorta**, ITT Corporation

In recent years quantum computing has emerged as an important area of research that promises to revolutionize information processing systems. Unfortunately, quantum devices are highly susceptible to the effects of environmental noise and decoherence. Nevertheless, the Threshold Theorem guarantees the possibility of fault tolerant quantum computation, as long as the probability of error of each physical component is below a certain threshold. However, it can be shown that uncorrected constant errors affect algorithmic complexity, even if they are arbitrarily small. Furthermore, these errors alter complexity classifications. In order to avoid such an algorithmic penalty, the uncorrected error probability has to scale with the characteristic variable of the problem (i.e. the number of qubits). More specifically, the uncorrected error probability needs to scale on both, the size of the circuit and the number of iterations necessary to implement a quantum algorithm. As such, error probability scaling plays a role on trade-offs between time and circuit size resources. Therefore, quantum error scaling consumes nontrivial amounts of time and circuit size resources and it does not reflect the theoretical difficulty of the underlying computational problem that needs to be solved. As such, quantum errors act as "complexity sinks".

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## Sombrero Adiabatic Quantum Computation: A Heuristic Strategy for Quantum Adiabatic Evolution

**Salvador Venegas-Andraca**, Tecnológico de Monterrey

In the adiabatic quantum computation (AQC) approach, the ground state of a quantum system is evolved by a time-dependent Hamiltonian toward a final ground state that encodes the answer to a computational problem. AQC initial Hamiltonians conventionally have a uniform superposition as ground state. We diverge from this practice by introducing a new strategy, in which the adiabatic evolution starts with an initial guess chosen at random or by following prior knowledge or intuition about the problem, followed by a "sombrero-like" perturbation, hence the name sombrero AQC (SAQC).

We provide a scheme to build initial Hamiltonians which encode initial guesses in their ground states, and we describe a proof-of-concept simulation of the SAQC protocol by performing an exhaustive numerical study on hard-to-satisfy instances of the satisfiability problem (3-SAT).

Our results show that about 35% of the initial 7 variable guesses have a significantly larger minimum gap compared to the minimum gap expected for conventional AQC (CAQC), possibly allowing for more efficient quantum algorithms. Finally, we propose serial and parallel versions of a quantum adiabatic algorithm based on SAQC.

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## Quantum Computational Resources to Generate Structure

**Karoline Wiesner**, University of Bristol

One way to measure information storage in quantum systems is to ask what resources does it take to generate the system's dynamics statistically accurate. The system's dynamics is represented as a measurement sequence. Classically, this question has a well-defined answer in terms of stochastic finite-state machines. Quantum mechanically, we show how quantum finite-state machines can be used to the same end. We present a general definition of quantum finite-state machines and discuss information theoretic measures of structure present in the dynamics.

# Choosing Resources

**Damien Woods**, Universidad de Sevilla

We discuss computational resources in a number of models. We begin by talking about resources that are important for optical computers. These are computers that process images in a single timestep, and use resources such as amplitude resolution and number of pixels. A few results that highlight the importance of our particular choice of optical resources will be mentioned, as well as some modelling traps that one should not fall into. Often we identify certain resources as important because they are expensive in some sense. However, it is also the case that there are resources that are important because they tell us something interesting about the structure of the problems that we wish to solve. This leads to a discussion about careful choices of resources and of claims of what can be solved in practice via various computing paradigms.