

Thursday 20 May 2010

— S Q M (stabiliser Q M)

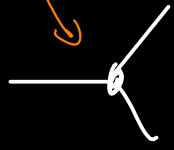
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$$P^n := \{X, Y, Z, I, \pm i, \pm 1\}$$

pauli-group

$$\sigma_i \sigma_j = \delta_{ij} I + i \epsilon_{ijk} \sigma_k$$

pauli-product

C.G. $\{P' \wedge$ 

→ pauli operators

copy

dagger

, $T, H, -\text{CNOT}, P\}$

Clifford group

H.P.

$$\psi_t = \psi_0$$

states constant

$$A_t = U^\dagger A_0 U$$

$\{Z_i, X_i\}$

"phase-gate" operators in

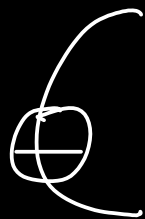
restrict U to the clifford group. follow 2^n the H.P.

$$\left\{ \text{---} \circ \text{---} , H, \text{---} \triangle \text{---} , \text{---} \text{---}^\dagger \right\}$$

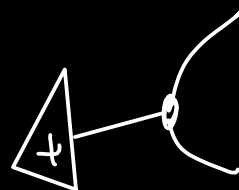
this presentation gives all circuits in $\oplus$ -L and GHZ-states

$$\text{---} \boxed{H} \text{---} \circ \begin{matrix} \boxed{H} \\ \boxed{H} \end{matrix} = \text{---} \bigoplus \text{---}$$

$$\left(\text{---} \circ \text{---} \right)^\dagger = \text{---} \bigoplus \text{---}$$

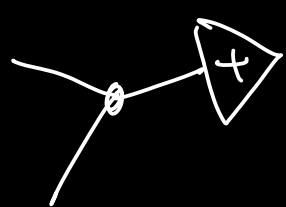


$$|01\rangle + |10\rangle$$



$$= ($$

cups and caps



$$=)$$



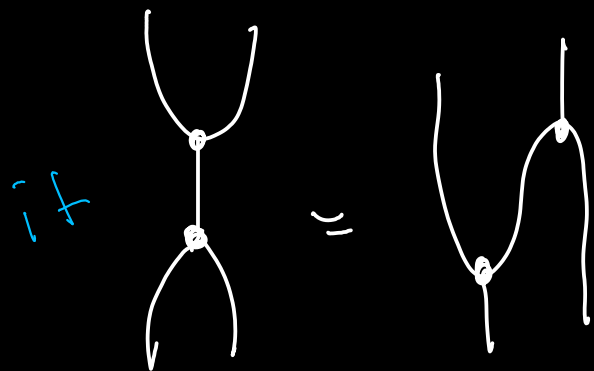
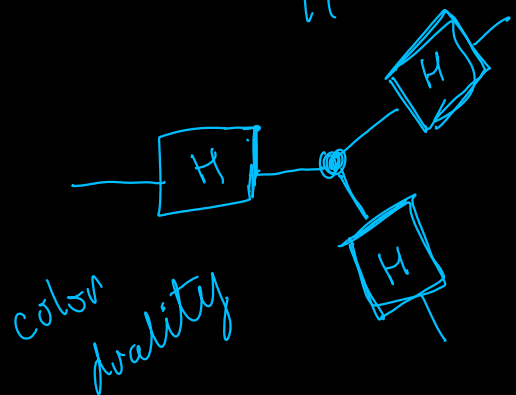
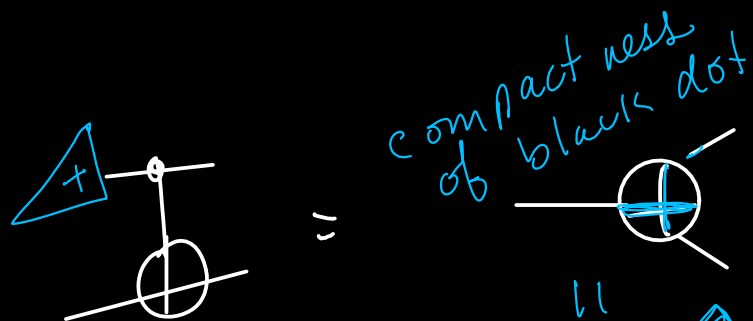
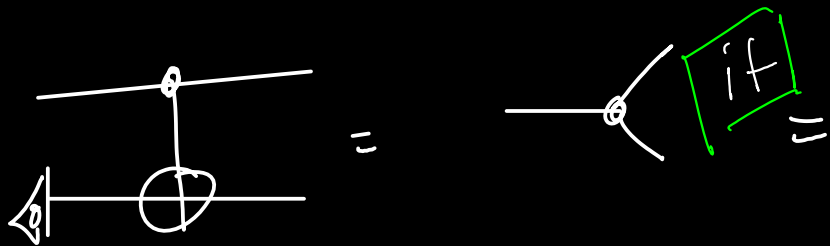
H

$|0\rangle$

$$\left\{ \text{---} \circ \text{---} , \text{---} \oplus \text{---} \right\} \Rightarrow \oplus\text{-L}$$

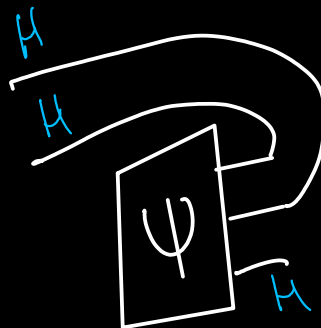
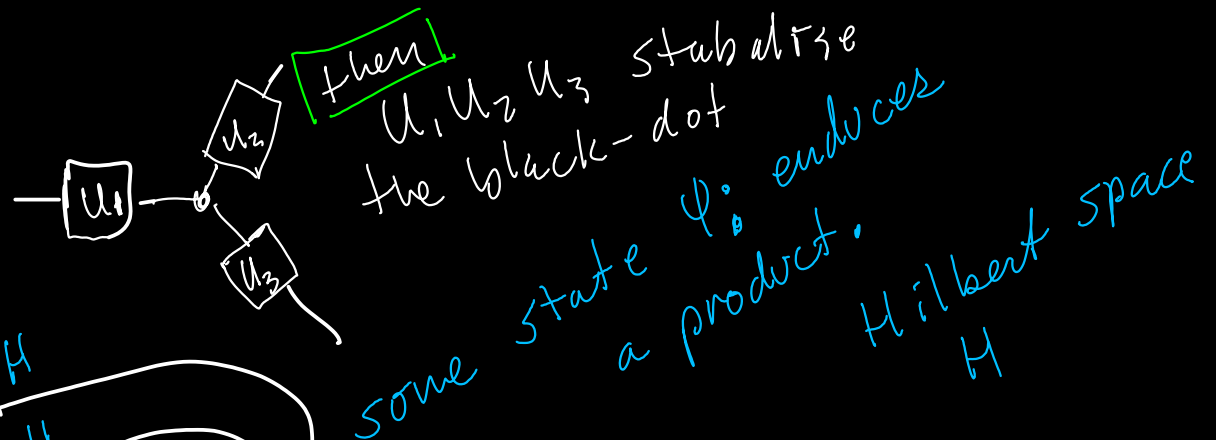
Linear functions

$$f(x_1, \dots, x_n) = c_0 \oplus c_1 x_1 \oplus \dots \oplus c_n x_n$$



$\Rightarrow FA$

{ black and plus dots are Frobenius } copy

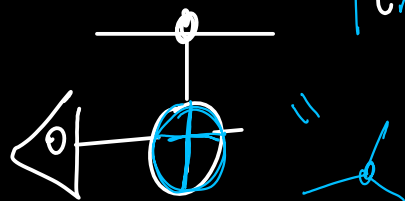


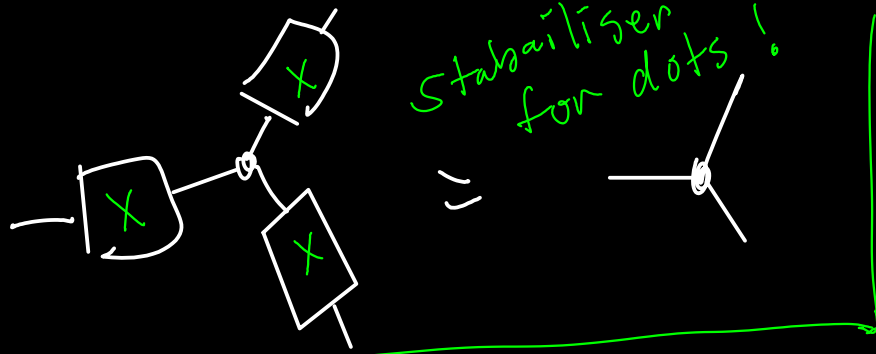
$$H \oplus H \rightarrow H$$

$$H \rightarrow H \oplus H$$

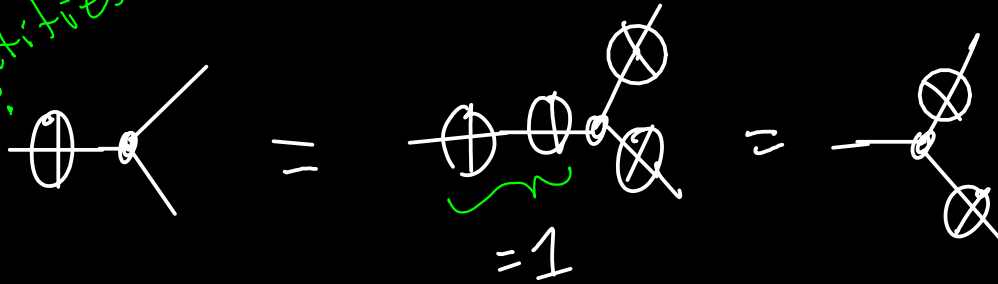
$$|i\rangle \rightarrow |\bar{i} i\rangle$$

$$\bar{i} \in \{0, 1\}$$



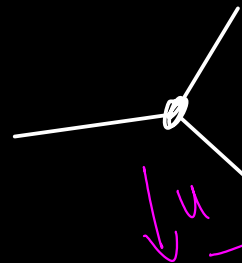


we can prove identities such as:

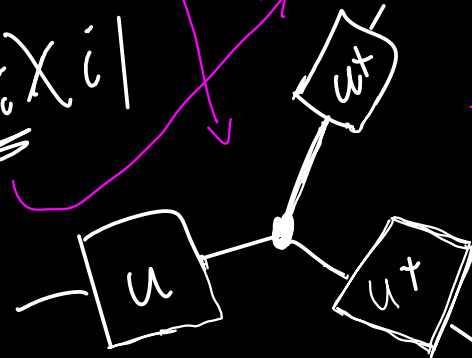


$$\text{copy} = \sum_{i=1}^d |ii\rangle\langle ii|$$

defining U in this way transforms dots in such a way that singular values are preserved $U =$



$$\sum_i |\phi_i\rangle\langle ii|$$



now this dot copies $|\phi_i\rangle$
 $\sum_{i=1}^d |\phi_i\rangle\langle ii|$

GHZ-state $\mathbb{1}$ all states are stabilised by $\mathbb{1}$

$$X_1 X_2 X_3$$

$$Z_1 Z_2$$

$$Z_2 Z_3$$

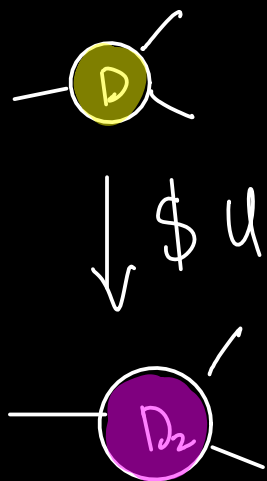
$$Z_1 Z_2 \cdot Z_2 Z_3 = Z_1 Z_3$$

copy new basis $|\phi_i\rangle$

since states induce products, we are now able to apply stabiliser theory to operators, etc.

$$\langle \phi_i | \phi_j \rangle = \delta_{ij}$$

dots change color under U



$$\begin{array}{ccc} A_1 & A_2 & A_3 \\ \downarrow \$U & & \downarrow \$U \\ A'_1 & A'_2 & A'_3 \end{array}$$

stabiliser
↓ transforms under U
to be a stabiliser for a new dot

$$\{Z_i, X_i\}$$

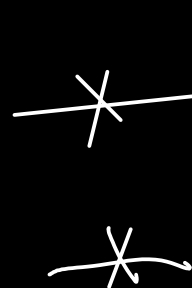
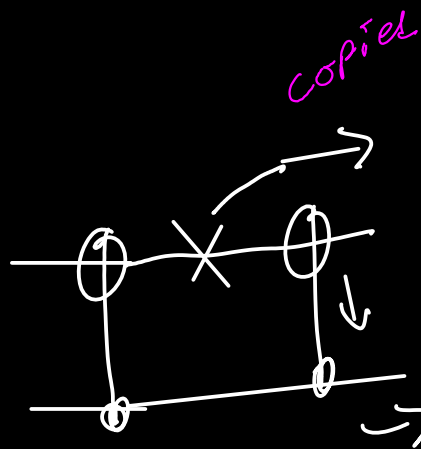
$$A_t = U^\dagger A_0 U$$

↑
CG

H, P, CN

ZZZ is a stabiliser for

$$\begin{pmatrix} 1 \\ \vdots \end{pmatrix}$$



as

$ZZZ =$
 $HHH(XXX)HHH$
and XXX is a stabiliser for black dots.

we need to determine how X, Z transform under the Clifford group

we can do this graphically

span $\{ |0\rangle, |1\rangle \}$
 spins written in
 basis $|0\rangle, |1\rangle$

some
Hamiltonian

$$H = |1\rangle\langle 1|$$



$$\lambda = 0, |0\rangle$$

$$\lambda = 1, |1\rangle$$

$$H = \frac{1}{2}(1 - Z)$$

Introduction to Spins

spin-logic in ground
states of Hamiltonians

we want
spin-1 to equal spin-2

$$S_1 = S_2$$

$$H = 1 - |00\rangle\langle 00| - |11\rangle\langle 11|$$

applies
an equality
penalty.

$$\text{span} \{ |00\rangle, |11\rangle \} \quad \lambda = 0$$

$$\text{span} \{ |01\rangle, |10\rangle \} \quad \lambda = 1$$

any time the spins are
equal, the are in
the ground state
of the Hamiltonian

x	y	$x \wedge y$
0	0	0
0	1	0
1	0	0
1	1	1

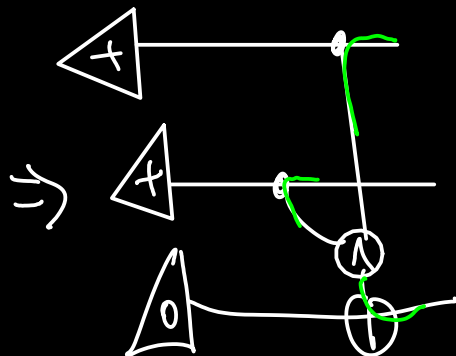
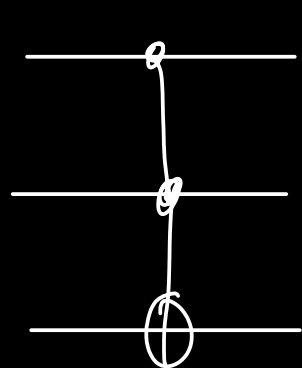
truth table for
AND

$$H_{\text{AND}} = 1 - (|000\rangle\langle 000| + |010\rangle\langle 010| + |100\rangle\langle 100| + |111\rangle\langle 111|)$$

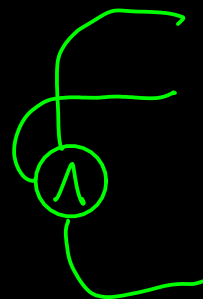
3-body

logic gate in ground state!

$$\psi_{AND} = |000\rangle + |010\rangle + |100\rangle + |111\rangle$$



=



example to show
the difference between
being in the span
of a ground state and
being a coherent
mixture of AND.



as top
bits are
sym.

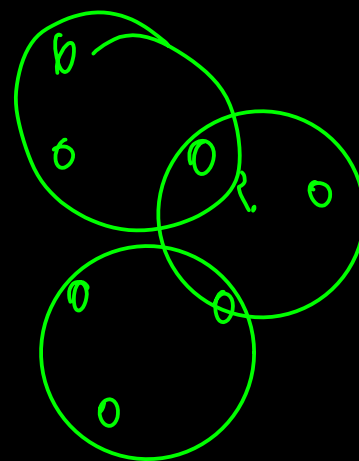
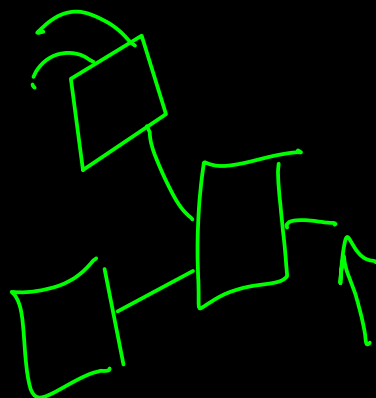
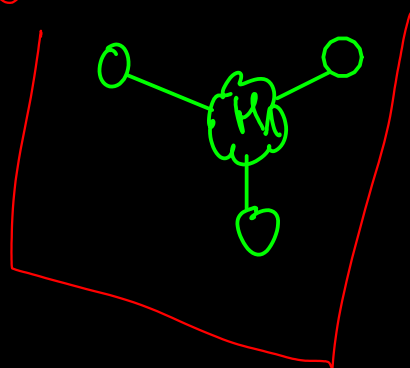
H_A

H_V

$H_{\oplus}$

Hamiltonians for AND, OR, XOR, etc.
these allow us to "embed" logic circuits into lattices.

$\text{span}\{ |x, y, x \wedge y\rangle \mid x, y \in \{0, 1\} \}$



$$H = \sum_{i,j,k} J_{ijk} \underbrace{Z_i Z_j Z_k}_{\text{3-body}} + \sum_{i,i} J_{ii} Z_i Z_i + \sum_i K_i Z_i$$

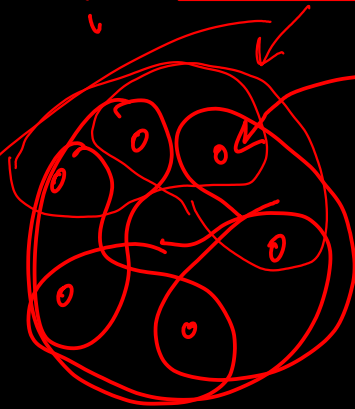
finding the ground state of H is as hard as finding an input to make a Boolean function

$$H_c = -J \sum_i Z_i X_{i+1} Z_{i+2} + B \sum_i Z_i$$

$f=1$, etc

Cluster model

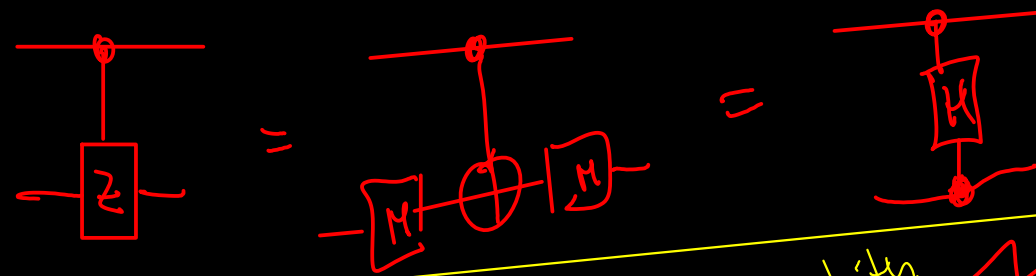
acts on triples



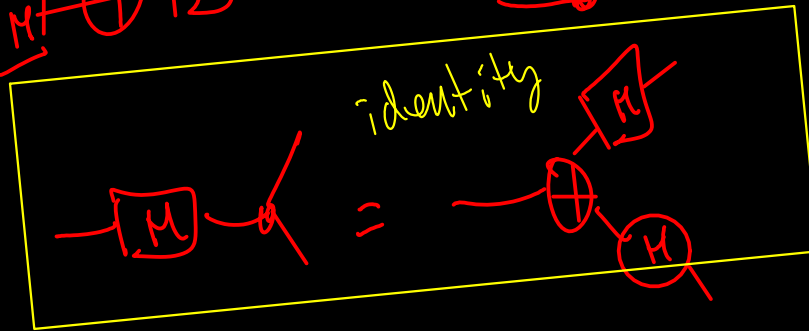
act on singles
 $\downarrow \bar{U} H_c U^+$
 find U to "localise" H_c !

$$\sum_i P_i \sigma_i = \sum_i P_i^x X + P_i^y Y + P_i^z Z$$

this is local and can be diagonalised by single qubit gates.

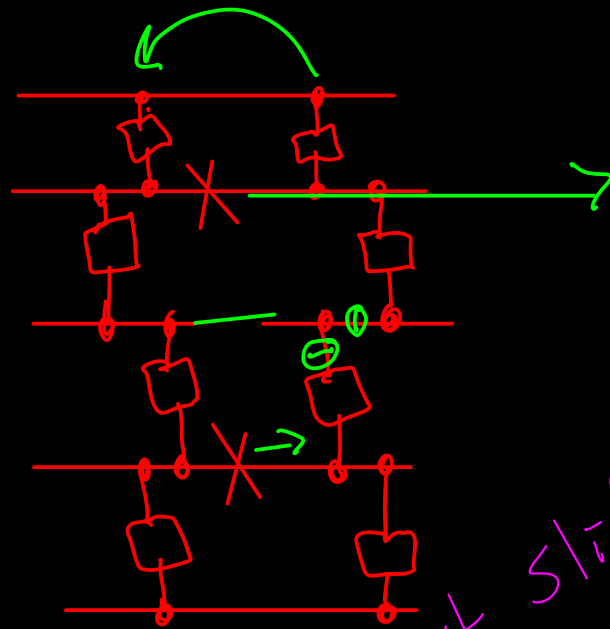


Controlled Z gates

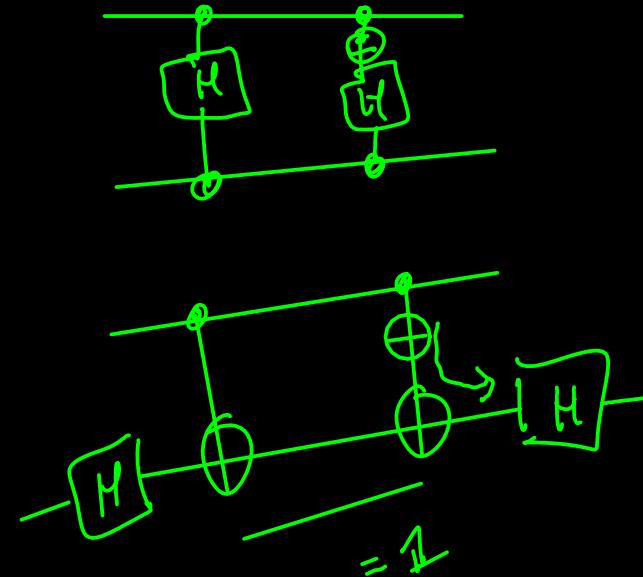


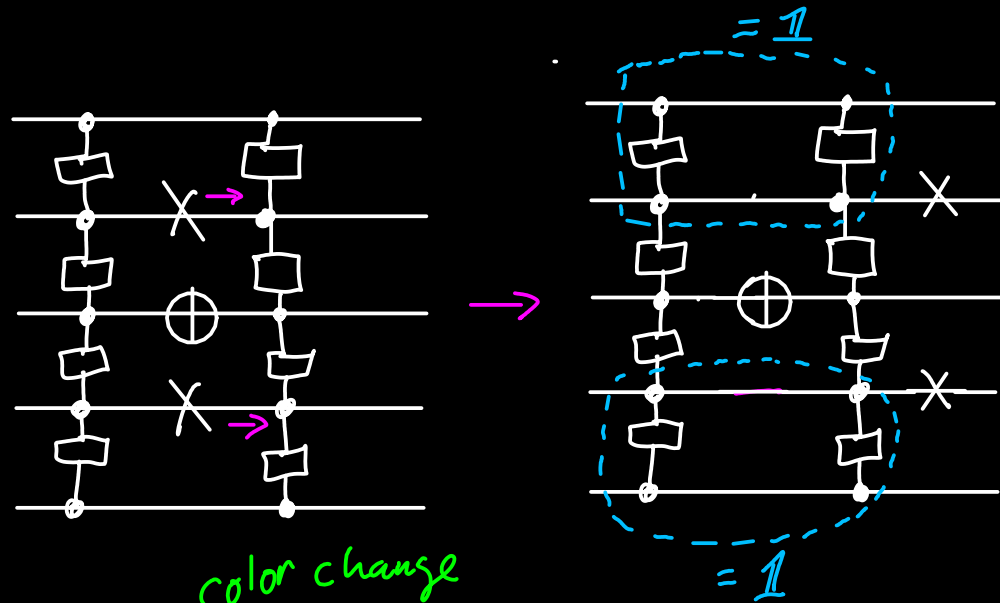
$$X \otimes I \rightarrow X \otimes Z$$

$$I \otimes X \rightarrow Z \otimes X$$

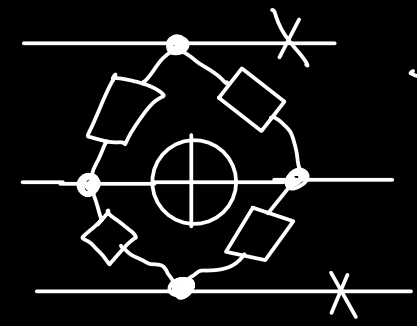
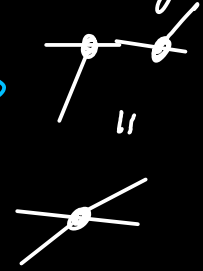


see the next slide!

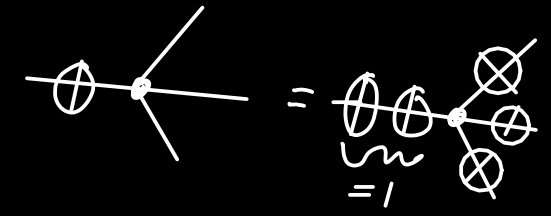




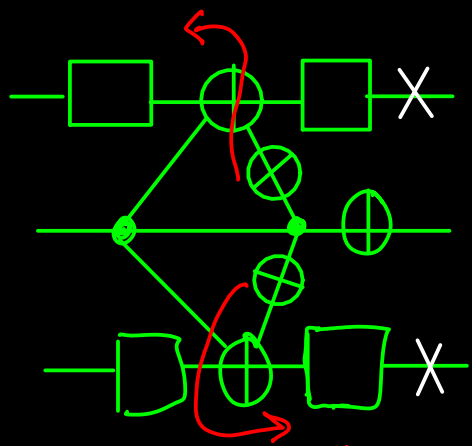
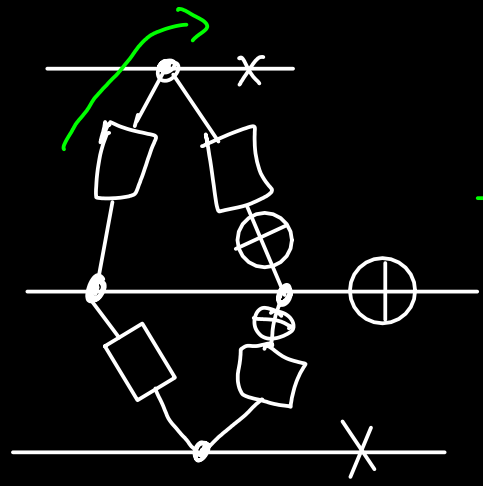
we group dots, eg



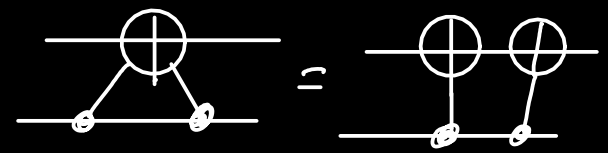
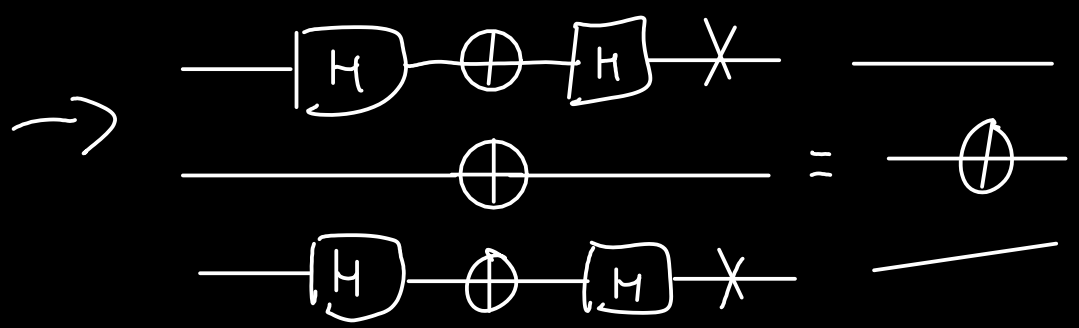
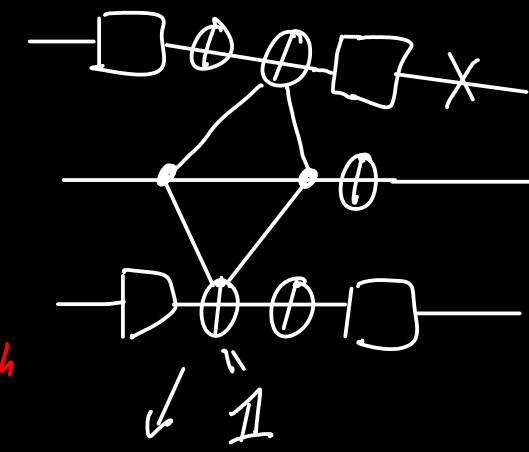
using
 $\text{---} \oplus \text{---} = \text{---} \oplus \oplus \oplus \oplus \text{---}$
 as $XXXX$ is a stab
 for
 $|0000\rangle + |1111\rangle$



color change



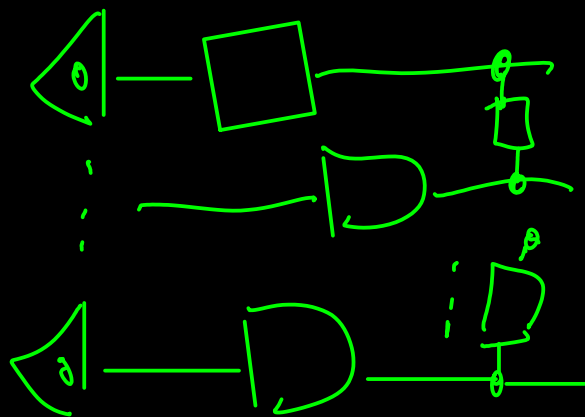
dots of same color move through each other.



$$Z_i X_{i+1} Z_{i+2}$$

$$\downarrow$$

$$X_{i+1}$$



now we
can use this
to prepare
ground states
to Hc.

$$\sum_i h_i = \sum_i -J X_i + B Z_i$$

$$\begin{pmatrix} B & -J \\ -J & -B \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$\lambda + \sqrt{-J^2 + B^2}$
nice application
of stabilisers!

