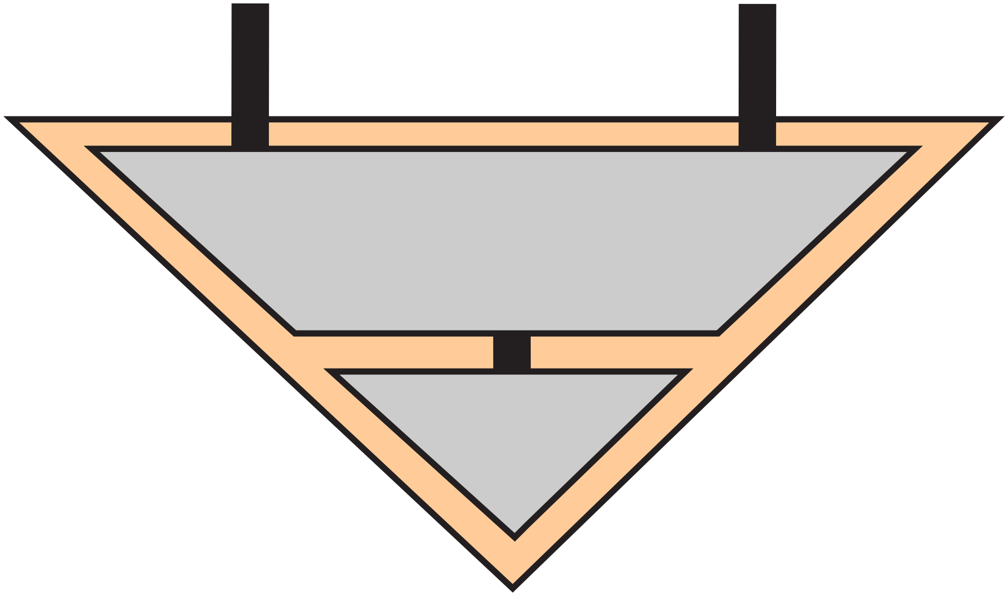
Bob Coecke
University of Oxford – Computer Science – Quantum Group



STRUCTURAL RESOURCES FOR $Q(x)$

— *Categorical Quantum Mechanics* —

- An attempt to formulate quantum mechanics in *symmetric monoidal categories*, i.e. using *CS methods*.

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- ***Intuitive purely graphical quantum reasoning*.**

— *security related* —

B. Coecke, B.-S. Wang, Q.-L. Wang, Y.-J. Wang and Q.-Y. Zhang (2010) *Graphical calculus for quantum key distribution*. ENTCS (QPL'09 volume).

B. Coecke and S. Perdrix (2010) *Environment and classical channels in categorical quantum mechanics*. Computer Science Logic, LNCS 6247. arXiv:1004.1598

A. Hillebrand (2011) *Quantum protocols involving multiparticle entanglement and their representations in the zx-calculus*. MSC thesis, University of Oxford, 2011. http://www.cs.ox.ac.uk/people/bob.coecke/Anne.pdf

CATEGORICAL Q.M. IN †-COMPACT CATEGORIES

S. Abramsky and B.C. – LiCS'04 – arXiv:0808.1023

— *(physical) data in monoidal category* —

Systems:

$$A \qquad B \qquad C$$

Processes:

$$A \xrightarrow{f} A \qquad A \xrightarrow{g} B \qquad B \xrightarrow{h} C$$

Compound systems:

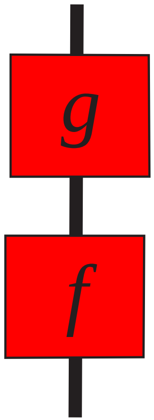
$$A \otimes B \qquad \mathrm{I} \qquad A \otimes C \xrightarrow{f \otimes g} B \otimes D$$

Temporal composition:

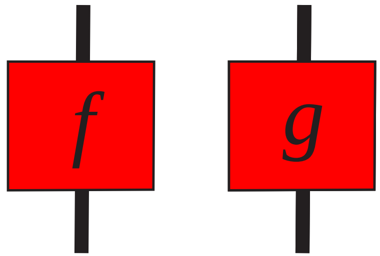
$$A \xrightarrow{h \circ g} C \ := \ A \xrightarrow{g} B \xrightarrow{h} C \qquad A \xrightarrow{1_A} A$$

— *graphical notation* —

$g \circ f \equiv$

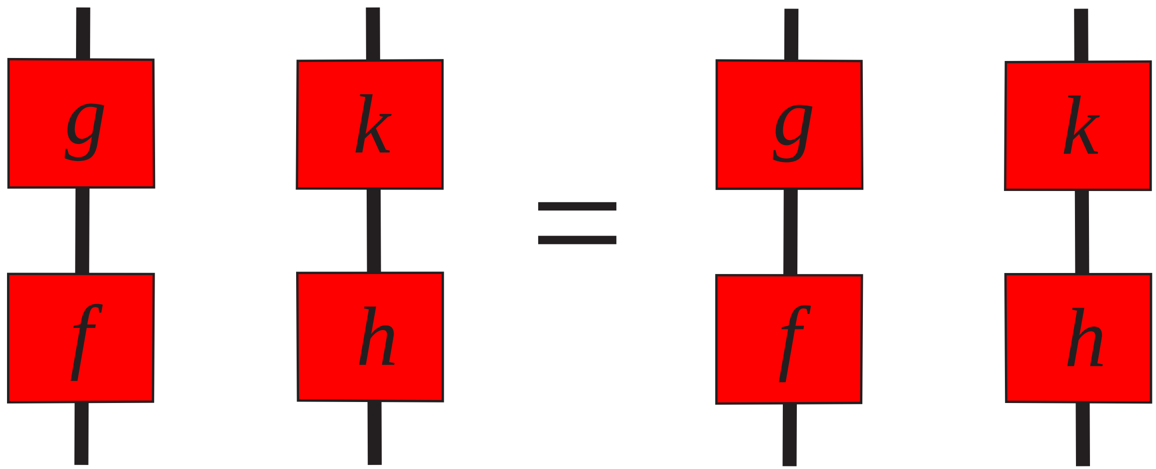


$f \otimes g \equiv$



— *merely a new notation?* —

$$(g \circ f) \otimes (k \circ h) = (g \otimes k) \circ (f \otimes h)$$

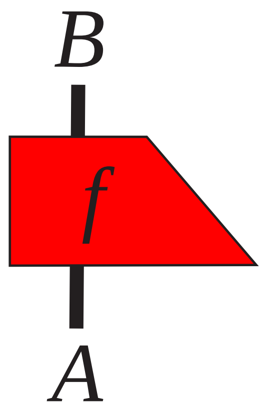


— *graphical notation* —

Thm. [Joyal & Street '91] ***An equational statement between expressions in symmetric monoidal categorical language holds if and only if it is derivable in the graphical notation via homotopy*.**

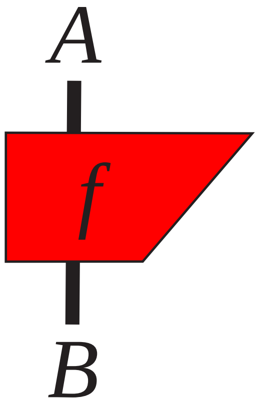
— *quantum metric* —

$$f : A \longrightarrow B$$



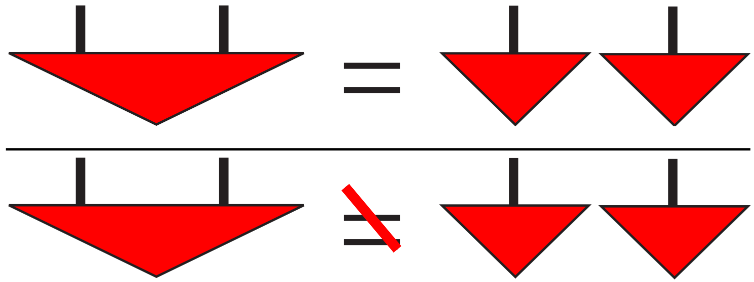
— *quantum metric* —

$$f^{\dagger} : B \to A$$

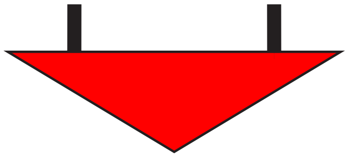


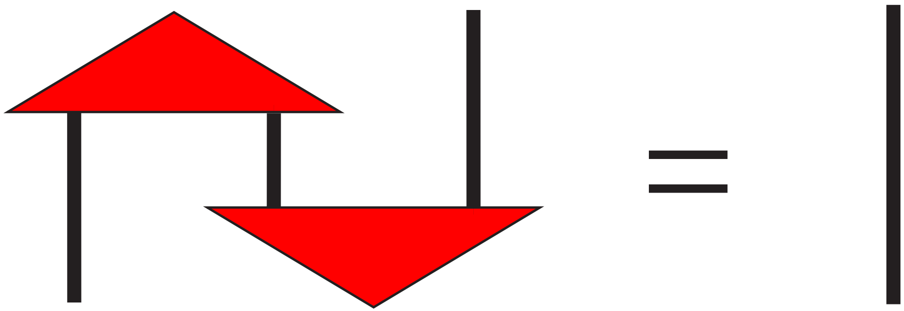
— *(pure) classical vs. quantum* —

$$\frac{classical}{quantum} =$$



— †-compact categories —





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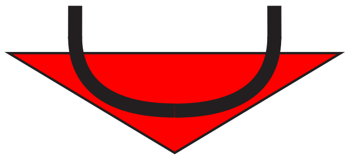

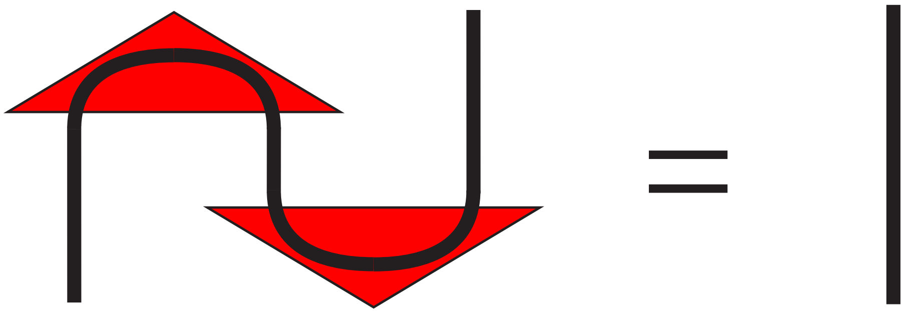

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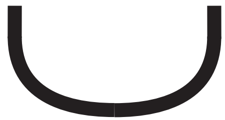

— †-*compact categories* —

Thm. [Selinger '05] *An equational statement between expressions in dagger compact symmetric monoidal categorical language holds if and only if it is derivable in the graphical notation via homotopy*.

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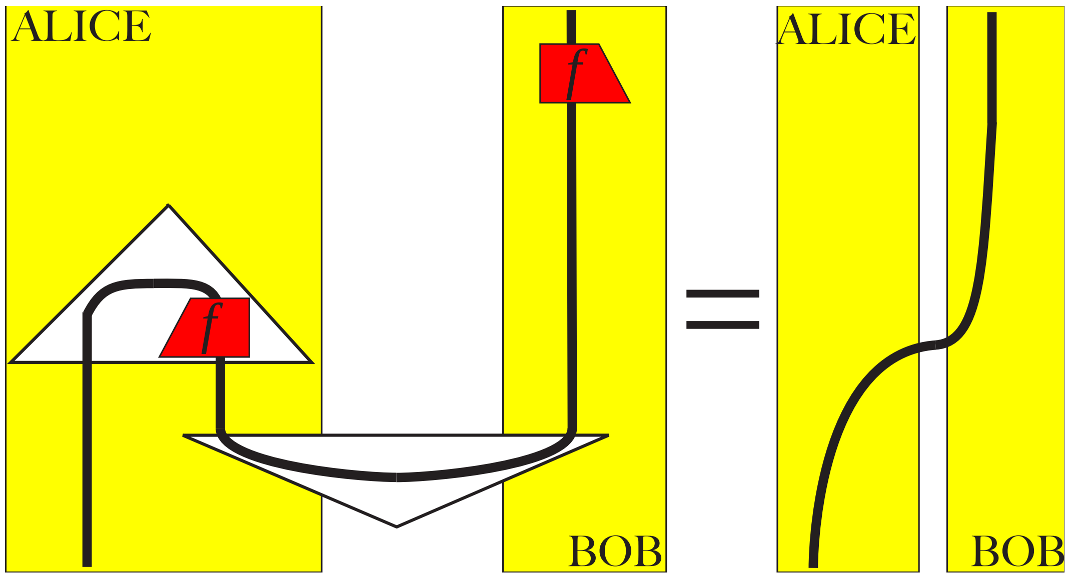
Thm. [Selinger '08] *An equational statement between expressions in dagger compact symmetric monoidal categorical language holds if and only if it is derivable in the category of finite dimensional Hilbert spaces, linear maps, tensor product, and adjoints*.

— *†-compact categories* —



$\Rightarrow$ **quantum teleportation**

CLASSICAL STRUCTURES IN CATEGORICAL Q.M.

B.C. and D. Pavlovic – arXiv:quant-ph/0608035

B.C., D. Pavlovic and J. Vicary – arXiv:0810.0812

B.C., E.O. Paquette and D. Pavlovic – arXiv:0904.1997

B.C. and S. Perdrix – arXiv:1004.1598

— *observables and classical data* —

quantum data cannot be copied nor deleted

— *observables and classical data* —

quantum data cannot be copied nor deleted

classical data CAN be copied and deleted

— *observables and classical data* —

NON-FEATURE:

quantum data cannot be copied nor deleted

FEATURE:

classical data CAN be copied and deleted

— *observables and classical data* —

NON-FEATURE:

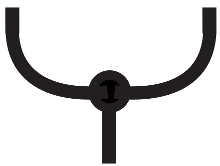
quantum data cannot be copied nor deleted

FEATURE:

classical data CAN be copied and deleted

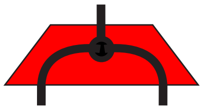
OBSERVABLE:

copying operation + deleting operation

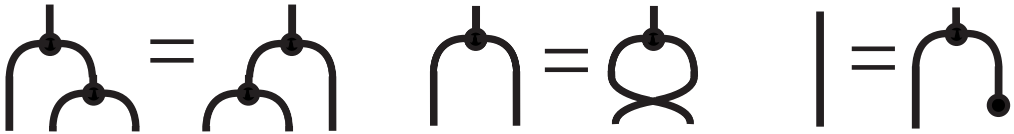


— *observables and classical data* —

A commutative monoid is object A with morphisms

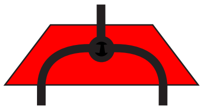
 $: A \otimes A \longrightarrow A$  $: \mathrm{I} \longrightarrow A$

s.t.

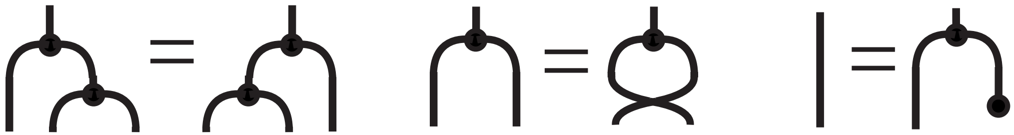


— *observables and classical data* —

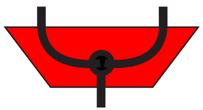
A commutative monoid is object A with morphisms

 $: A \otimes A \to A$  $: \mathrm{I} \to A$

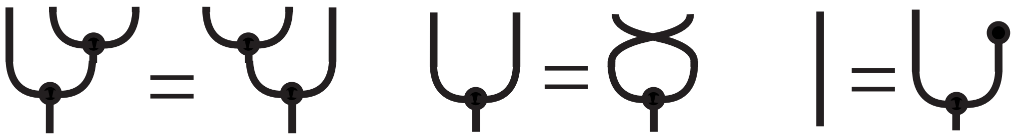
s.t.



A cocommutative comonoid is object A with morphisms

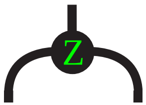
 $: A \to A \otimes A$  $: A \to \mathrm{I}$

s.t.

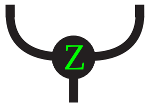


— *observables and classical data* —

FdHilb:



$$:: \begin{cases} |00\rangle \mapsto |0\rangle \\ |11\rangle \mapsto |1\rangle \end{cases}$$



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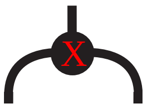
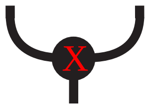
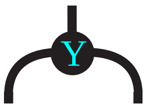


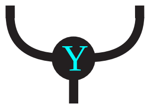
$$:: \begin{cases} |++\rangle \mapsto |+\rangle \\ |--\rangle \mapsto |-\rangle \end{cases}$$



$$:: \begin{cases} |+\rangle \mapsto |++\rangle \\ |-\rangle \mapsto |--\rangle \end{cases}$$



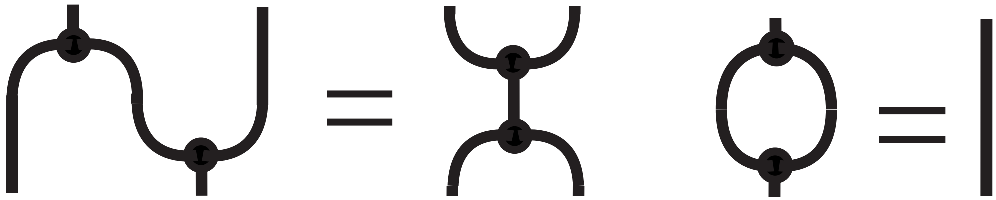
$$:: \begin{cases} |\sharp\sharp\rangle \mapsto |\sharp\rangle \\ |==\rangle \mapsto |=\rangle \end{cases}$$



$$:: \begin{cases} |\sharp\rangle \mapsto |\sharp\sharp\rangle \\ |=\rangle \mapsto |==\rangle \end{cases}$$

— *observables and classical data* —

Theorem. **Special dagger commutative Frobenius algebras** (†-SCFAs) in **FHilb**, that is,



Frobenius *special*

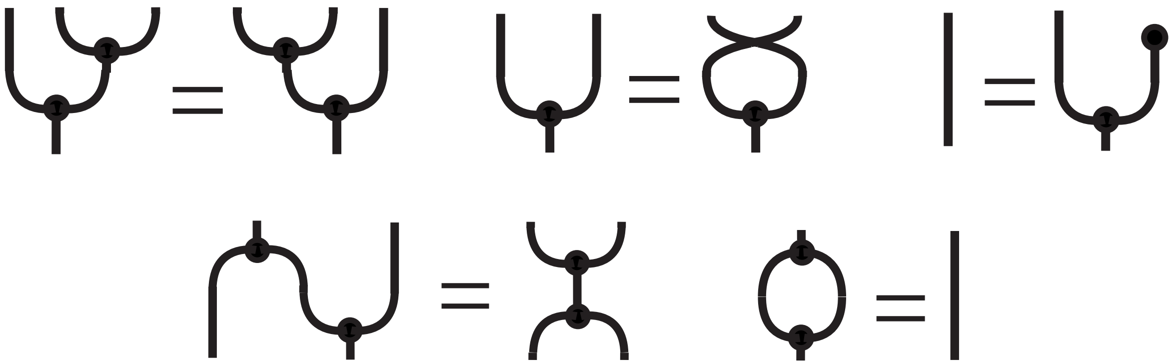
exactly correspond with **orthonormal bases** on the underlying Hilbert space via the correspondence:

$$\{|\,i\,\rangle\}_i \quad \longleftrightarrow \quad |\,i\,\rangle \mapsto |\,ii\,\rangle$$

— *observables and classical data* —

A **†SCFA** is a pair:

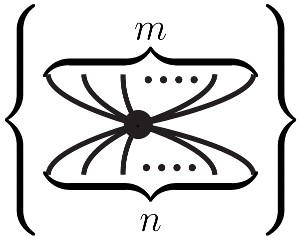
which is such that:



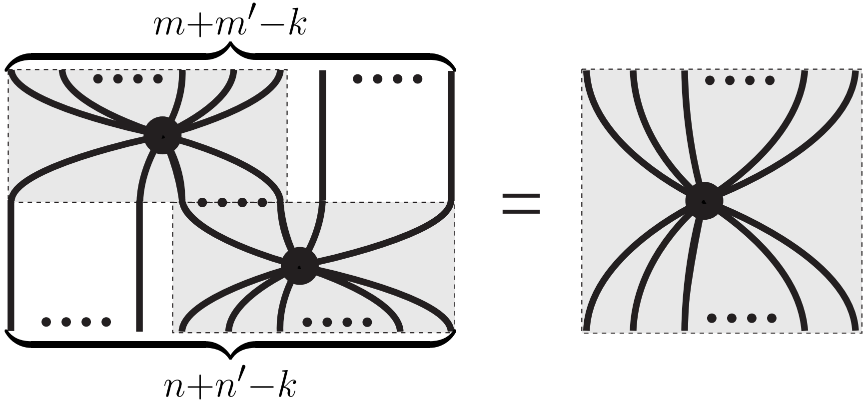
— *observables and classical data* —

A **†SCFA** is a family:

‘spiders’ =

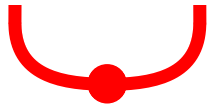


which is such that, for $k > 0$:

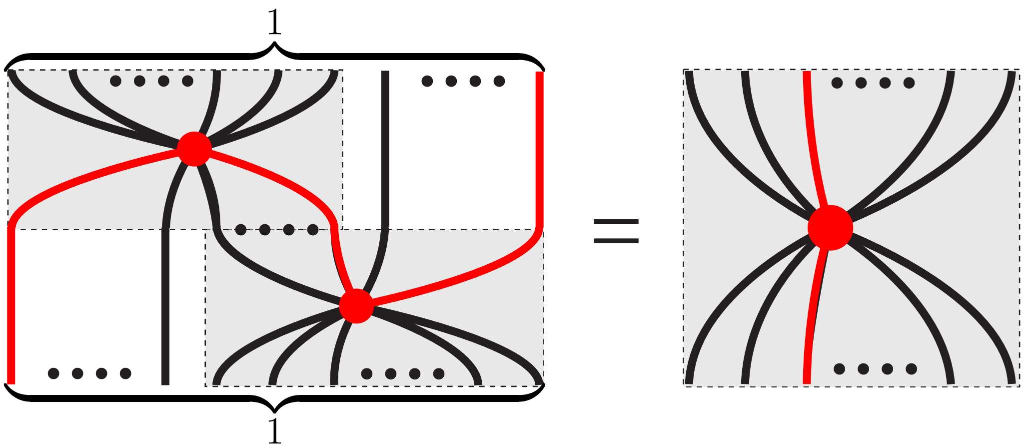


— *observables and classical data* —

Prop. The 2/0-spider ~ compactness:



Proof.



COMPLEMENTARITY IN CATEGORICAL Q.M.

B.C. and R. Duncan – ICALP'08 – arXiv:0906.4725

— *complementarity* —

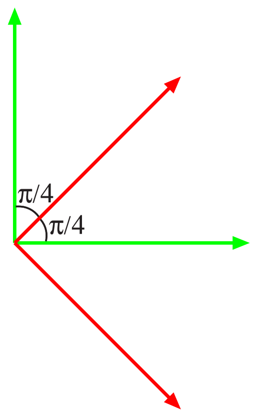
Two bases

$$\{|0\rangle, \ldots, |n\rangle\} \quad \text{and} \quad \{|0\rangle, \ldots, |n\rangle\}$$

are **complementary** (or **unbiased**) if

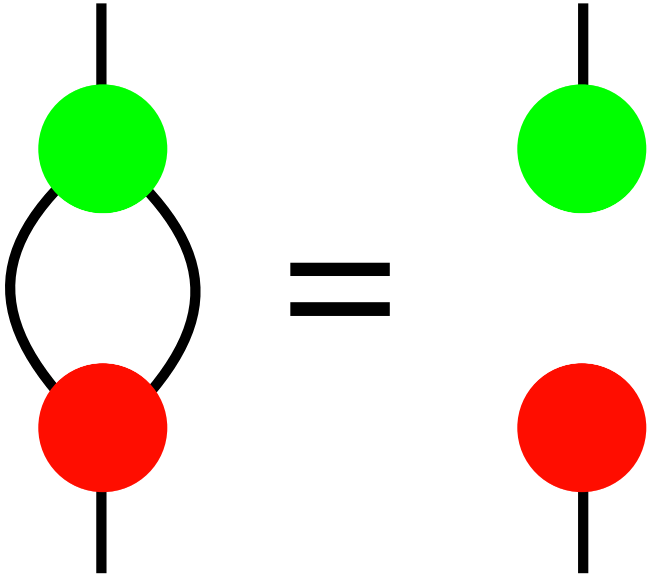
$$|\langle\, i \,\|\, j \,\rangle| = \frac{1}{\sqrt{\dim}}$$

yielding equal transition probabilities.



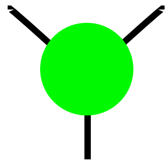
— *complementarity* —

Thm. Complementarity means:



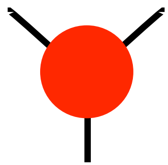
— *complementarity* —

Z-spin:



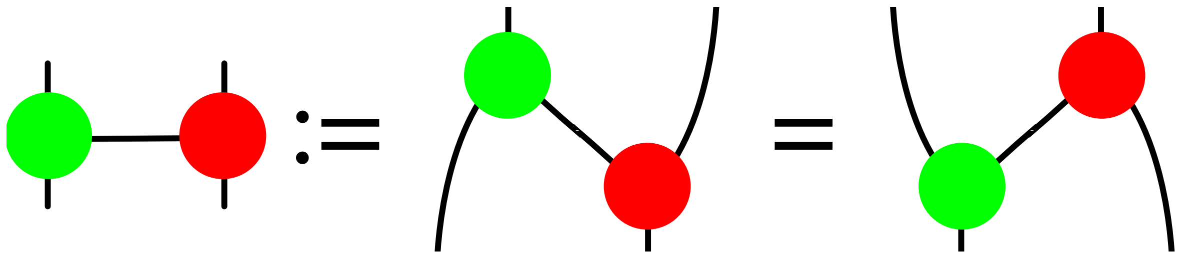
$$\delta_Z : |i\rangle \mapsto |ii\rangle$$

X-spin:



$$\delta_X : |\pm\rangle \mapsto |\pm\pm\rangle$$

— *complementarity* —



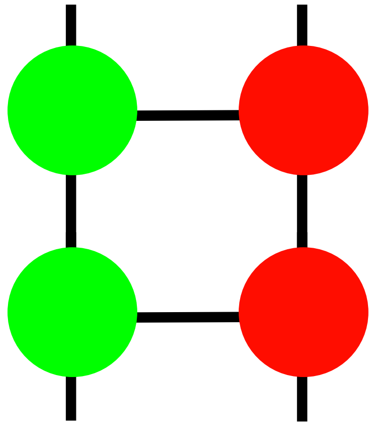
i.e.

$$(\delta_Z^\dagger \otimes 1) \circ (1 \otimes \delta_X) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = CNOT$$

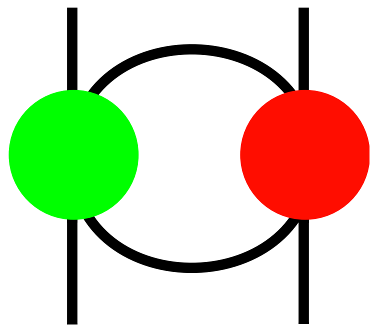
— *complementarity* —

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \circ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = ?$$

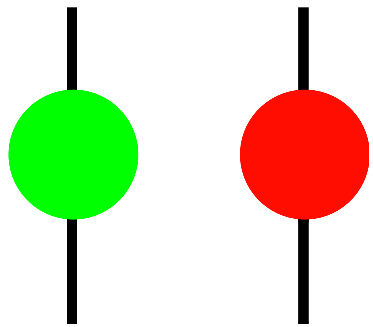
— *complementarity* —



— *complementarity* —



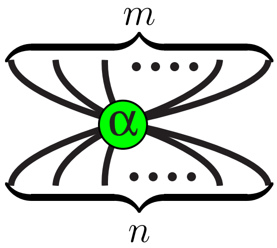
— *complementarity* —



— *complementarity* —

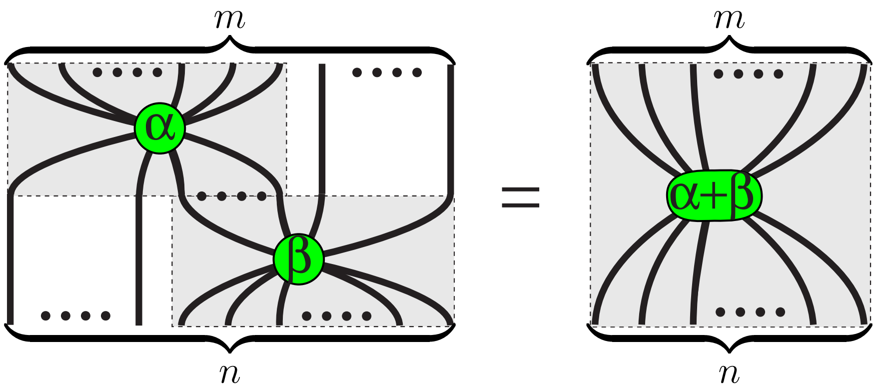
— *phases* —

Thm. Unbiased states of an observable always constitute an Abelian group with conjugates as inverses.



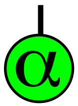
$$\left\{ \quad \middle| \; n, m \in \mathbb{N}_0, \alpha \in G \right\}$$

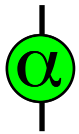
such that:



— *phases* —

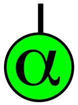
For qubits in **FHilb** with green $\equiv \{|0\rangle, |1\rangle\} \equiv Z$:


$$= \begin{pmatrix} 1 \\ e^{i\alpha} \end{pmatrix}_Z$$

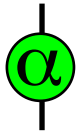

$$= Z_\alpha = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}_Z$$

— ***phases*** —

For qubits in **FHilb** with green $\equiv \{|0\rangle, |1\rangle\} \equiv Z$:

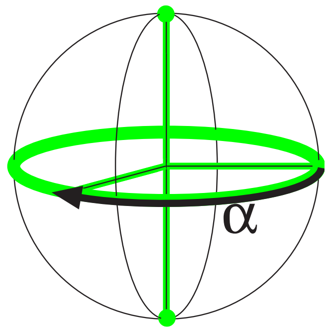


$$= \begin{pmatrix} 1 \\ e^{i\alpha} \end{pmatrix}_Z$$



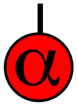
$$= Z_\alpha = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}_Z$$

These are relative phases for Z, hence in X-Y:

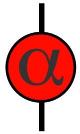


— *phases* —

For qubits in **FHilb** with red $\equiv \{|+\rangle, |-\rangle\} \equiv X$:

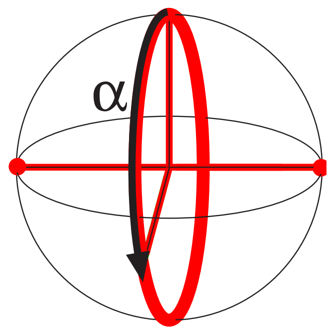


$$= \begin{pmatrix} 1 \\ e^{i\alpha} \end{pmatrix}_X$$



$$= X_\alpha = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}_X$$

These are relative phases for X, hence in Z-Y:

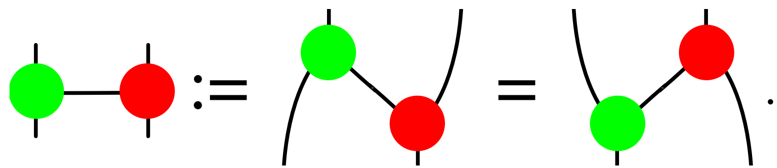


— *phases* —

Thm. Every linear map in $\mathbf{FHilb}_2$ can be expressed in the language of a pair of complementary observables and the corresponding phases, that is, it can be written down using only red and green decorated spiders.

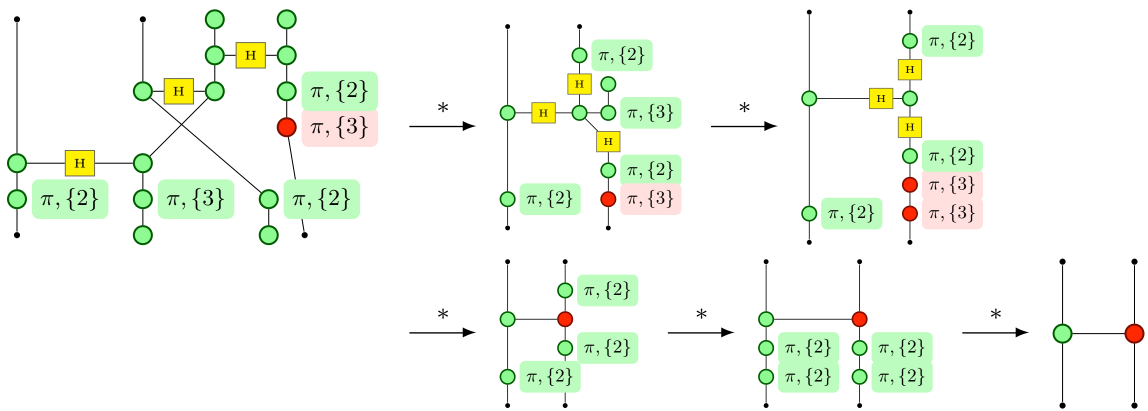
$$\Lambda^Z(\gamma) \circ \Lambda^X(\beta) \circ \Lambda^Z(\alpha) =$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} =$$



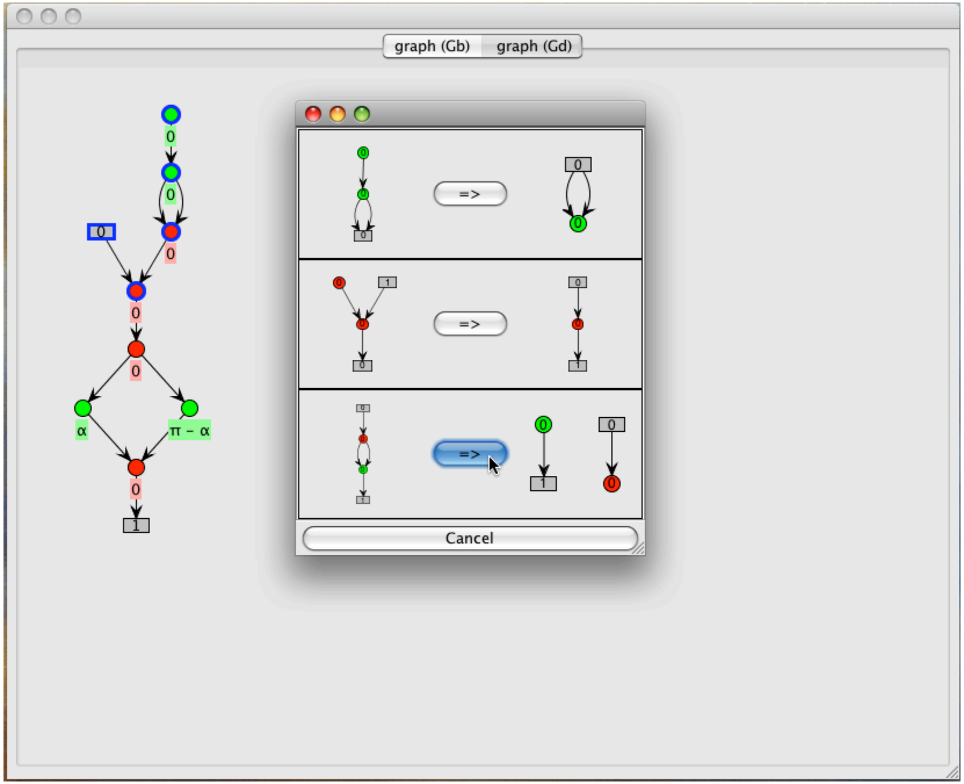
— *MBQC* —

New results on resource requirements and complexity of translations between Q-computational models:



R. Duncan and S. Perdrix ICALP'10

`quantomatic` – *Dixon / Duncan / Frot / Kissinger / Merry / Soloviev*



`http://sites.google.com/site/quantomatic/home`

ENVIRONMENT AND CLASSICAL CHANNELS IN CATEGORICAL Q.M.

B.C. and S. Perdrix – CSL'10 – arXiv:1004.1598

classical Q-mixtures := density matrices & CP-maps

classical Q-mixtures := density matrices & CP-maps

Selinger QPL'05: pure cats to mixed cats construction

classical Q-mixtures := density matrices & CP-maps

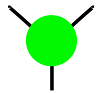
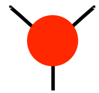
Selinger QPL'05: pure cats to mixed cats construction

C QPL'06: axiomatic account on mixed cats via:

classical Q-mixtures := density matrices & CP-maps

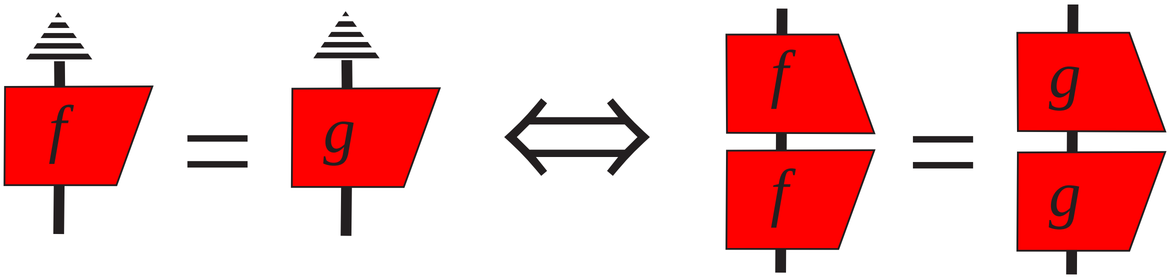
Selinger QPL'05: pure cats to mixed cats construction

C QPL'06: axiomatic account on mixed cats via:

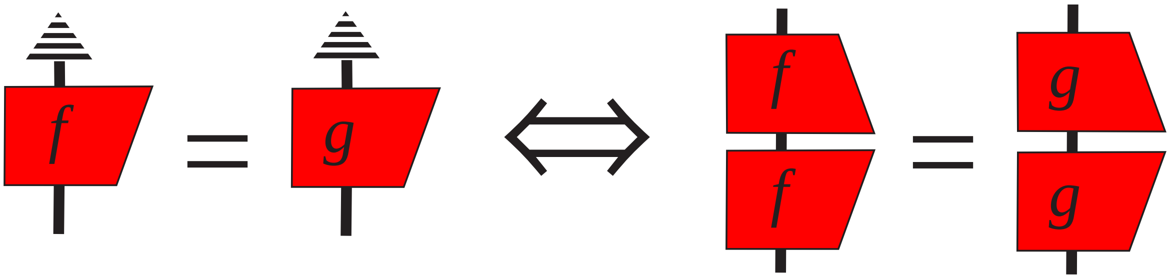
C and Perdrix '10: interaction of  with  &  :

- **Decoherence**
- **Classical channels**
- **Complex control structure**
- **Elementary derivation of general protocols**

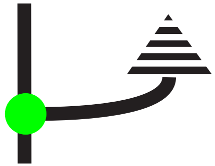
$\{\text{⏚}\}_A$ is **environment** iff for all (pure) f, g:

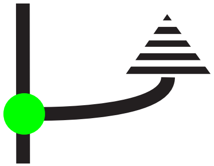


$\{\text{⏚}\}_A$ is **environment** iff for all (pure) f, g:

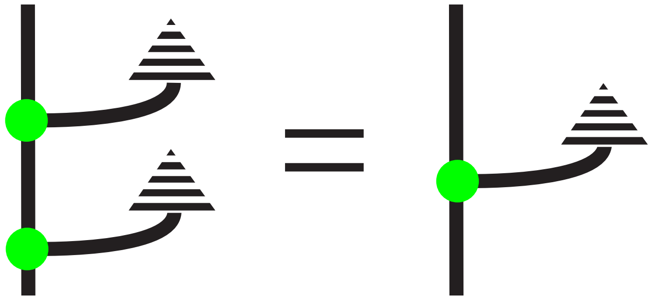


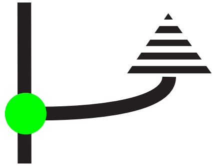
and:

 **= classical channel = decoherence**

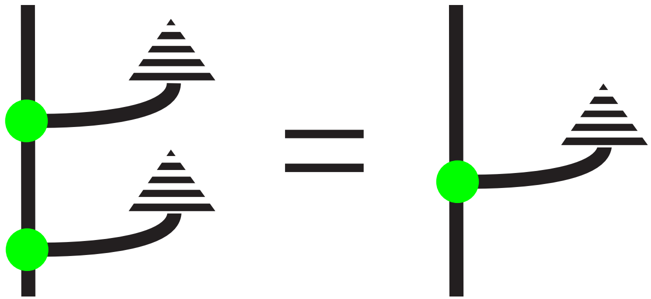
 **= classical channel = decoherence**

Prop 1:

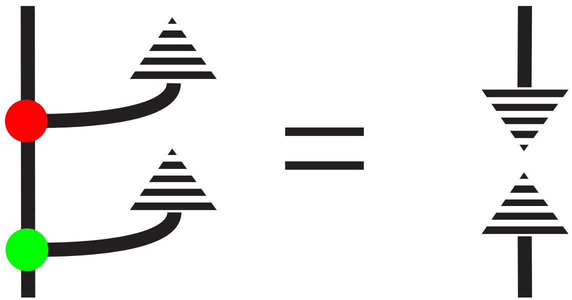


 **= classical channel = decoherence**

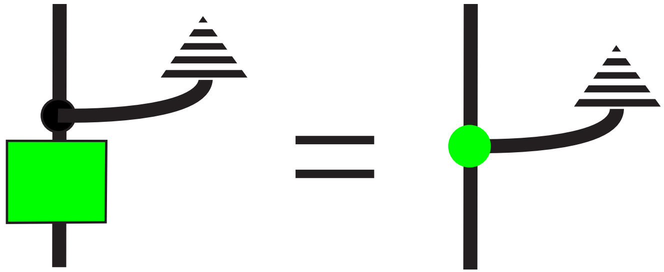
Prop 1:



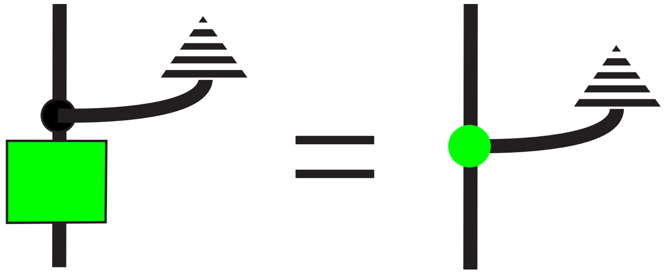
Prop 2:



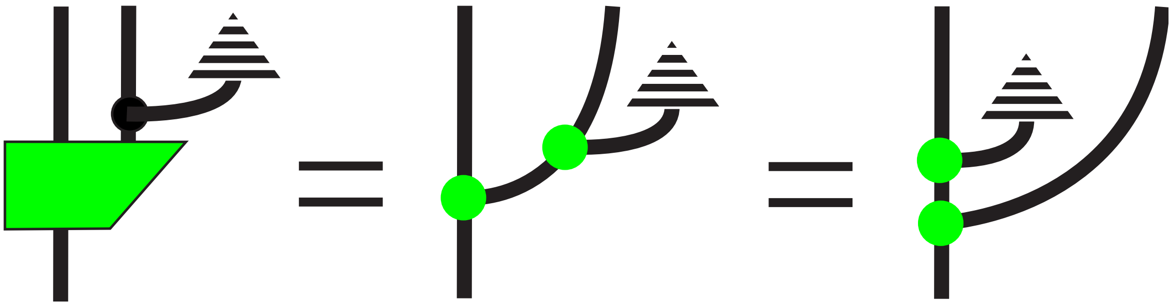
Destructive measurement:



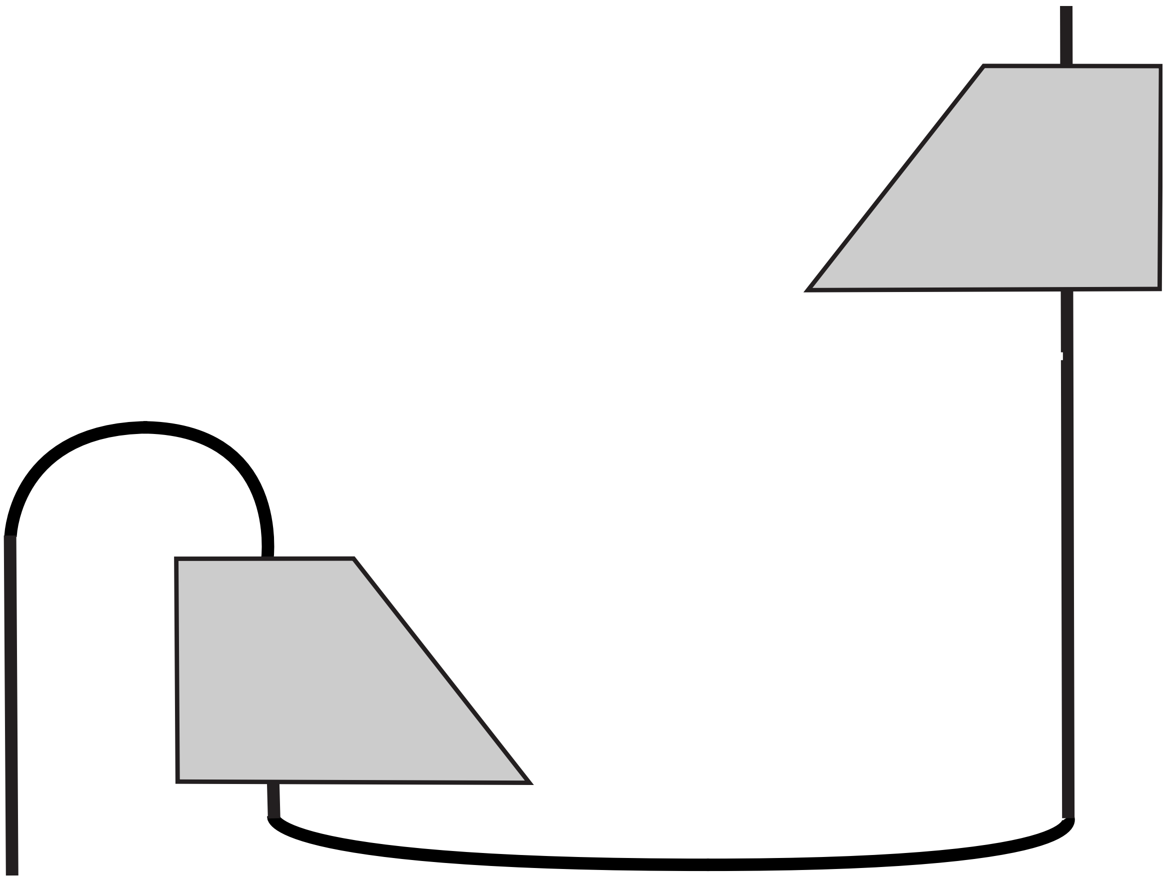
Destructive measurement:

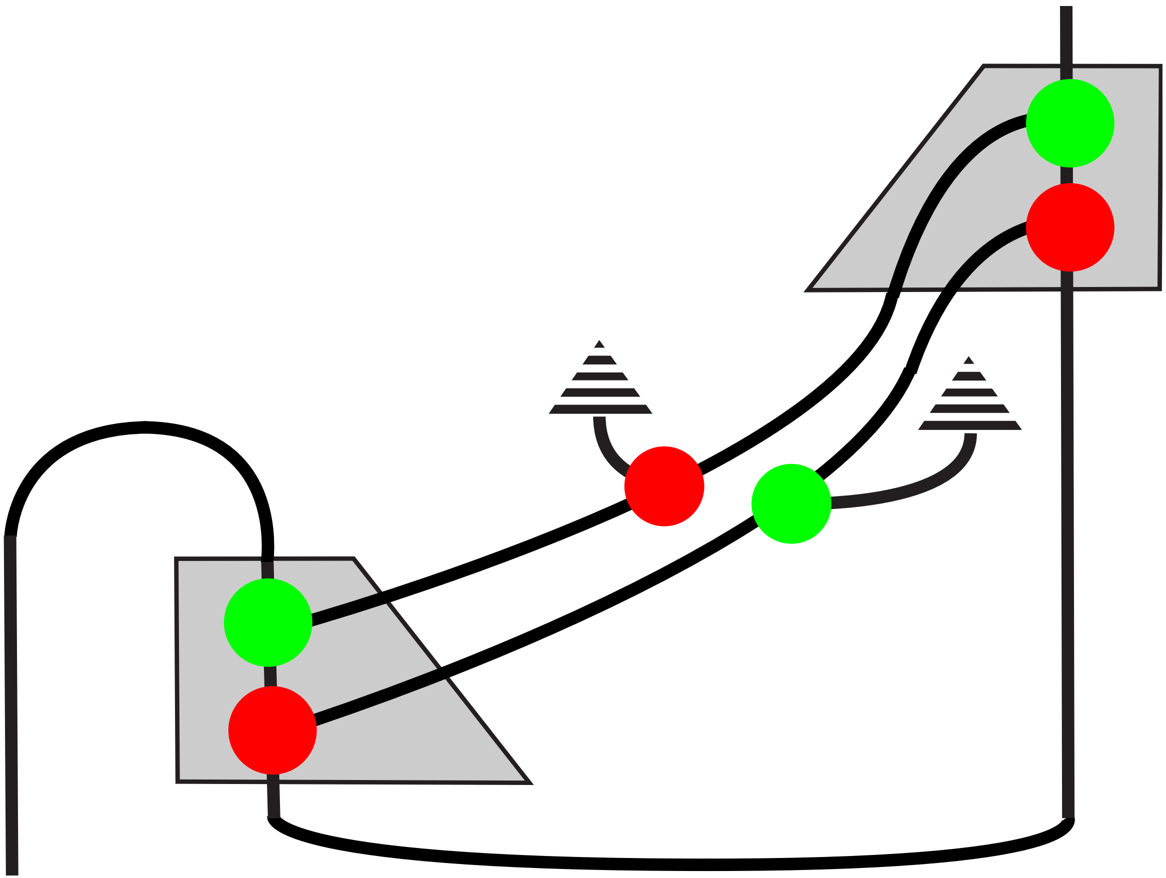


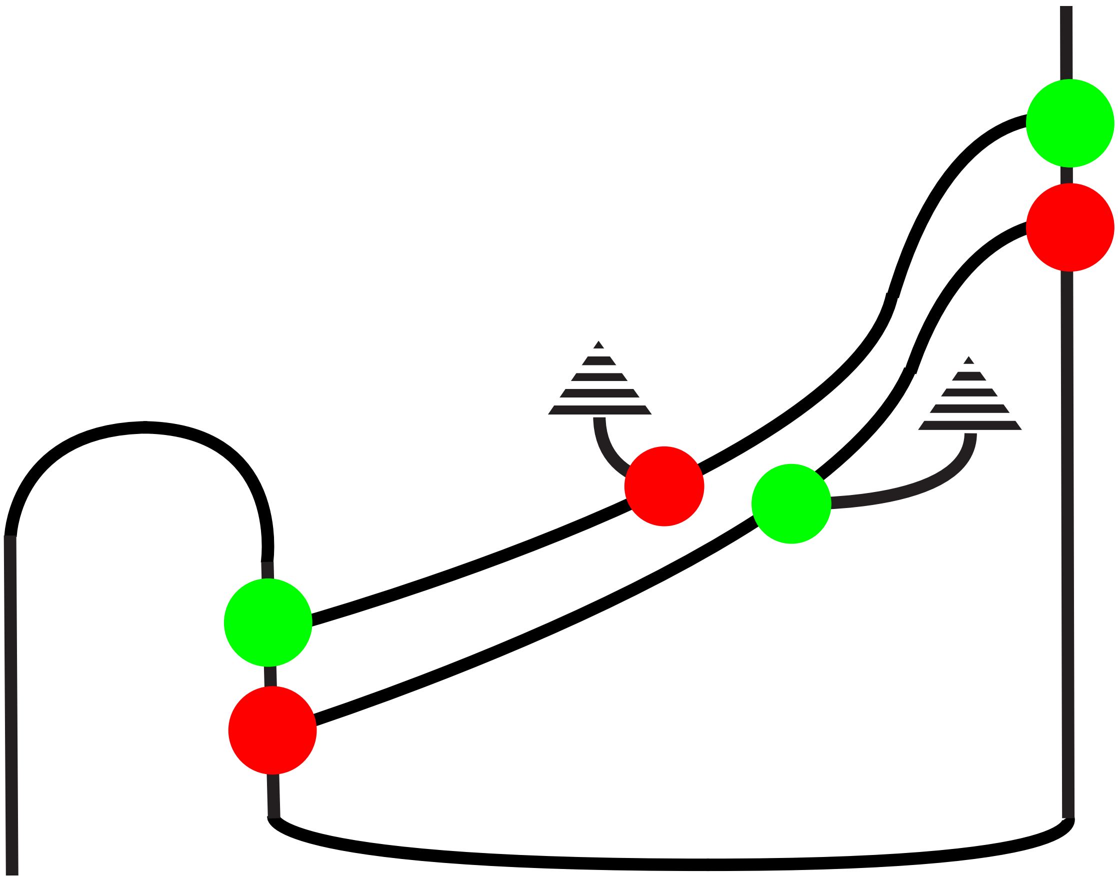
Non-destructive measurement:

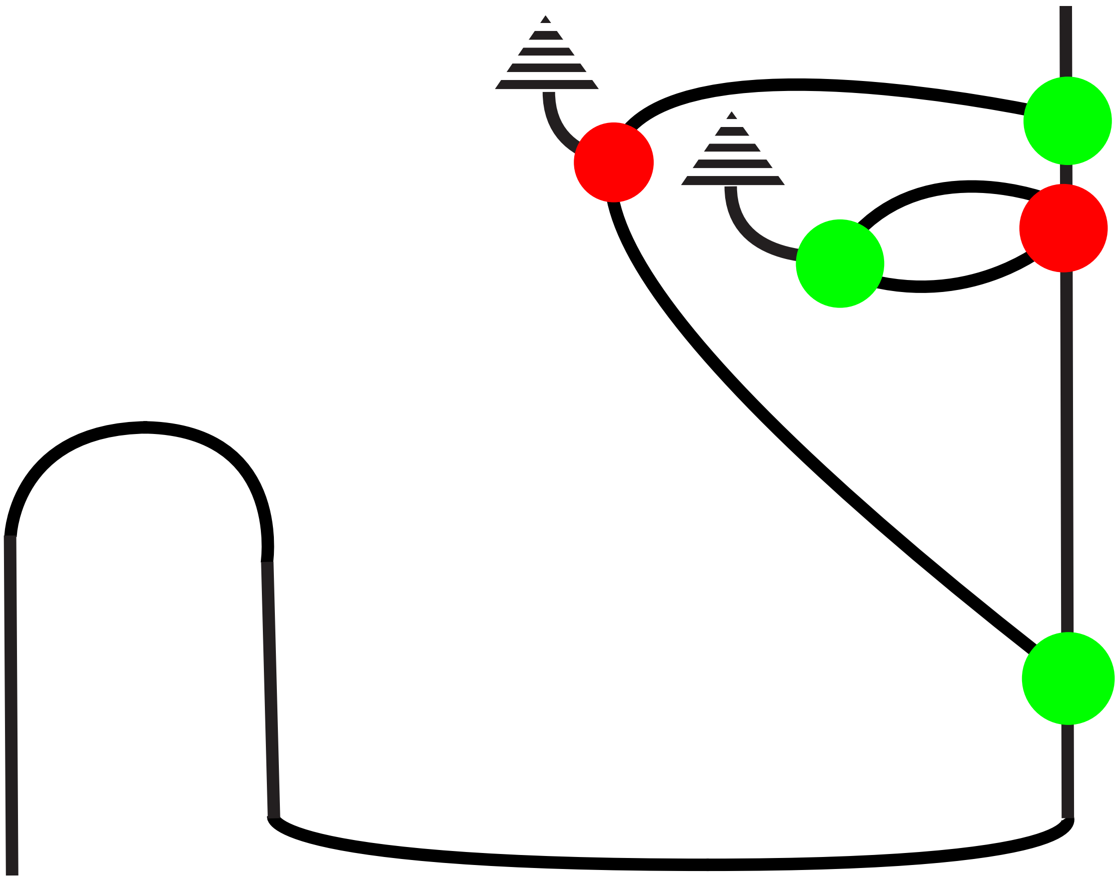


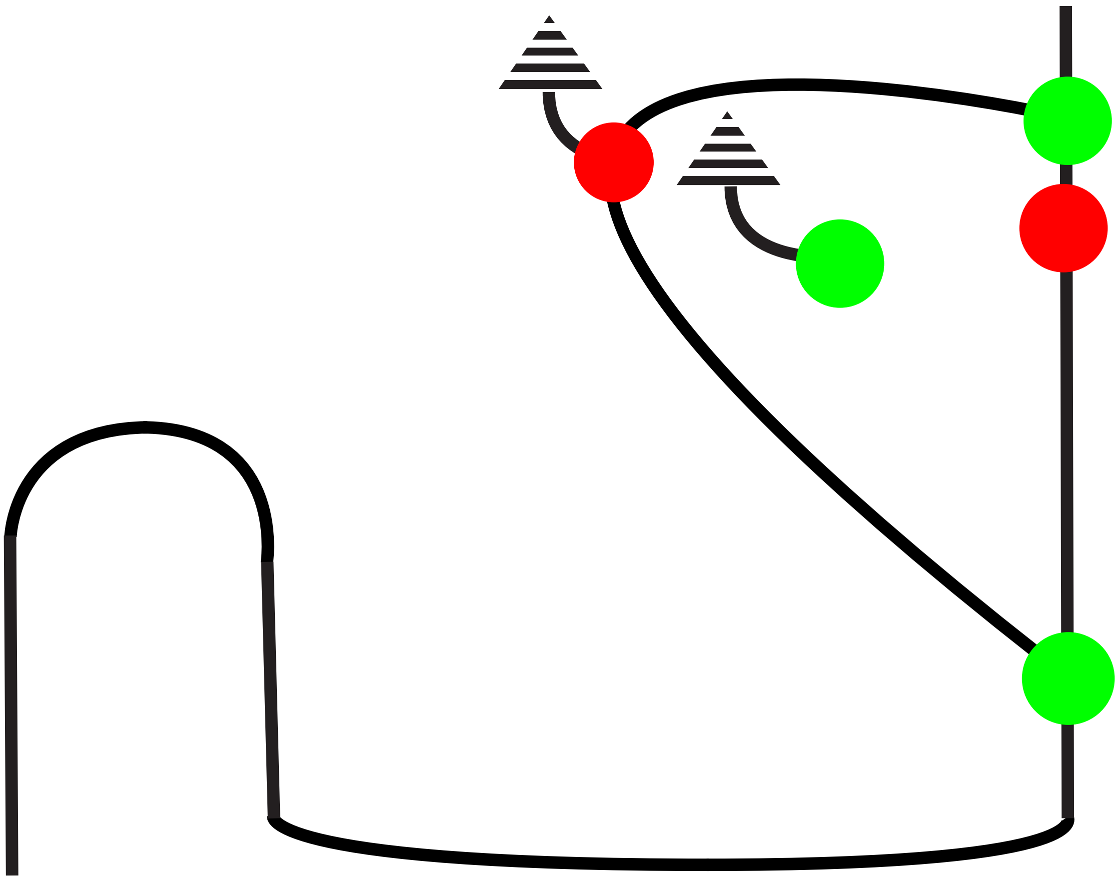
1st example:
QUANTUM TELEPORTATION

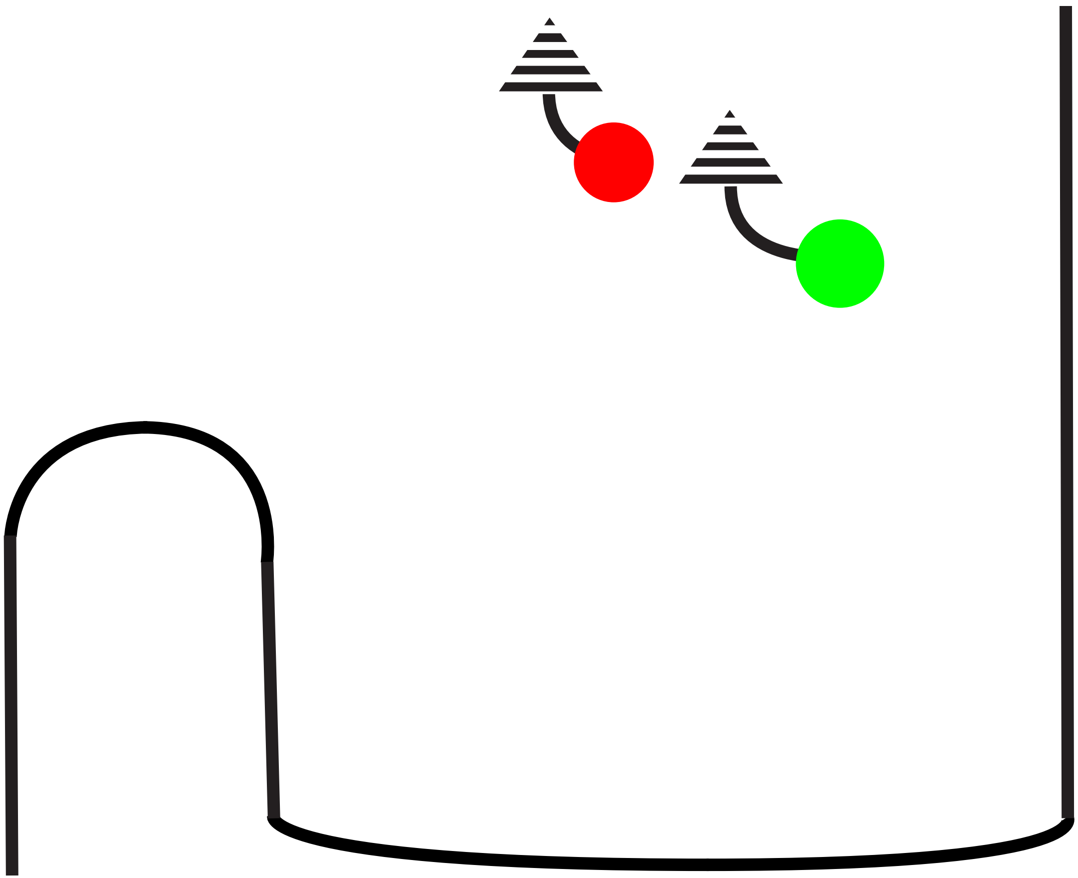




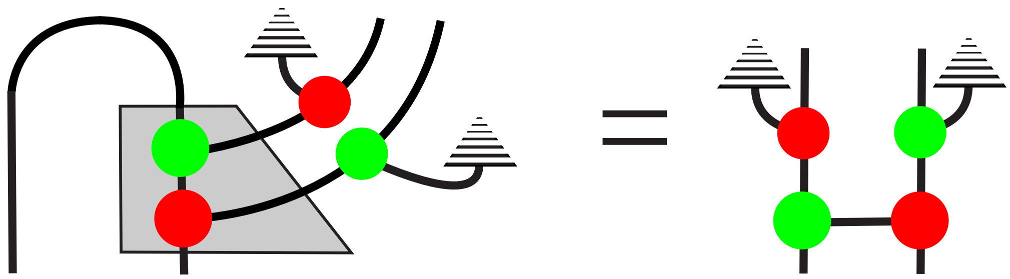




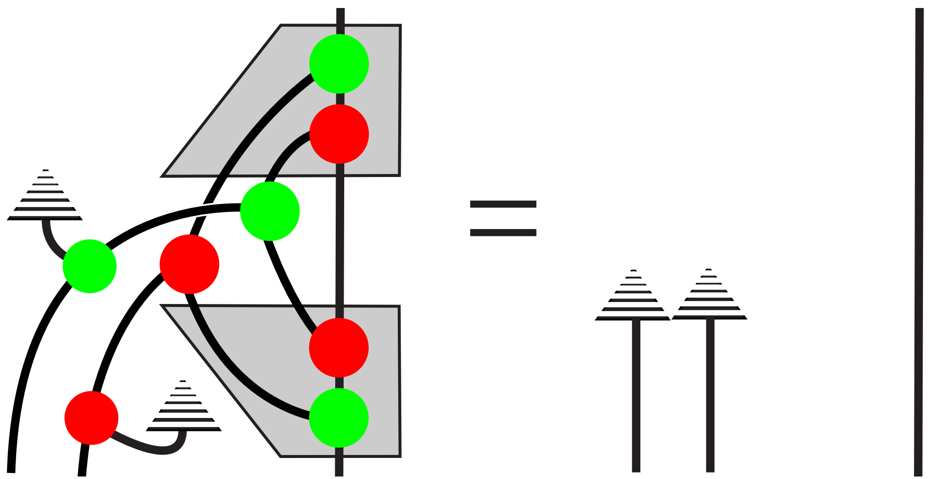




Indeed measurement:



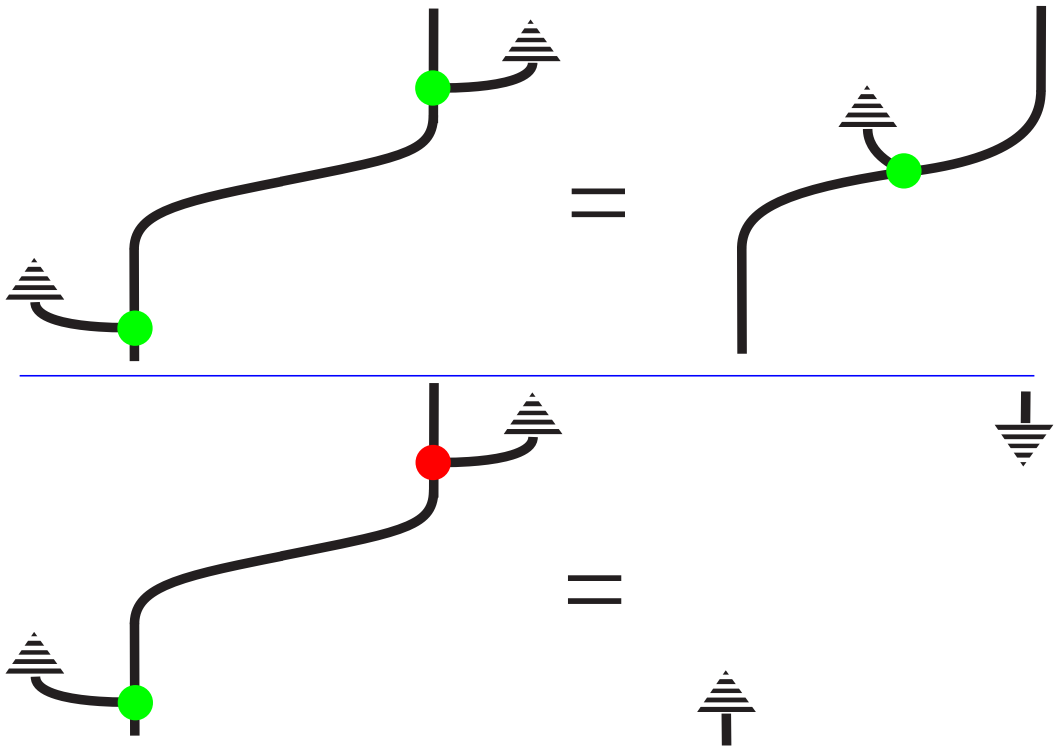
Indeed controlled unitary:



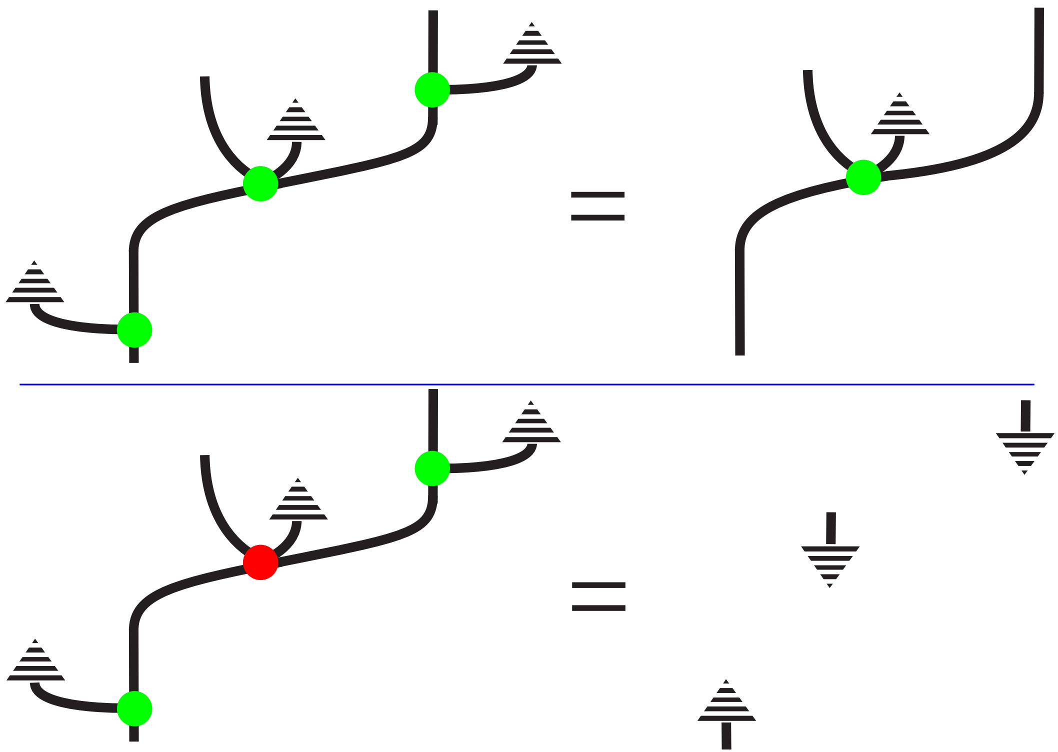
2nd example:

QUANTUM KEY DISTRIBUTION

— *key distribution* —



— *key distribution* —

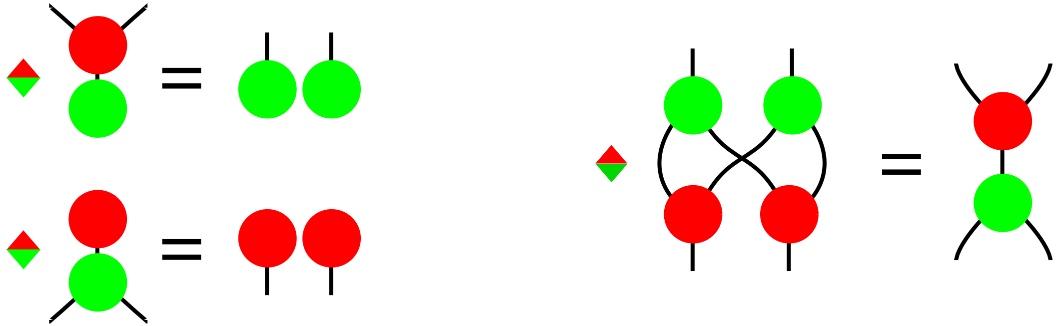


STRONG COMPLEMENTARITY

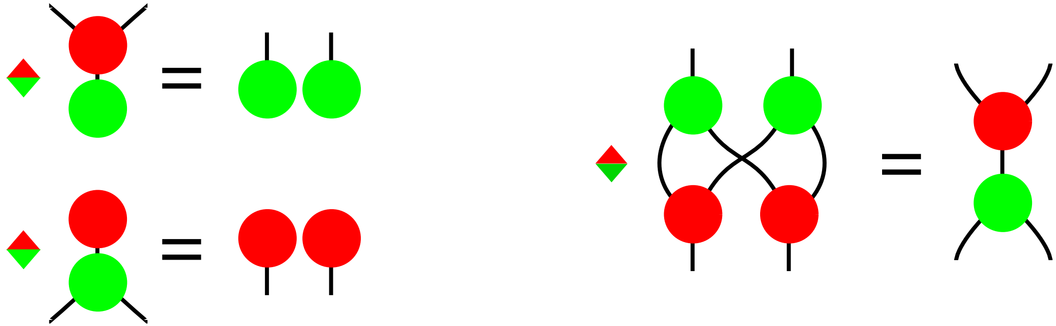
B.C., Ross Duncan, Quanlong Wang

Anne Hillebrand

Def. Strong complementarity := (scaled) bialgebra

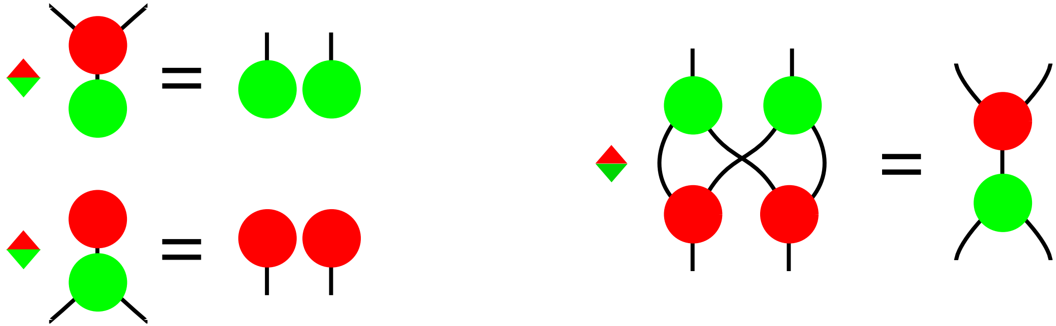


Def. Strong complementarity := (scaled) bialgebra



Prop. Strong complementarity ⇒ Complementarity

Def. Strong complementarity := (scaled) bialgebra



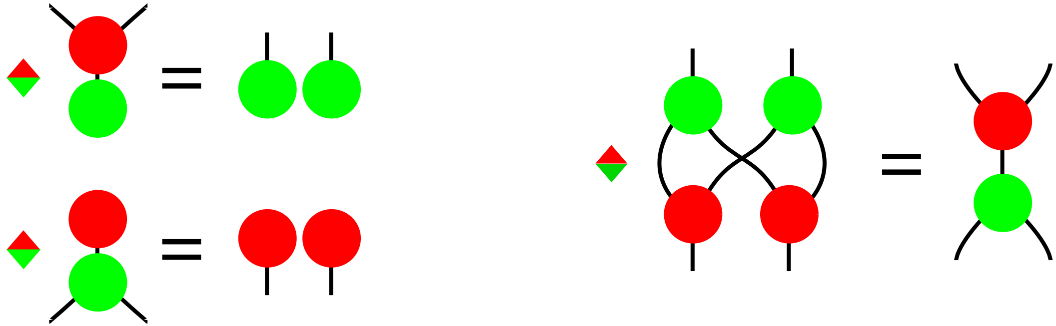
Prop. Strong complementarity ⇒ Complementarity

Conj. Strong complementarity, $\pi/2$-phases, H-decomp. is complete with respect to stabilizer qubit theory.

R. Duncan and S. Perdrix (2009) *Graph states and the necessity of Euler decomposition*. CiE'09, LNCS 5635. arXiv:0902.0500

Alex Lang and B.C. (2011) *Trichromatic open digraphs for understanding qubits*. QPL'11 proceedings.

Def. Strong complementarity := (scaled) bialgebra



Prop. Strong complementarity $\Rightarrow$ Complementarity

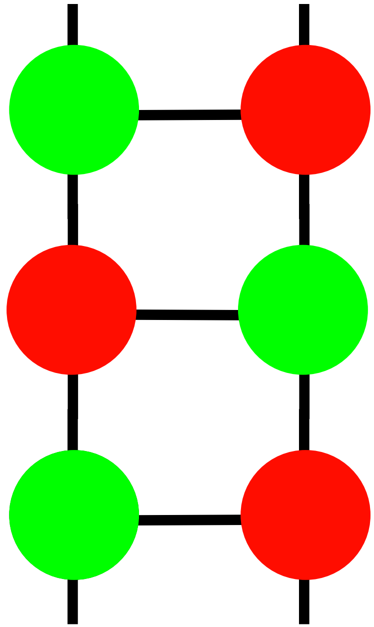
Conj. Strong complementarity, $\pi/2$-phases, H-decomp. is complete with respect to stabilizer qubit theory.

Claim. Strong complementarity is more fundamental as a structural resource than complementarity.

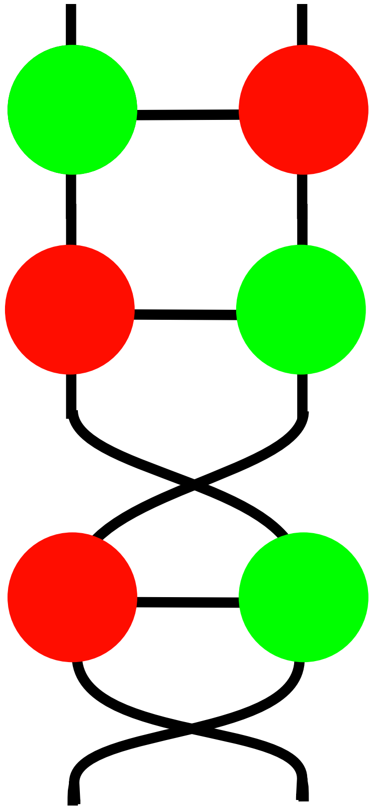
— *quantum gates* —

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \circ \sigma \circ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \circ \sigma \circ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = ?$$

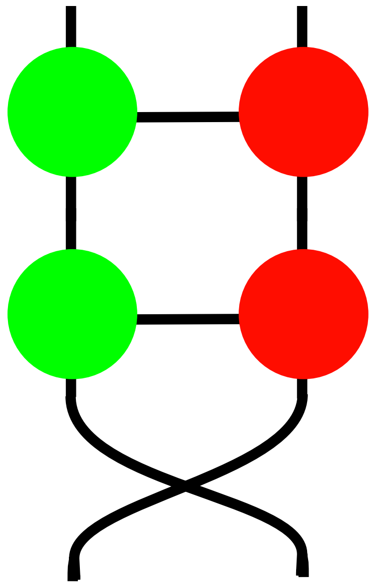
— *quantum gates* —



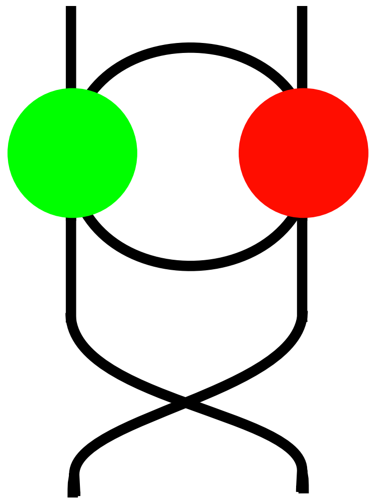
— *quantum gates* —



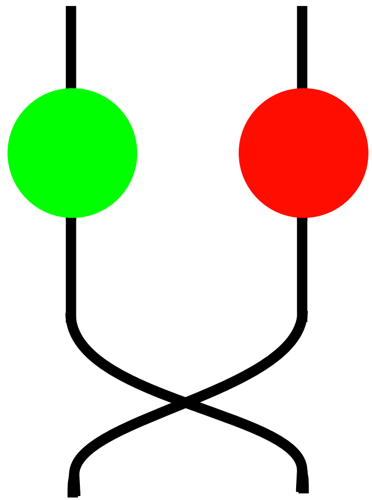
— *quantum gates* —



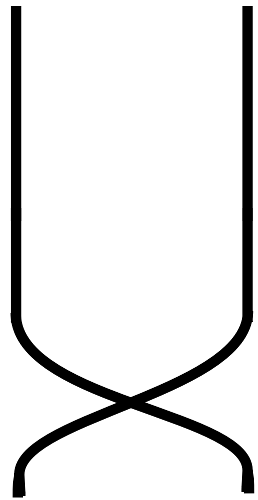
— *quantum gates* —



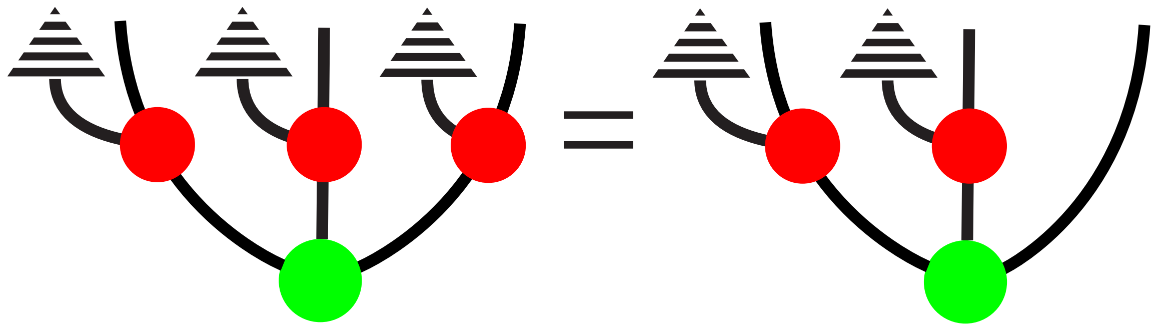
— *quantum gates* —



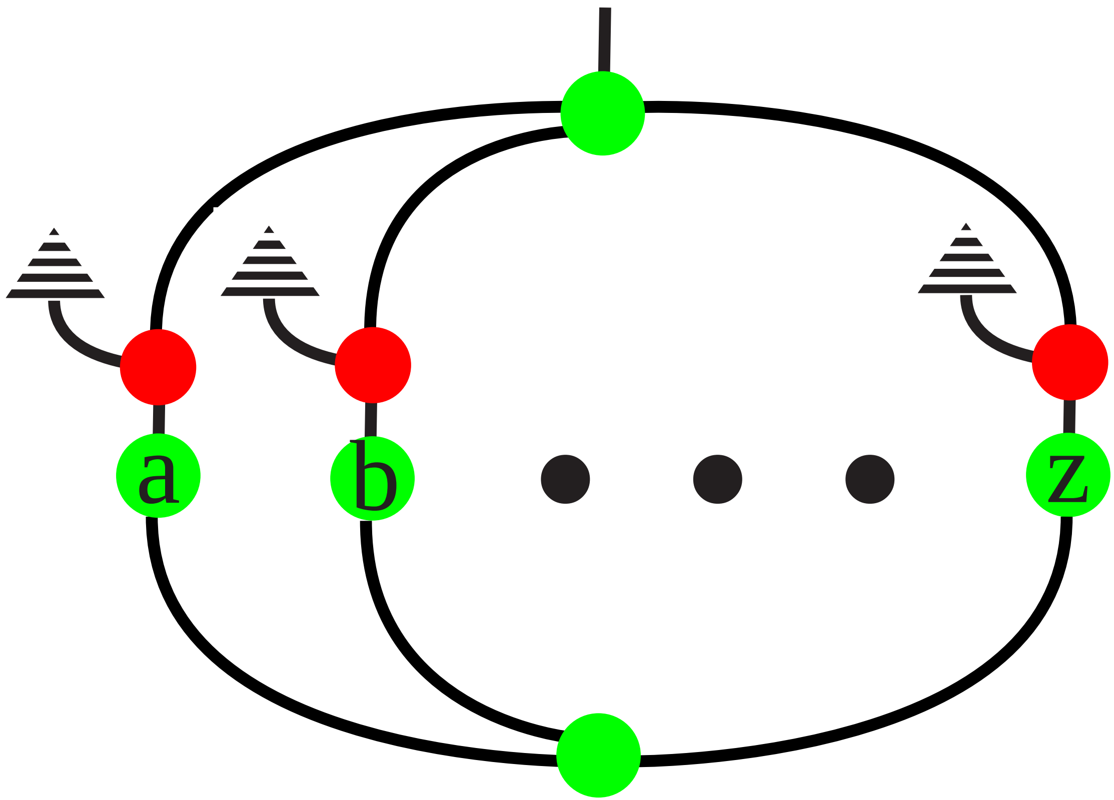
— *quantum gates* —



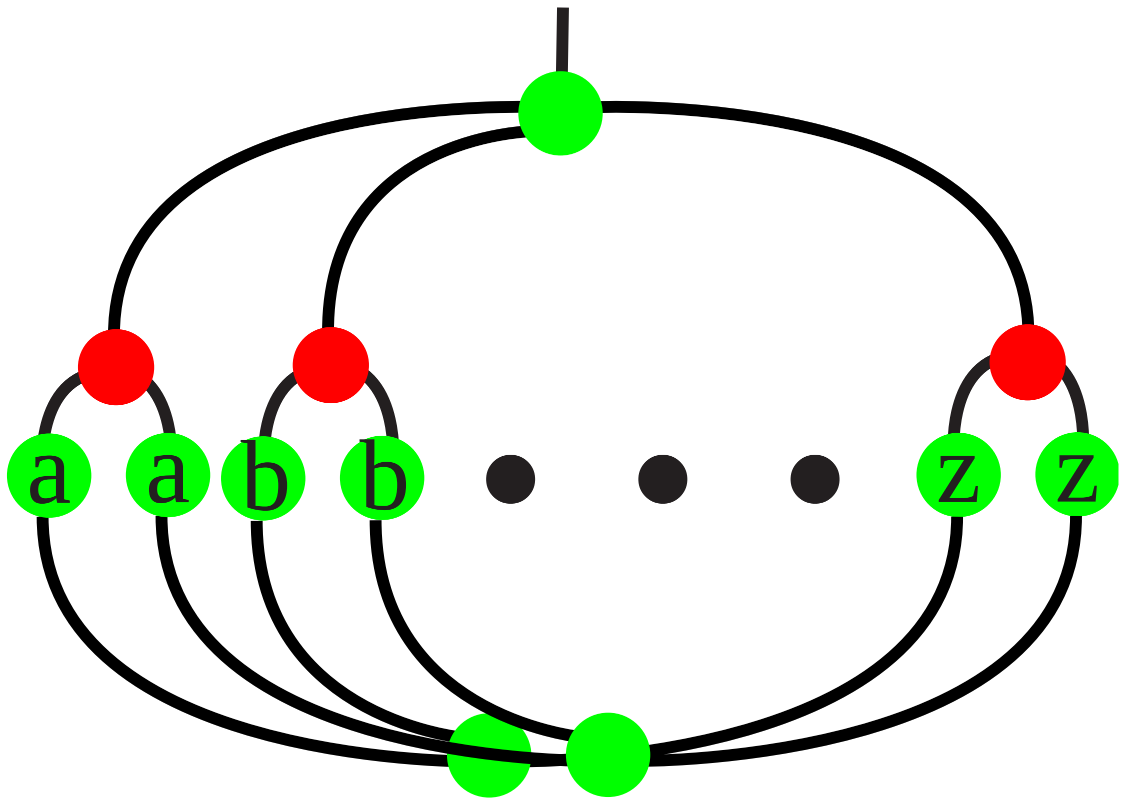
— *quantum correlations* —



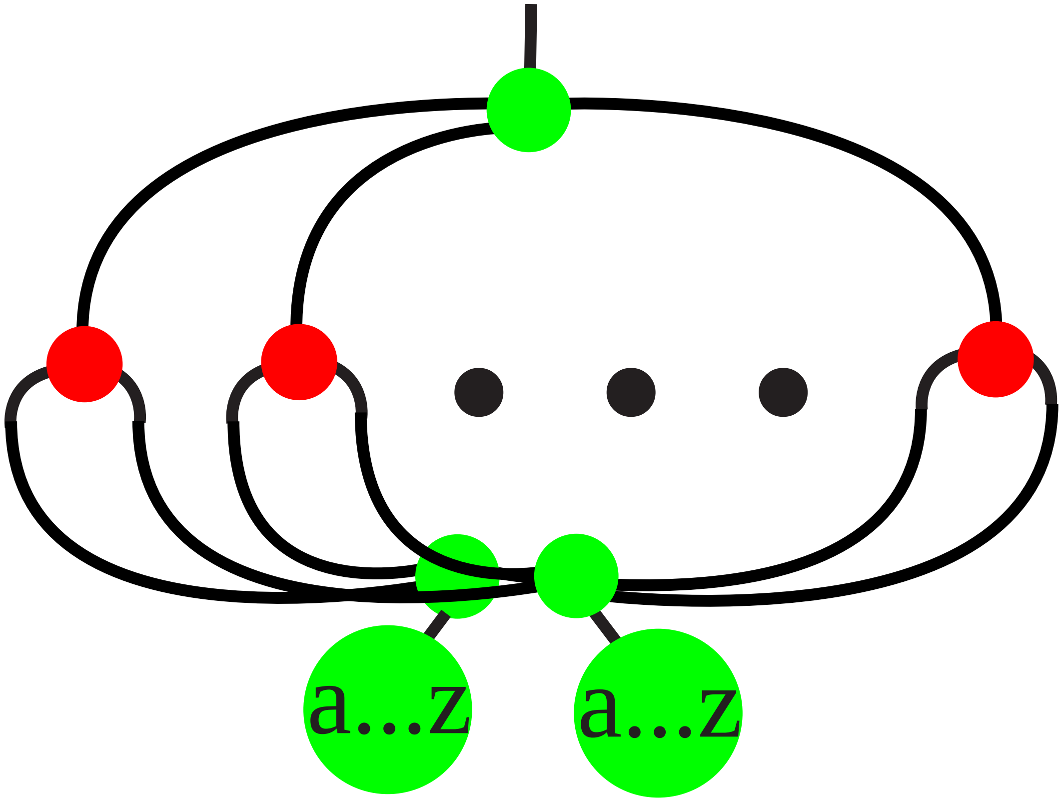
— *quantum correlations* —



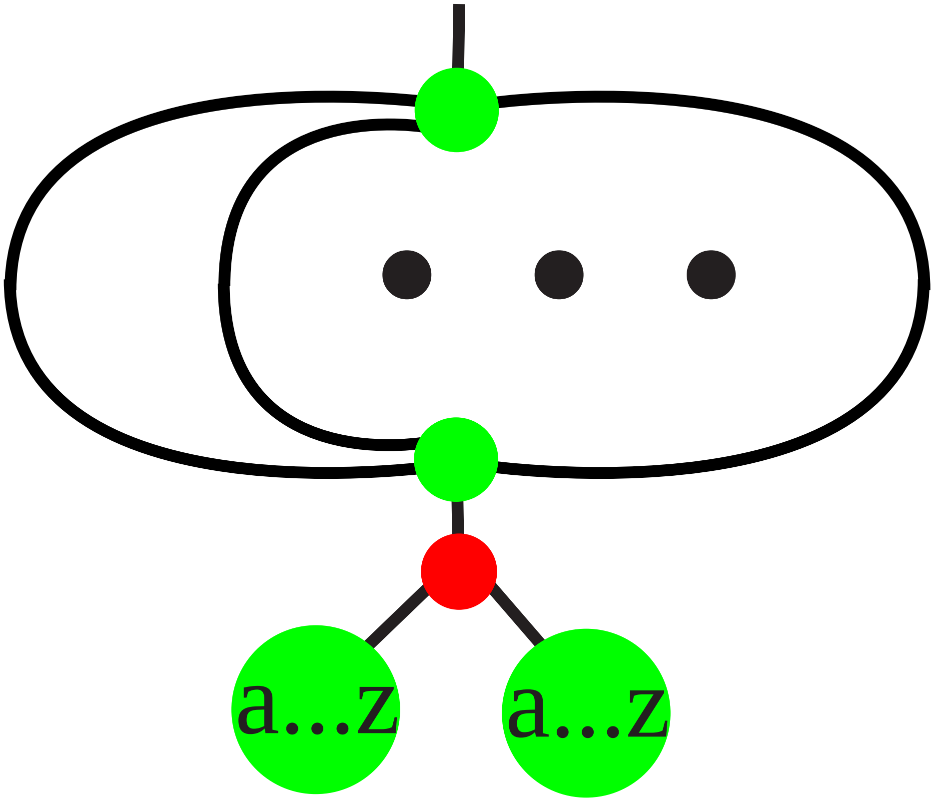
— *quantum correlations* —



— *quantum correlations* —



— *quantum correlations* —



— *quantum correlations* —