

1.1

$$\psi, |\psi\rangle, |\bar{1} + \bar{2}\rangle = |\bar{1}\rangle + |\bar{2}\rangle$$

Tuesday May 11 2010

Jacob.Biamonte@comlab.ox.ac.uk

$$|\sum_i a_i \psi_i\rangle = \sum_i a_i |\psi_i\rangle$$

$$\langle \psi, \phi \rangle = \sum_i \bar{\psi}_i \phi_i$$

<http://www.comlab.ox.ac.uk/activities/quantum/course/>

$$\psi \phi, \psi \otimes \phi, |\psi\rangle \otimes |\phi\rangle, |\psi \phi\rangle, |\psi, \phi\rangle$$

$$|\psi\rangle, |\phi\rangle \mid |u\psi\rangle |u\phi\rangle$$

$$\langle \psi, u^\dagger u \phi \rangle = \langle u \psi, u \phi \rangle$$

$$- [\mathbb{C}^2]^n \quad P^n := \{1, x, y, z\}^n \rightarrow n\text{-wise tensor products}$$

- Show that linear sums of p^n form a vector space and define an inner product

- 1.2 evolution

$$|\psi\rangle$$

$$i\hbar \frac{d}{dt} \psi = H\psi$$

↑
real linear sums of p^n

$$H = H^\dagger$$

$$H\psi = E\psi$$

$$i\hbar \frac{d\psi}{dt} = E\psi, \quad \int_0^t \frac{d\psi}{\psi} = -\frac{i}{\hbar} E \int_0^t dt'$$

$$\ln \psi(t') \Big|_0^t = \ln \psi(t) - \ln \psi(0) = -\frac{i}{\hbar} Et$$

$$\ln \frac{\psi(t)}{\psi(0)} = \frac{-i}{\hbar} t E$$

$$\hbar = 1$$

$$E$$

$$\hbar \omega \rightarrow r/s$$

$$\psi(t) = e^{-i t E / \hbar} \psi_0$$

$$t = 2\pi / E$$

$$H = \sum_i \hbar \omega_i z^i + \frac{1}{2} \sum_{i,j} J_{ij} \underline{z^i z^j} + \sum_i \Delta^i x^i$$

$$[\sigma_i, \sigma_j] = 2i \epsilon_{ijk} \sigma_k$$

$$\{\sigma_i, \sigma_j\} = 2\delta_{ij}$$

$$x^2 = y^2 = z^2 = -ixyz$$

$$\sigma_i \sigma_j = 2\delta_{ij} + i \epsilon_{ijk} \sigma_k$$

- ϵ_{ijk} is Levi-Civita

$$[A, B] = AB - BA$$

$$\{A, B\} = AB + BA$$

1.4 Polarisation Vectors

$$\underline{P} = (P_1, P_2, P_3) \in \mathbb{R}^3$$

$$\underline{\sigma} = (X, Y, Z) \quad \underline{P} \cdot \underline{\sigma} = \underline{\sigma} \cdot \underline{P}$$

$$- \underline{\sigma} \cdot \underline{P} = P_1 X + P_2 Y + P_3 Z$$

$$- \text{Ex: Show } \text{tr}[(\underline{\sigma} \cdot \underline{A})(\underline{\sigma} \cdot \underline{B})] = \underline{A} \cdot \underline{B}$$

$$- E_X^{\text{show}} (\underline{\sigma} \cdot \underline{A}) (\underline{\sigma} \cdot \underline{B}) = \underline{A} \cdot \underline{B} + i \underline{\sigma} \cdot \begin{matrix} \nearrow \\ i \in i, k \end{matrix} \underline{A} \wedge \underline{B}$$

$$\text{if } \underline{A} = \underline{B} \quad (\underline{\sigma} \cdot \underline{A}) = \underline{A} \cdot \underline{A} \quad \Rightarrow \quad \lambda = \pm |A|$$

$$\psi = c_0 | + | A | \rangle + c_1 | - | A | \rangle$$

$$|A| = 1$$

$$= c_0 | 1 \rangle + c_1 | -1 \rangle$$

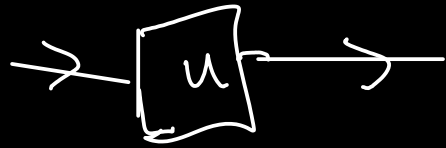
$$S_i \in \pm 1$$

$$S_i = 1 - 2\chi_i$$

$$\overline{S}_i = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\underline{\psi} = c_0 |0\rangle + c_1 |1\rangle$$

- Gates 1.5



$$H = P_0 \underline{1} + \underline{P} \cdot \underline{\sigma}$$

↑
global phase term

$$\exp(-itH) = \exp[-itP_0] \underbrace{\exp(-it\underline{P} \cdot \underline{\sigma})}_{U(\exp(-itD))U^\dagger}$$

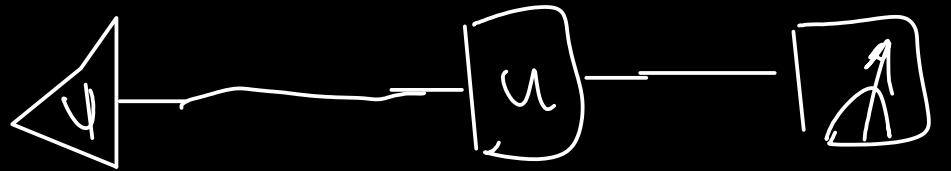
$$R_{\underline{P}}(\theta) = e^{-i\theta \underline{P} \cdot \underline{\sigma}} = \underline{1} \cos \theta - i \underline{P} \cdot \underline{\sigma} \sin \theta$$

- Ex show that $A^2 = 1 \Rightarrow$ using $e^A = \sum_{k=0}^{\infty} \frac{1}{k!} A^k$

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

A basis $\{|i\rangle\}$

$$\{|i\rangle\}_i \quad \langle i | j \rangle = \delta_{ij}$$



$t=0$

$t=1$

$t=2$

$U\psi$

$P_\phi |U\psi\rangle$

$$X \otimes X |01\rangle = |10\rangle$$

$$\phi^- = |01\rangle - |10\rangle$$

$$U = \sum_{k=0}^{\infty} \frac{1}{k!} (-itH)^k$$

$$\simeq 1 - itH + O(t^2)$$

$$H = X \otimes X$$

$$U|01\rangle = (1 - i t X \otimes X) |01\rangle + O(t^2)$$

$$\underline{|01\rangle - i t |10\rangle + O(t^2)}$$

$$e^{-i t X \otimes X} = 1 \cos t - i X \otimes X \sin t$$

$$(X \otimes X)^2 = 1$$

pick $t = \pi/4$

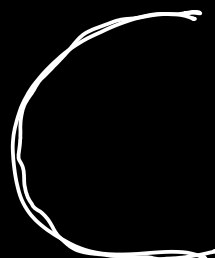
$$U|01\rangle = \frac{|01\rangle - i |10\rangle}{\sqrt{2}}$$

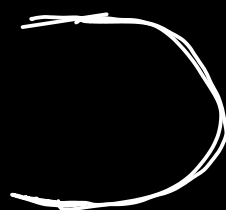
$$e^{-i |0\rangle\langle 0| t} = e^{-i t/2} (\underline{1} + Z) \quad \text{act 2nd bit}$$

$t = 2\pi$ $\begin{pmatrix} e^{-i\pi} & \\ & 1 \end{pmatrix}$

so using we construct $\phi^- = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$

H  H wire

 cap

 cap

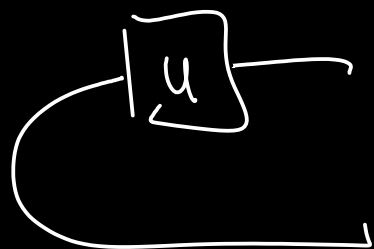
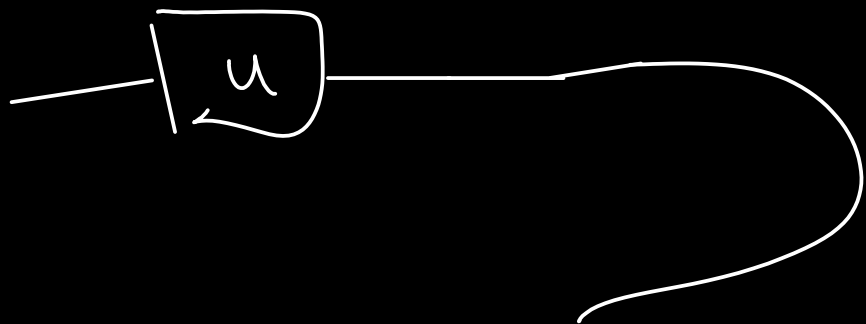
$$\eta = \sum_i |i\rangle |i\rangle$$

$$\epsilon = \sum_i \langle i | \langle i |$$

$$\bigcirc = \sum_{i=1}^d S_{ii} = d \quad (\epsilon \circ \eta)$$

$= 1$

$\nearrow$ i_b normalised $= 1$



can use E, M
to send wires on
gates — these can
be realised as
physical processes

$$E \circ U = \sum_{i,j,k} \langle i | \langle i | U_{jk} | j \rangle | k \rangle$$

$$= \sum_{i,j,k} U_{jk} \underbrace{\langle i | j \rangle \langle i | k \rangle}$$

$$\psi = \sum_i c_i |i\rangle$$

$\uparrow$
 $\langle i | \psi \rangle$

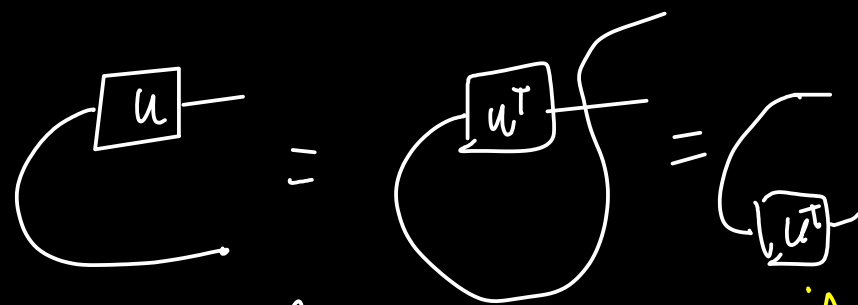
$$= \sum_{j,k} U_{jk} \langle j | \langle k |$$

A diagram showing a unitary gate U represented by a box with the letter U inside. A single horizontal line enters the box from the left and exits to the right, representing a single-qubit operation. A yellow arrow points from the equation above to the output wire.

$$\sum_{i,j,k} \langle i | U_{jk} | i \rangle \langle k | \langle i |$$

$$= \sum_{i,j,k} U_{jk} \langle i | i \rangle \langle k | \langle i |$$

$$= \sum_{j,k} U_{jk} \langle k | \langle i | = \epsilon_0 U^T$$



call this
transpose
duality

(Statistical independence)

$$\langle A \otimes B \rangle = \langle \psi, A \otimes B \psi \rangle$$

$$\langle A \otimes B \rangle_\psi \text{ or } \langle AB \rangle$$

$$\langle AB \rangle = \langle A \otimes 1 \rangle \langle 1 \otimes B \rangle$$

$$\Rightarrow | \langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle | \leq 2$$

$$\Rightarrow | \underline{\langle A_1 \otimes 1 \rangle} \underline{\langle 1 \otimes B_1 \rangle} + \underline{\langle A_1 \otimes 1 \rangle} \underline{\langle 1 \otimes B_2 \rangle} |$$

$$+ |\langle A_2 \otimes 1 | X | 1 \otimes B_1 \rangle - \langle A_2 \otimes 1 | X | 1 \otimes B_2 \rangle|$$

$$\stackrel{\leq 1}{|\langle A_1 \otimes 1 | (\langle 1 \otimes B_1 \rangle + \langle 1 \otimes B_2 \rangle)|}$$

$$+ |\langle A_2 \otimes 1 | (\langle 1 \otimes B_1 \rangle - \langle 1 \otimes B_2 \rangle)|$$

$$\leq \underbrace{|\langle 1 \otimes B_1 \rangle + \langle 1 \otimes B_2 \rangle|}_{T_1} + \underbrace{|\langle 1 \otimes B_1 \rangle - \langle 1 \otimes B_2 \rangle|}_{T_2}$$

B_1	B_2	T_1	T_2
+	+	2	0
+	-	0	2
-	+	0	2
-	-	2	0

$$\Rightarrow \leq 2$$

$$\phi^- = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle) \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

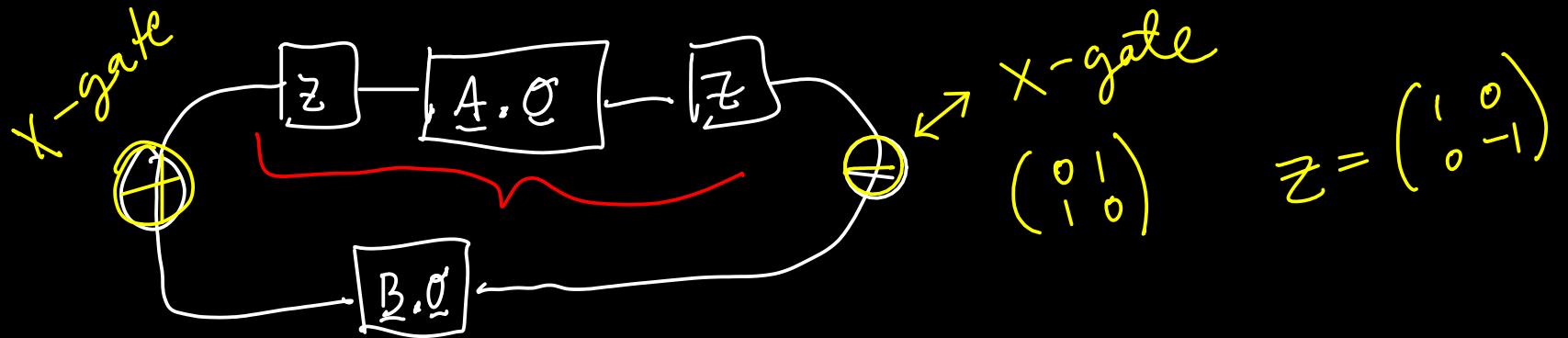
$$\text{---} \oplus \text{X} \text{---} \rightarrow \text{---} \oplus \text{X} \text{---} \quad |10\rangle - |01\rangle$$

$$- E_X \quad \text{if } |A| \approx 1, \quad \psi, \psi^\perp \approx \pm 1$$

$\psi, \psi^\perp$ are eigenvectors
+1, -1

Show that $|01\rangle - |10\rangle = |\psi, \psi^\perp\rangle - |\psi^\perp, \psi\rangle$
up to a phase

$$\langle \phi^-, \underline{A} \cdot \underline{\sigma} \otimes \underline{B} \cdot \underline{\sigma} \phi^- \rangle \quad \text{general } \underline{A} \cdot \underline{\sigma} \text{ and } \underline{B} \cdot \underline{\sigma}$$



$$\underline{A} \cdot \underline{\theta} = (A_1, A_2, A_3) \cdot \underline{(x, y, z)}$$

$$xz (x \ y \ z) z x \quad i \neq j$$



$$\theta_i \bar{\theta}_j; \theta_i = -\theta_j$$

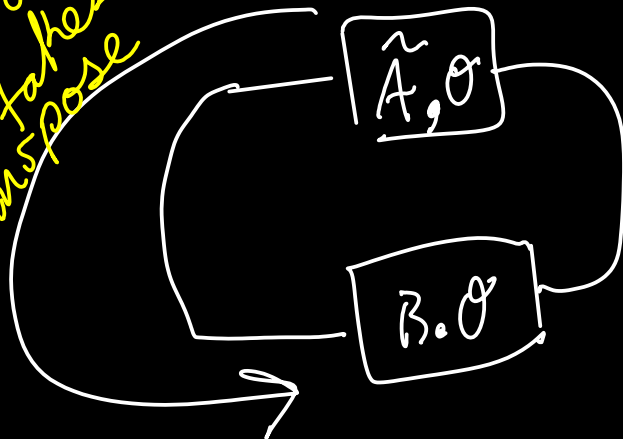
overbar
is complex conjugation

$$x (-x, -y, z) x$$



$$(\underline{-x}, \underline{y}, \underline{-z})$$

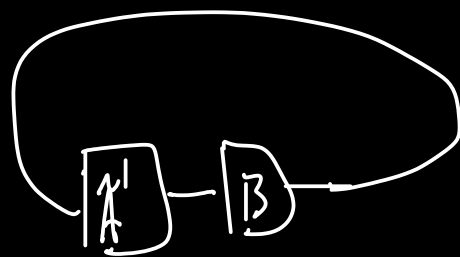
this sends
 $x \rightarrow x$
 $y \rightarrow -y$!
 $z \rightarrow z$
 moving box
 around takes
 a transpose



$$\boxed{u} = \boxed{u^T}$$

note, terms like

$$\bigcirc \oplus = 0$$



$$(-x, -y, -z) \cdot A$$

sign changed sending $A \rightarrow -A$ on all basis vectors

$$(A, \sigma)(B, \sigma) = \boxed{-A \cdot B} + i \sigma(A \wedge B)$$

$$\begin{aligned} & \langle i | \langle i | \overset{=-A \cdot B}{K} | j \rangle | j \rangle \\ & = K \end{aligned}$$

$$\boxed{-A \cdot B = -\cos \theta}$$

$$\bigcirc \oplus = 0$$

show using ϵ, μ that this must be zero.

$$A_1 = (1, 0, 0) \quad (\sigma^x)$$

$$A_2 = (0, 1, 0) \quad (\sigma^y)$$

$$B_1 = \frac{1}{\sqrt{2}} (1, 1, 0) \quad (\sigma^x + \sigma^y)$$

$$B_2 = \frac{1}{\sqrt{2}} (1, -1, 0) \quad (\sigma^x - \sigma^y)$$

here we show violation of Bell inequality

$$\underline{A}_1 \cdot \underline{B}_1 = 1 \mid \sqrt{2}$$

$$\underline{A}_1 \cdot \underline{B}_2 = 1 \mid \sqrt{2}$$

$$\underline{A}_2 \cdot \underline{B}_1 = 1 \mid \sqrt{2}$$

$$\underline{A}_2 \cdot \underline{B}_2 = -1 \mid \sqrt{2}$$

$$\Rightarrow 4 \mid \sqrt{2} = 2\sqrt{2}.$$