

Soft Constraints for Optimisation

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Oxford Configuration Workshop

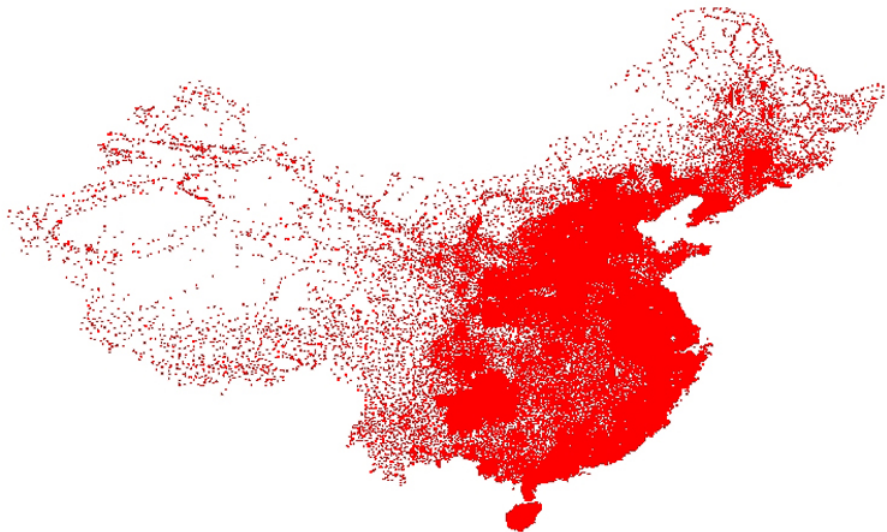
12/01/2012

Interesting problems are **hard**!

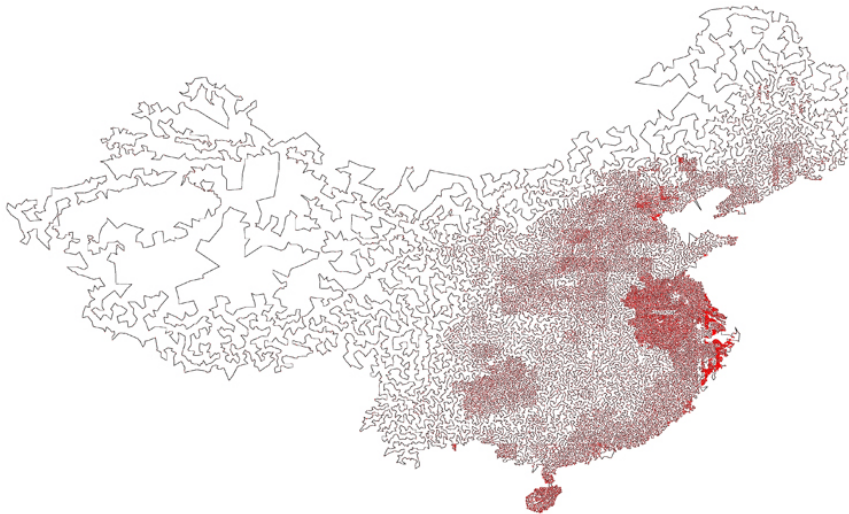
Optimal Tour around London



Optimal Tour around China?



Almost Optimal Tour around China!



How to Cope with Hardness?



How to Cope with Hardness?



How to Cope with Hardness?

SAT

SMT

How to Cope with Hardness?

SAT

LP

SMT

How to Cope with Hardness?

SAT

IP

LP

SMT

How to Cope with Hardness?

SAT

MIP

IP

LP

SMT

How to Cope with Hardness?

SAT

MIP

CP

IP

LP

SMT

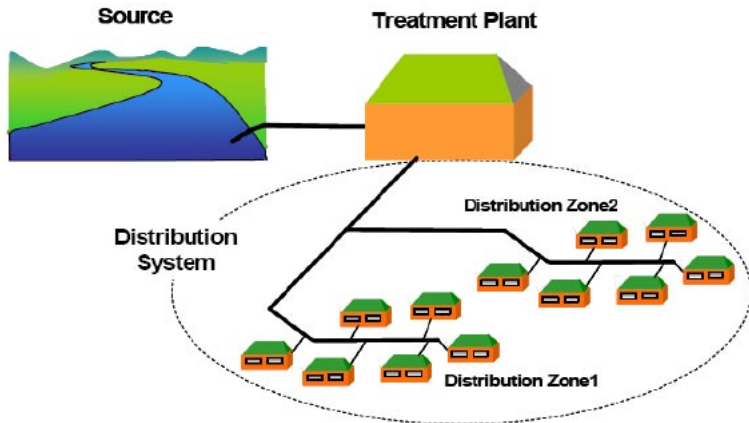
Decision vs Optimisation



CP deals with decision problems.

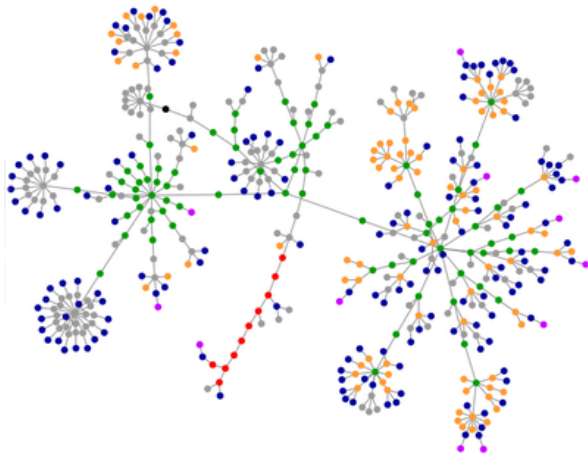
Many problems involve **optimisation!**

Example 1: Sustainability



Where to place sensors to quickly detect contamination in water distribution networks?

Example 2: Blogs



Which blogs on the Web do you read to learn about the biggest stories and yet read only few?

Soft Constraints



Constraint $f : D^k \rightarrow \{0, \infty\}$

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$$D = \{0, 1\}$$

$$k = 3$$

\mathbf{t}	$f(\mathbf{t})$
000	0
001	∞
010	0
011	0
100	∞
101	∞
110	0
111	0

Soft Constraints



$$\begin{array}{ll} \text{Constraint} & f : D^k \rightarrow \{0, \infty\} \\ \text{Soft Constraint} & f : D^k \rightarrow \{0, \infty\} \cup \mathbb{Q}_+ \end{array}$$

$$D = \{0, 1\}$$

$$k = 3$$

\mathbf{t}	$f(\mathbf{t})$
000	1
001	30
010	0
011	0.2
100	∞
101	∞
110	4
111	$\frac{3}{2}$

Soft Constraints



$$\begin{array}{ll} \text{Constraint} & f : D^k \rightarrow \{0, \infty\} \\ \text{Soft Constraint} & f : D^k \rightarrow \{0, \infty\} \cup \mathbb{Q}_+ \end{array}$$

$D = \{0, 1\}$ $k = 3$	\mathbf{t}	$f(\mathbf{t})$
	000	1
	001	30
	010	0
	011	0.2
	100	∞
	101	∞
	110	4
	111	$\frac{3}{2}$

Goal: to **minimise** the **sum** of all soft constraints

Configurations



Soft Constraints $\xrightarrow{?}$ Configurations

Area 1: Algebraic Theory

Algebraic theory of complexity for soft CSPs parametrised by the set of allowed constraint types [MFCS'11].



Classification of Boolean soft CSPs [CP'11].

Area 2: Conservative Soft CSPs

Hardness technique for studying certain classes of soft CSPs.



Classification of conservative soft CSPs [SODA'12].

(including all unary constraints $u : D \rightarrow \{0,1\}$)

Area 3: Expressibility

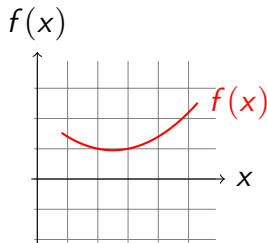
Usefulness of a constraint system via what it can **express**.



Expressibility of various soft CSPs [CP'07, TCS'08, IPL'09].

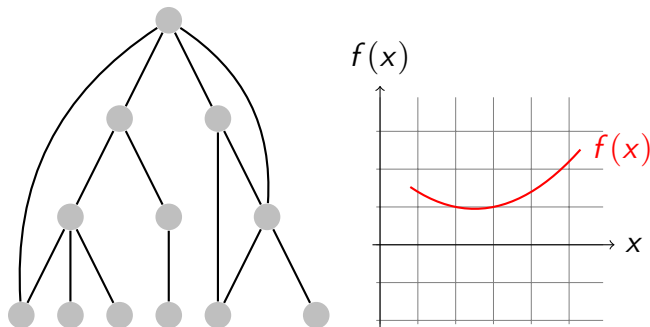
Area 4: Submodularity

Exploiting **submodularity**.



Which submodular soft CSPs are solvable by min-cut max-flow [CP'08,MFCS'09,DAM'09,Constraints'10]?

Area 5: Hybrid Soft CSPs



Tractable soft CSPs without a tree-like structure or a condition on soft constraints [CP'10,AIJ'10,CP'11].

Could these be useful to configurations?

1. algebraic techniques
2. conservativity
3. notion of expressibility
4. submodularity
5. hybrid tractability

Thank you



Questions?

(now or over dinner @ Balliol College)

<http://zivny.cz/>