Exercises for Chapter 2 of *An Introduction to Description Logic*

October 2, 2017

1 Exercises for Section 2.1

Exercise 1 The goal of this exercise is to make sure that you understand the notion of an *interpretation*.

1. How many elements does the smallest domain of an interpretation contain?
2. Can an interpretation domain be infinite?
3. In an interpretation $\mathcal{I}$ and for a concept name A, how many elements can/must $A^{\mathcal{I}}$ have?
4. In an interpretation $\mathcal{I}$ and for a role name r, how many pairs of elements can/must $r^{\mathcal{I}}$ have?
5. For an element $e \in \Delta^{\mathcal{I}}$, can it be the case that $(e, e) \in r^{\mathcal{I}}$?
6. For two elements $e, f \in \Delta^{\mathcal{I}}$, can it be the case that $\{(e, f), (f, e)\} \subseteq r^{\mathcal{I}}$?

Exercise 2 Formulate $\mathcal{ALC}$ concepts: for each of the following concepts, build a suitable $\mathcal{ALC}$ concept description, using only the concept names

Person, Happy, Animal, Cat,Old, Fish

and the role name *owns*.

1. happy person
2. happy pet owner
3. person who owns only cats
4. unhappy pet owners who own an old cat
5. pet owners who only own cats and fish

Exercise 3 For each of the concepts formulated as answers of Exercise 2, draw an interpretation that has an element in the extension of that concept.

2 Exercises for Section 2.2

Exercise 4 Build an $\mathcal{ALC}$ knowledge base: capture each of the following statements in a suitable GCI, equivalence axioms, or assertion, using only the concept names

Vehicle, Boat, Bicycle, Car, Device, Wheel, Engine, Axle , Rotation, Water
Human, Driver, Adult, Child

and the role names

hasPart, *poweredBy*, *capableOf* , *travelsOn controls*.

1. Cars are exactly those vehicles that have wheels and are powered by an engine.
2. Bicycles are exactly those vehicles that have wheels and are powered by a human.
3. Boats are exactly those vehicles that travel on water.
4. Boats have no wheels.
5. Cars and bicycles do not travel on water.
6. Wheels are exactly those devices that have an axle and are capable of rotation.
7. Drivers are exactly those humans who control a vehicle.
8. Drivers of cars are adults.
9. Humans are not vehicles.
10. Wheels or engines are not humans.
11. Humans are either adults or children.
12. Adults are not children.
13. Bob controls a car.
14. Bob is a human.
15. Bob controls QE2.
16. QE2 is a vehicle that travels on water.

Exercise 5 Which of the statements in your answer to Exercise 4 are GCIs, equivalence axioms, concept assertions, or role assertions? Is the TBox of your knowledge base acyclic? If yes, can you unfold it into the ABox of you knowledge base?

Exercise 6 Draw a model of your answer to Exercise 6. Modify it such that it is no longer a model, in three different ways.

3 Exercises for Section 2.3

Exercise 7 Which of the following concepts is satisfiable?

1. $A \sqcap \neg A$
2. $A \sqcup \neg A$
3. $A \sqcap \exists r.B \sqcap \exists r.\neg B$
4. $A \sqcap \exists r.B \sqcap \forall s.\neg B$
5. $A \sqcap \exists r.B \sqcap \forall r.\neg B$
6. $A \sqcap \exists r.B \sqcap \forall r.(\neg B \sqcup \exists r..A)$
7. $A \sqcap \exists r.(B \sqcap C) \sqcap \forall r.\neg B$

Exercise 8 Which of the following statements is true?

1. $A \sqcap \neg A$ is subsumed by B
2. B is subsumed by $A \sqcup \neg A$
3. $A \sqcap \exists r.B$ is subsumed by $A \sqcap \exists r.\top$
4. $A \sqcap \exists r.(B \sqcap C)$ is subsumed by $A \sqcap \exists r.B$
5. $A \sqcap \exists r.(B \sqcup C)$ is subsumed by $A \sqcap \exists r.B$
6. $A \sqcap \forall r.B$ is subsumed by $A \sqcap \exists r.B$
7. $A \sqcap \exists r.B$ is subsumed by $A \sqcap \forall r.B$
8. $A \sqcap \exists r.A \sqcap \forall r.B$ is subsumed by $A \sqcap \exists r.B$

Exercise 9 Consider again the knowledge base $\mathcal{K}$ given as solution to Exercise 4. Which of the following statements is true?

1. $\mathcal{K}$ is consistent.
2. the concept $\mathsf{Boat} \sqcap \exists \mathit{hasPart}.\mathsf{Wheel}$ is satisfiable w.r.t. $\mathcal{K}$.
3. the concept $\mathsf{Boat} \sqcap \exists \mathit{poweredBy}.\mathsf{Engine}$ is satisfiable w.r.t. $\mathcal{K}$.
4. the concept $\mathsf{Car} \sqcap \mathsf{Bicycle}$ is satisfiable w.r.t. $\mathcal{K}$.
5. the concept $\mathsf{Driver} \sqcap \mathsf{Vehicle}$ is satisfiable w.r.t. $\mathcal{K}$.
6. the concept $\mathsf{Driver} \sqcap \mathsf{Child}$ is satisfiable w.r.t. $\mathcal{K}$.
7. the concept $\exists \mathit{controls}.\mathsf{Car} \sqcap \mathsf{Child}$ is satisfiable w.r.t. $\mathcal{K}$.
8. the concept $\exists \mathit{controls}.\mathsf{Car} \sqcap \mathsf{Child} \sqcap \mathsf{Human}$ is satisfiable w.r.t. $\mathcal{K}$.

9. Bob is an instance of Adult w.r.t. $\mathcal{K}$.
10. Bob is an instance of Driver w.r.t. $\mathcal{K}$.
11. Bob is an instance of (Adult $\sqcap$ Driver) w.r.t. $\mathcal{K}$.
12. QE2 is an instance of Boat w.r.t. $\mathcal{K}$.
13. Driver is subsumed by Human w.r.t. $\mathcal{K}$.
14. Adult is subsumed by Human w.r.t. $\mathcal{K}$.
15. Human $\sqcap \exists$*controls*.(Vehicle $\sqcap \exists$*hasPart*.Wheel $\sqcap \exists$*poweredBy*.Engine) is subsumed by Adult w.r.t. $\mathcal{K}$ (this is a difficult one!).
16. $\exists$*controls*.Car is subsumed by Adult w.r.t. $\mathcal{K}$ (this is another difficult one!).

4 Exercises for Section 2.5

Exercise 10 Extend the knowledge base you built in Exercise 4 to capture the following statements (you may need more than one axiom for some of the statements below).

1. Cars have between three and four wheels.
2. Bicycles have exactly two wheels.
3. A human who legally controls a car holds a driving license and is an adult (this is a difficult one!).
4. A vehicle is controlled by exactly one human.
5. A thing's parts' parts are that thing's parts.
6. A car with a broken part is broken.
7. Bob controls a car with a wheel that has a broken axle.

Exercise 11 Consider the knowledge base $\mathcal{K}'$ that is the result of your answers to Exercise 4 and 10: which of the following statements is true?

1. $\mathcal{K}$ is consistent.
2. $\exists$*legallyControls*.$\top$ is subsumed by $\exists$*controls*.$\top$ w.r.t. $\mathcal{K}'$.
3. the concept Car $\sqcap$ Bicycle is satisfiable w.r.t. $\mathcal{K}'$.
4. Bob is an instance of $\exists$*controls*.(Car $\sqcap$ Broken) w.r.t. $\mathcal{K}'$.
5. the interpretation given in Figure ch2-fig2 is a model of $\mathcal{K}'$.

5 Exercises for Section 2.6

Exercise 12 Translate the knowledge base given as answer to Exercise 4 into Modal Logic.

Exercise 13 Translate the knowledge base given as answer to Exercise 4 into First Order Logic.