EXTRA MATHEMATICS ADMISSIONS TEST

December 2021

Time allowed: 1 hour

Surname	
Other names	

This paper contains 10 multiple choice questions.
Calculators are not permitted.

For each question on pages 2–11 you will be given **five** possible answers, just one of which is correct. Indicate for each question **A-J** which answer (a), (b), (c), (d), or (e) you think is correct with a tick (✓) in the corresponding column in the table below.

	(a)	(b)	(c)	(d)	(e)
A					
B					
C					
D					
E					
F					
G					
H					
I					
J					

A. Which of the following expressions has the largest value? Note that all angles are given in degrees.

(a) $\cos(10^\circ)$,

(b) $\sin(115^\circ)$,

(c) $\cos(375^\circ)$,

(d) $\sin(85^\circ)$,

(e) $\cos(-20^\circ)$.

Please turn over

B. In the expansion of $(x^2+xy+y^2)^n$, where n is a positive whole number, the coefficient of x^3y^{2n-3} is

(a) $\binom{n}{3}$

(b) $\binom{n}{3} \times \binom{n}{2}$

(c) $\binom{n}{3} + 2 \times \binom{n}{2}$

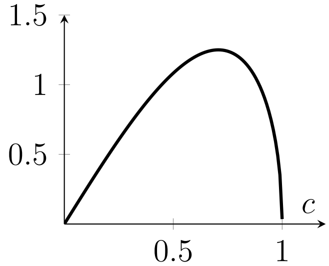
(d) $2 \times \binom{n}{2}$

(e) $\binom{n}{3} + \binom{n}{2}$

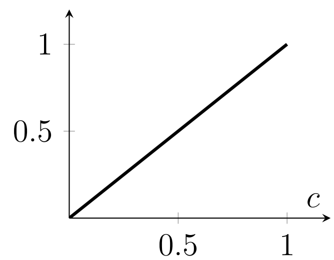
Please turn over

C. Given a real number c with $0 < c < 1$, the line $y = c$ intersects the circle $x^2 + y^2 = 1$ at two points. These two points, together with $(1, 0)$ and $(-1, 0)$, form a quadrilateral. Which of the following graphs is a plot of the area of that quadrilateral against c?

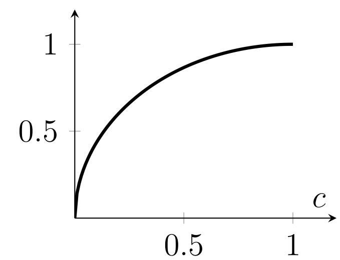
(a)



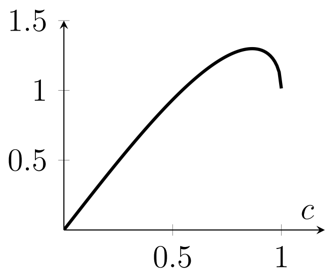
(b)



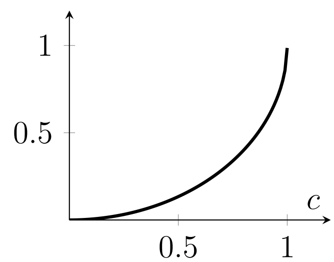
(c)



(d)



(e)



Please turn over

D. A particle moves along the x-axis. At time $t = 0$ the particle starts at $(0, 0)$ with initial speed 1, moving towards $x = 1$. When the particle reaches $x = n$ for any positive integer n, its speed immediately changes to 2^{-n} but its direction is unchanged. What is the particle's position at time $t = 100$?

(a) $x = \dfrac{89}{16}$,

(b) $x = \dfrac{105}{16}$,

(c) $x = \dfrac{3200}{32}$,

(d) $x = \dfrac{421}{64}$,

(e) The particle has escaped to infinity.

Please turn over

E. The polynomial equation $x^4 - (2k+1)x^2 + 2x + k^2 - 1 = 0$ has exactly four real solutions x if and only if

(a) $k > 1$,

(b) $k > -\dfrac{5}{4}$,

(c) $k > \dfrac{3}{4}$,

(d) $k < -\dfrac{5}{4}$ or $k > \dfrac{3}{4}$,

(e) $\dfrac{3}{4} < k < 1$ or $k > 1$.

Please turn over

F. The point A has coordinates $(3, 4)$. The origin $(0, 0)$ and the point A both lie on the circumference of a circle $\mathcal{C}$. The diameter of $\mathcal{C}$ through A also meets $\mathcal{C}$ at another point B. The distance between B and the origin is 10. It follows that the coordinates of B could be either

(a) $\left(-5\sqrt{2}, 5\sqrt{2}\right)$ or $\left(5\sqrt{2}, -5\sqrt{2}\right)$,

(b) $(-4, 3)$ or $(4, -3)$,

(c) $\left(-5, 5\sqrt{3}\right)$ or $\left(5, -5\sqrt{3}\right)$,

(d) $(-8, 6)$ or $(8, -6)$,

(e) $\left(-5\sqrt{3}, 5\right)$ or $\left(5\sqrt{3}, -5\right)$.

Please turn over

G. Without calculating it directly, which of the following numbers is the square of $123,456,789$?

(a) 15,241,578,710,190,521,

(b) 15,241,578,730,190,521,

(c) 15,241,578,750,190,521,

(d) 15,241,578,770,190,521,

(e) 15,241,578,790,190,521.

Please turn over

H. A function $f(x)$ satisfies the following equation

$$f(x) + f(y) = \frac{1}{f(xy)}$$

for any real positive numbers x and y, and also satisfies $f(x) > 0$ for all real positive numbers x. It follows that $f(2021)$ is

(a) 1,

(b) 2021,

(c) $\log_e 2021$,

(d) $\dfrac{1}{\sqrt{2}}$,

(e) $\dfrac{1}{\log_e 2021}$.

[Hint: try substituting $x = 1$ and $y = 1$ into the given expression.]

Please turn over

I. Given that there are positive real numbers a, b, c that satisfy

$$\int_a^b \log_c\left(\sin^4 x \tan^2 x\right)\,\mathrm{d}x = 1 \quad \text{and} \quad \int_a^b \log_c\left(\sin^2 x \cos^2 x\right)\,\mathrm{d}x = 3,$$

it follows that the value of

$$\int_a^b \log_c\left(\sin^4 x \cos^2 x\right)\,\mathrm{d}x$$

must be equal to

(a) 4,

(b) 5,

(c) 6,

(d) 7,

(e) 8.

[Note that $\sin^4 x$ means $(\sin x)^4$.]

Please turn over

J. There is a straight line that is normal to the curve $y = x^3 - kx$ at two different points if and only if

(a) $k \geq \sqrt{3}$,

(b) $k^2 \geq 3$,

(c) $k^2 \geq 1$,

(d) $k \geq 1$,

(e) $k \geq \sqrt{3}$ or $k \leq -1$.

End of last question