

"Many-Body Entanglement"

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(note: This is a copy of what was written on the board during lecture)

1) $\sum p_i: \rho_1^i \otimes \dots \rho_n^i$ $E=0$

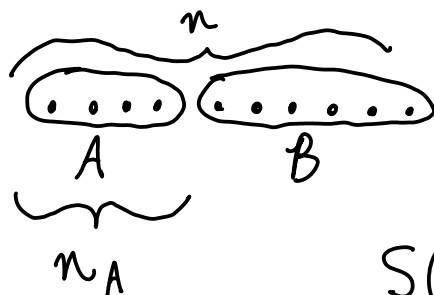
2) $|GHZ\rangle = |0\dots 0\rangle + |1\dots 1\rangle$ $E=1$

3) $|W\rangle = |00\dots 11\rangle + |0101\dots\rangle + \dots + |11\dots 00\rangle$ $E = \log N$

4) Cluster state $E = N/2$ $|\psi^-\rangle$ $|\psi^-\rangle$ $|\psi^-\rangle$. etc

5) Max entangled state scales as N (random state will have close to this!)

$\rho = |\psi\rangle\langle\psi|$



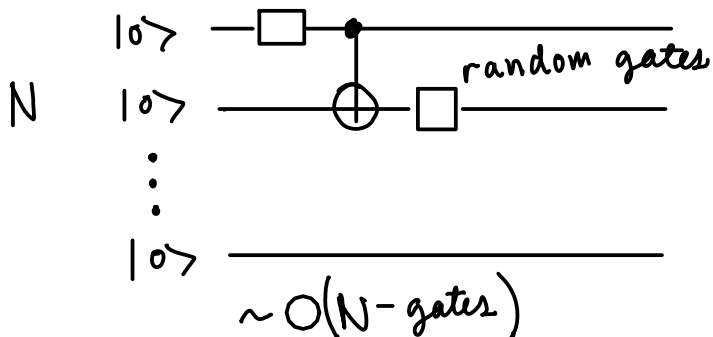
$\rho_A = \text{tr}_B |\psi\rangle\langle\psi|$

$S(\rho_A) = -\text{tr} \rho_A \log \rho_A \sim n_A$

$\text{Tr} \rho_A^2 \sim \frac{1}{2^n}$

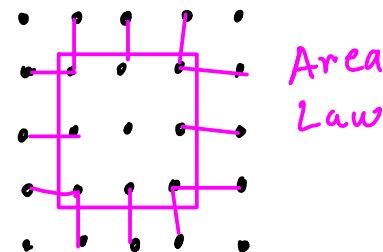
first order of entropy

2^{n_A}



$E \sim N \Rightarrow$ easy to generate

$\begin{pmatrix} \frac{1}{2^{n_A}} & & \\ & \frac{1}{2^{n_A}} & \\ & & \ddots \\ & & & \frac{1}{2^{n_A}} \end{pmatrix}$

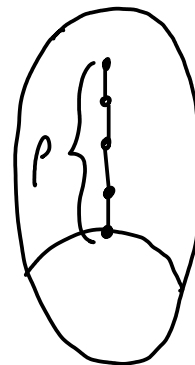


Relative entropy

$$S(\rho \parallel \rho_{\text{sep}}) = \text{tr}(\rho \log \rho - \rho \log \rho_{\text{sep}})$$

compute entropy and show = 1

$$\frac{1}{\sqrt{2}} (|00\dots 0\rangle + |11\dots 1\rangle) \rightarrow \rho_{\text{sep}} = \frac{1}{2} (|0\dots 0\rangle\langle 0\dots 0| + |1\dots 1\rangle\langle 1\dots 1|)$$



$$|W\rangle = |001\rangle + |010\rangle + |100\rangle$$

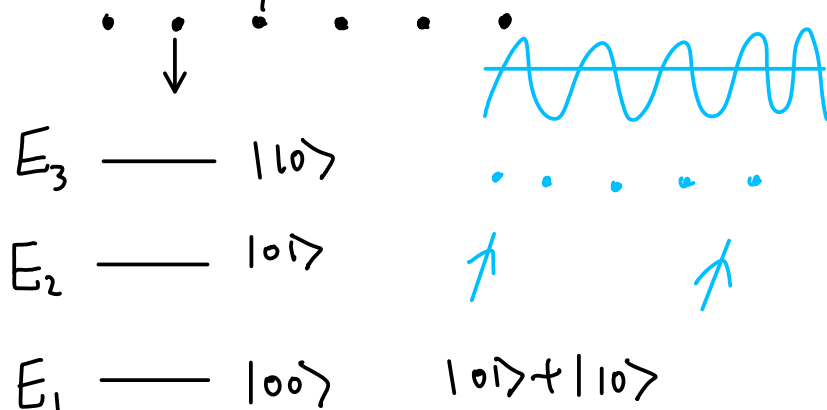
$$|e\rangle^{\otimes 3} = \left(\sqrt{2/3} |0\rangle + e^{i\phi} \sqrt{1/3} |1\rangle \right)^{\otimes 3}$$

$$\rho_{\text{sep}} = \frac{1}{2\pi} \int |e\rangle |e\rangle |e\rangle \langle e| \langle e| \langle e| d\phi$$

$$S \sim \log N$$

Note: JW-transform
as Home work.

$$\mathcal{H} = \mathcal{J} \sum_i [\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y] \stackrel{\text{JW}}{=} \sum_n E_n |E_n\rangle \langle E_n| \rightarrow F = -kT \ln Z$$



$$M = \frac{2F}{2B}$$

$$\chi = \frac{2^2 F}{2B^2}$$

$$Z = \sum_n e^{-E_n/kT}$$

free energy
internal energy

$$V = \frac{2F}{2T}$$

heat capacity $C = \frac{2^2 F}{2T^2}$

$$Z = \text{tr} e^{-\hat{H}/\mu T}$$

$$\sim \underbrace{\text{tr} \hat{H} e^{-\hat{H}/\mu T}}_{\rho} = \langle \hat{H} \rangle$$

$$\chi \sim \sum \langle \sigma^i \sigma^j \rangle$$

2-point function

$$\langle \sigma_1^x \sigma_2^x + \sigma_1^y \sigma_2^y \rangle_{\text{sep}}$$

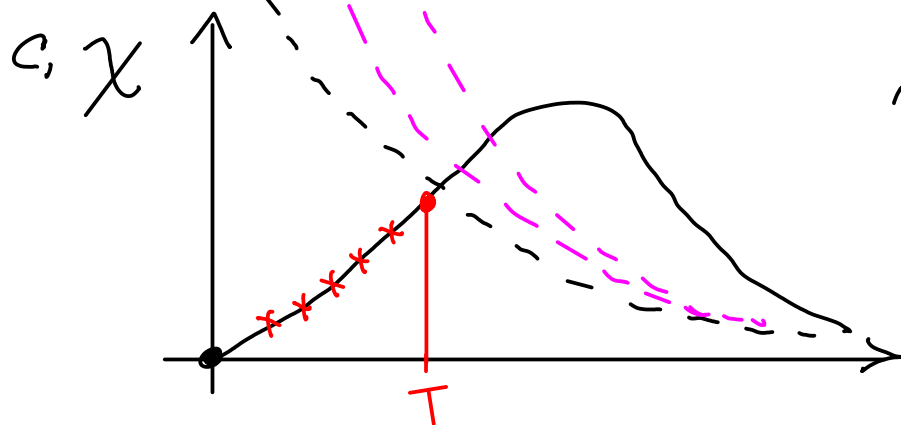
$\hat{O}$

$$\sum p_i \text{tr} (\hat{O} \rho_1^i \otimes \rho_2^i)$$

$$\sum p_i (\langle \sigma_1^x \chi \sigma_2^x \rangle + \langle \sigma_1^y \chi \sigma_2^y \rangle)$$

$$\leq 1$$

$$S_1^x S_2^x + S_1^y S_2^y \leq \sqrt{(S_1^x)^2 + (S_1^y)^2} \sqrt{(S_2^x)^2 + (S_2^y)^2} \leq 1$$



$$\chi_{\text{sep}} \sim \frac{\text{const}}{T}$$

$$\frac{2^n F}{2 T^n}$$

$$\chi < \chi_{\text{sep}}$$

$\Rightarrow$ state entangled

What type of entanglement?