

# A Two-Factor Error Model for Quantitative Steganalysis

Security, Steganography and Watermarking of Multimedia Contents  
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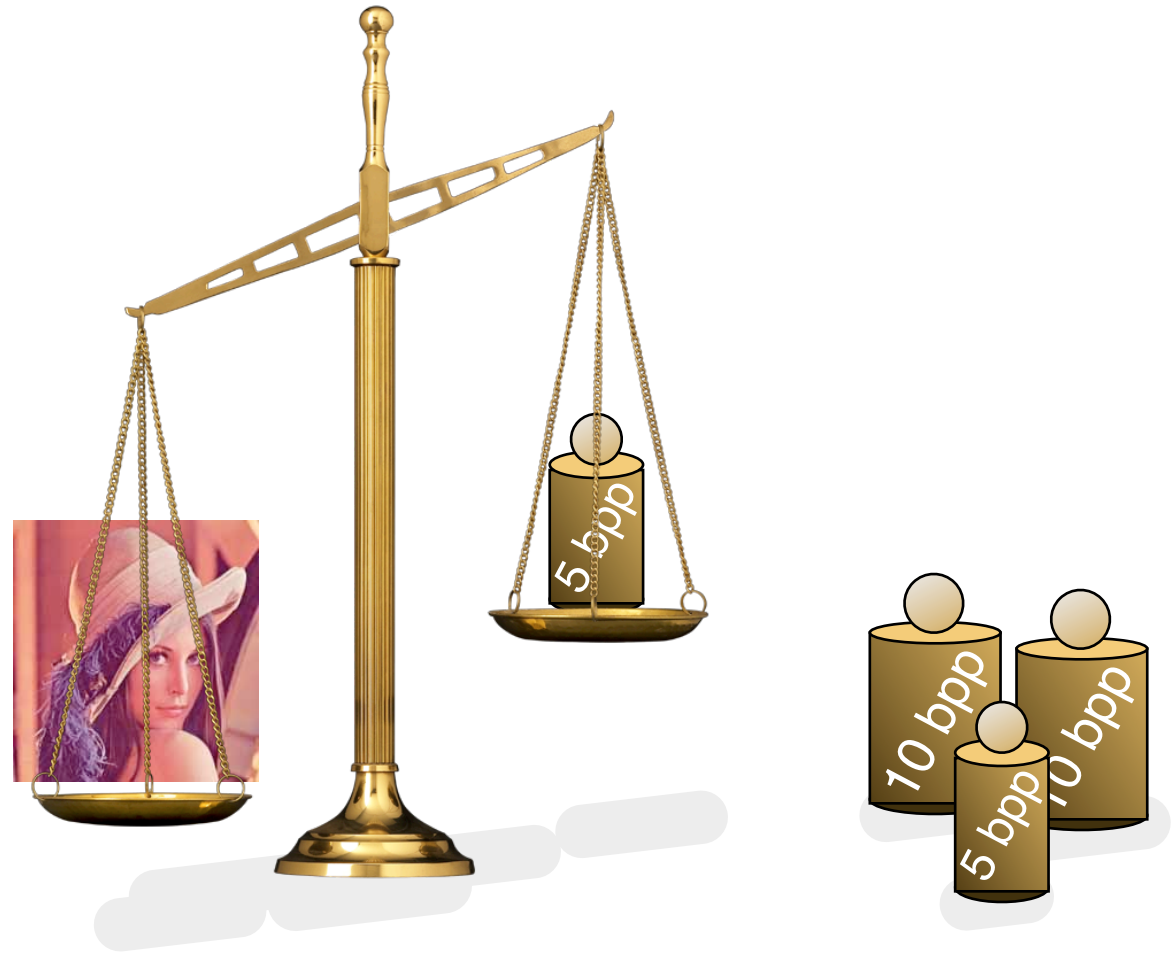
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# Recall Quantitative Steganalysis

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

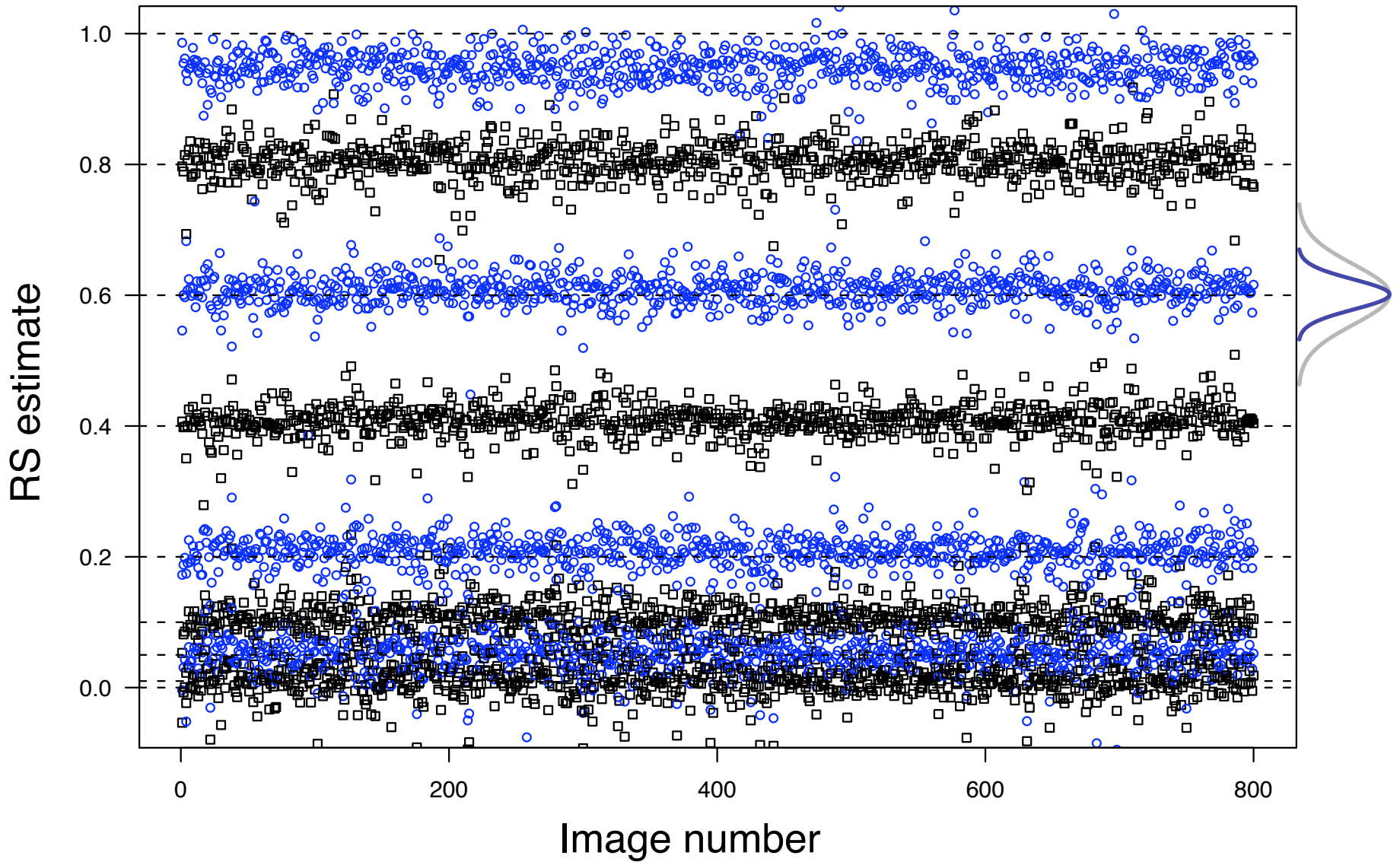


A number of different estimators has been proposed for LSB embedding.

Ker, 2004

# Typical Results from Secret Message Length Estimation

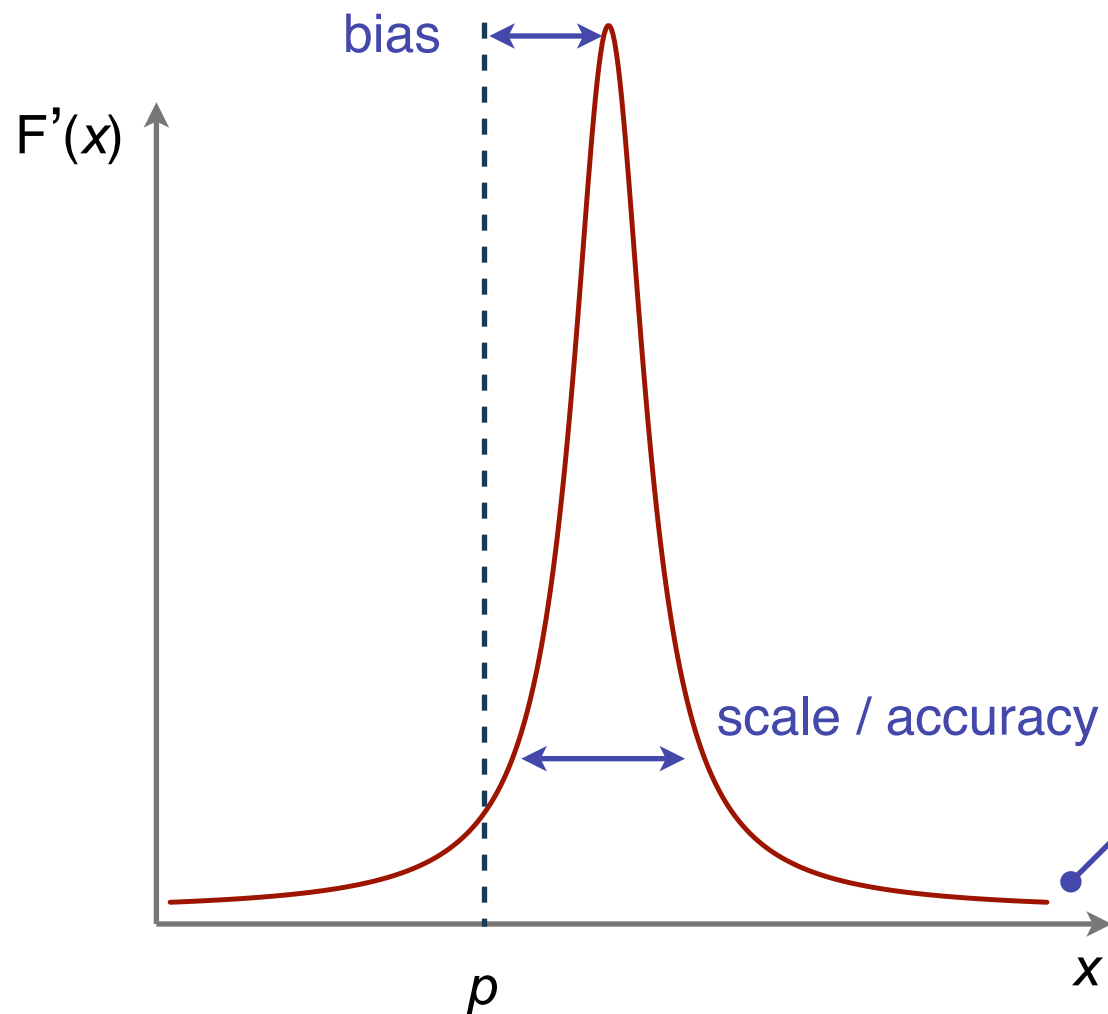
A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS



Results from 7200 attacks on 800 never-compressed grayscale images

# Error Distribution of Estimates

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS



$$\hat{p} = p + e$$

Distribution function

$$F(x) = P(e < x) = ???$$

$$1 - F(x) \sim x^{-k}$$

heavy tails

simulation results from images  
with randomly chosen message

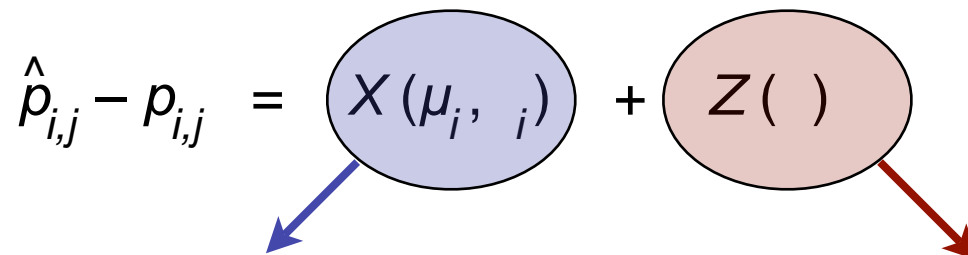
Error distribution has previously been modelled as Cauchy distribution.

Boehme, 2005

# A Two-Factor Error Model

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

$$\hat{p}_{i,j} = p_{i,j} + e$$

$$\hat{p}_{i,j} - p_{i,j} = X(\mu_i, \mu_j) + Z(\cdot)$$


Within-image error due to random correlation with the message

Between-image error due to characteristics of the image

$$e \sim D(X) \circ D(Z)$$

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## Symbols

$i$  .. cover image index  
 $j$  .. message index  
 $\hat{p}$  .. estimation result  
 $p$  .. actual embedding rate

## Random variables

$X$  .. within-image error  
 $Z$  .. between-image error  
 $D(\cdot)$  .. distribution function operator  
 $\circ$  .. convolution operator

# New Research Questions

## A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

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Shape of  $D(X)$  and  $D(Z)$



Relative magnitude



Re-examine influencing factors for error components



Similarities and differences between different quantitative steganalysis methods



*We use a large-scale experiment to explore the relationship for LSB detectors/estimators empirically.*

# Experimental Setup

## A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

RS	WS	SPA	SPA/LSM	Triples
Fridrich, Goljan & Du, 2001	Fridrich & Goljan 2004	Dumitrescu, Wu & Wang 2002	Lu, Luo, Tang & Shen, 2004	Ker, 2005

**800** .. never-compressed images 640 x 458

**200** .. secret messages per image

**5** .. detectors







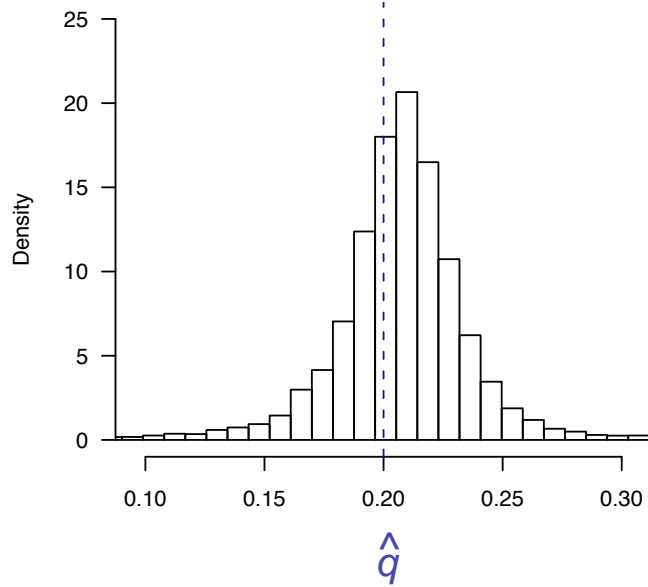




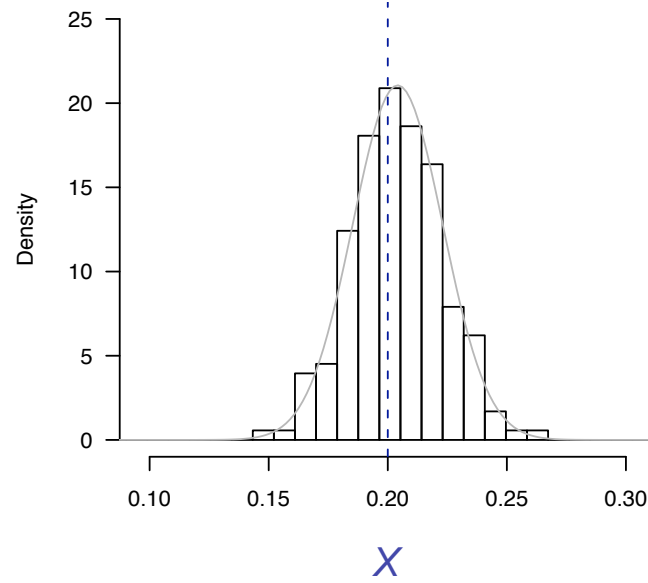
# Error Components for RS Analysis

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

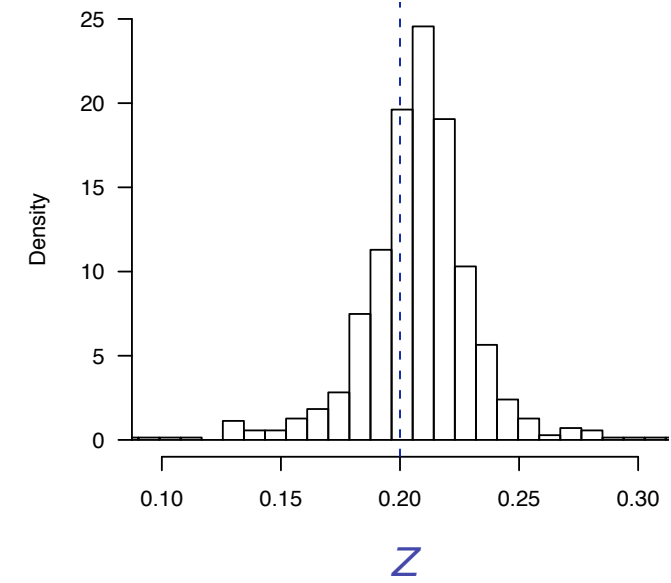
Compound estimation error



Within-image error



Between-image error



Data from 800 never-compressed grayscale images with embedding ratio  $p = 0.2$

# Shape of Within- and Between-Image Distributions

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

## Within-image error

Empirical evidence for Normality from a series of Shapiro-Wilk tests (see paper).

## Between-image error

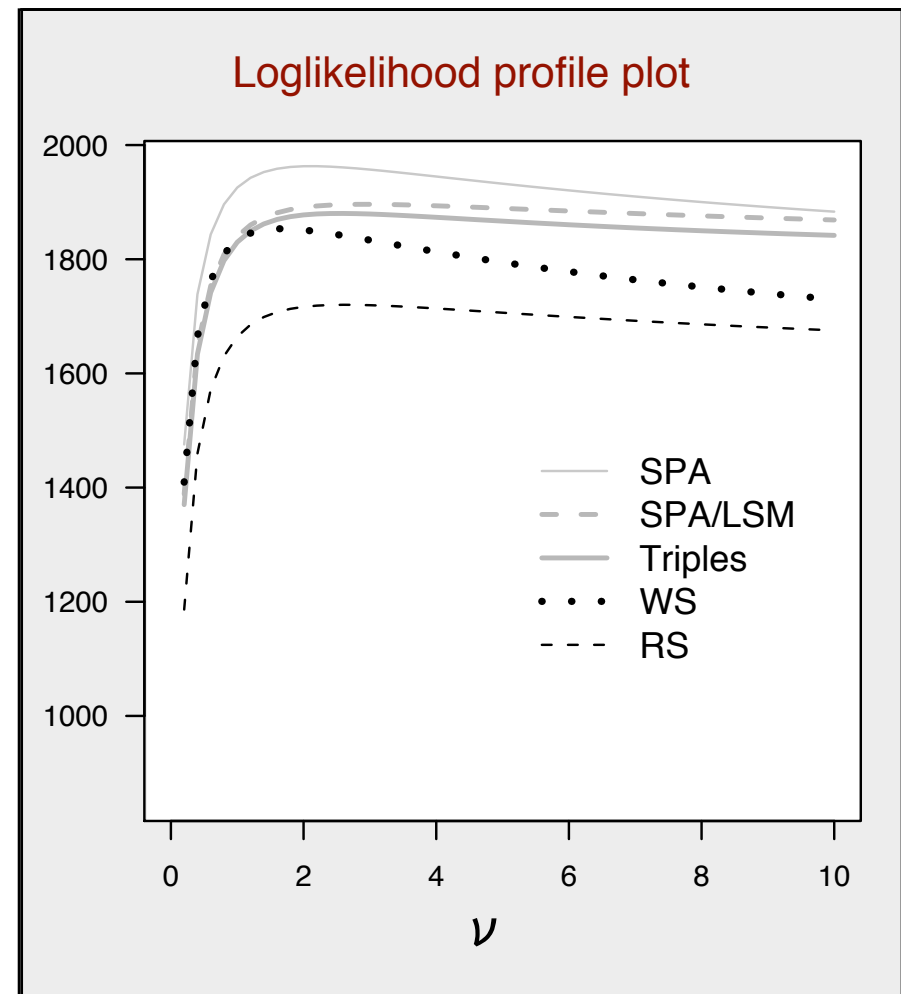
Good fit for heteroscedastic Student  $t$  distribution:

$$t_{\nu}(x, \sigma^2) = \frac{1}{\sqrt{\sigma^2}} \frac{\Gamma((1+\nu)/2)}{\Gamma(\nu/2)} \frac{\nu}{(\nu + x^2)^{(1+\nu)/2}}$$

heavy tails  
with tail index  $\nu$

.. scale parameter

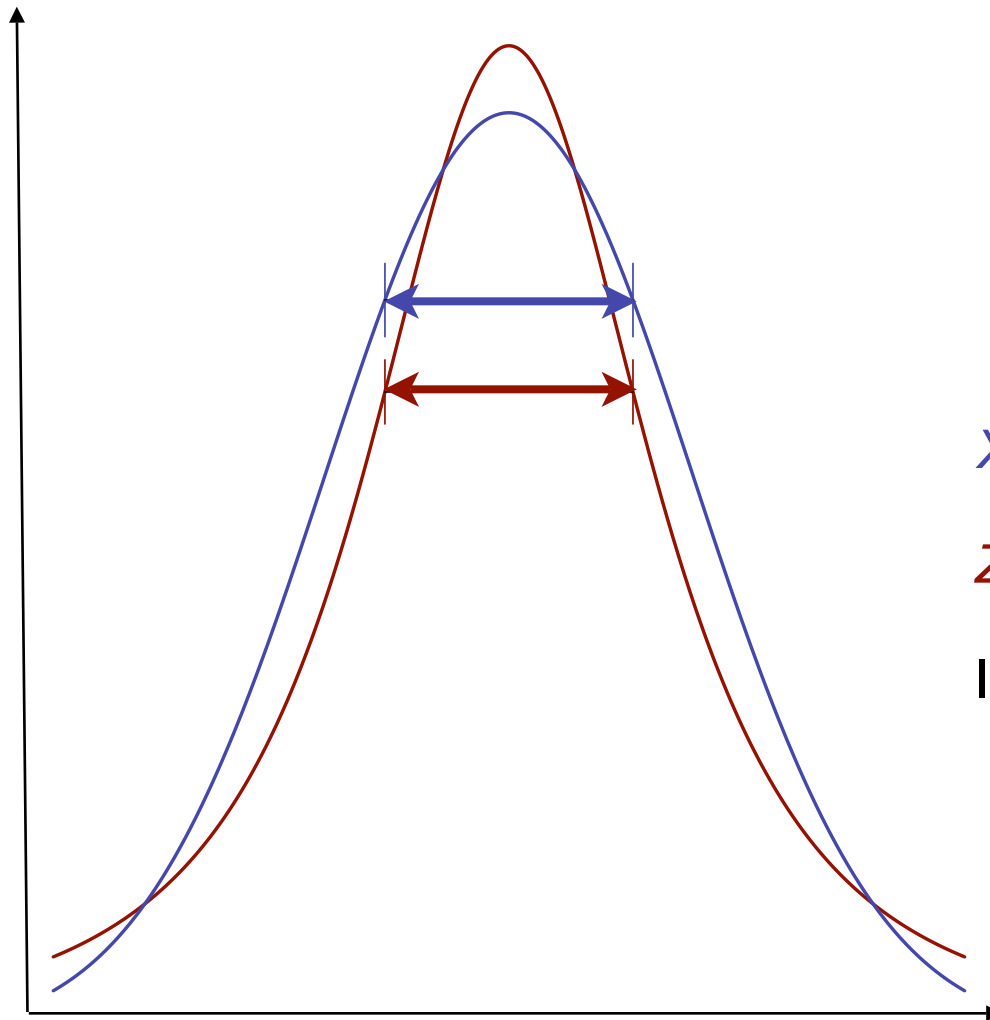
$\nu$  .. degrees of freedom parameter



# Robust Comparison of Distribution Spread

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

Inter-quartile ranges (IQR)



$$\bar{X} = 1.2105$$

$$\bar{Z} = 1, \nu = 2$$

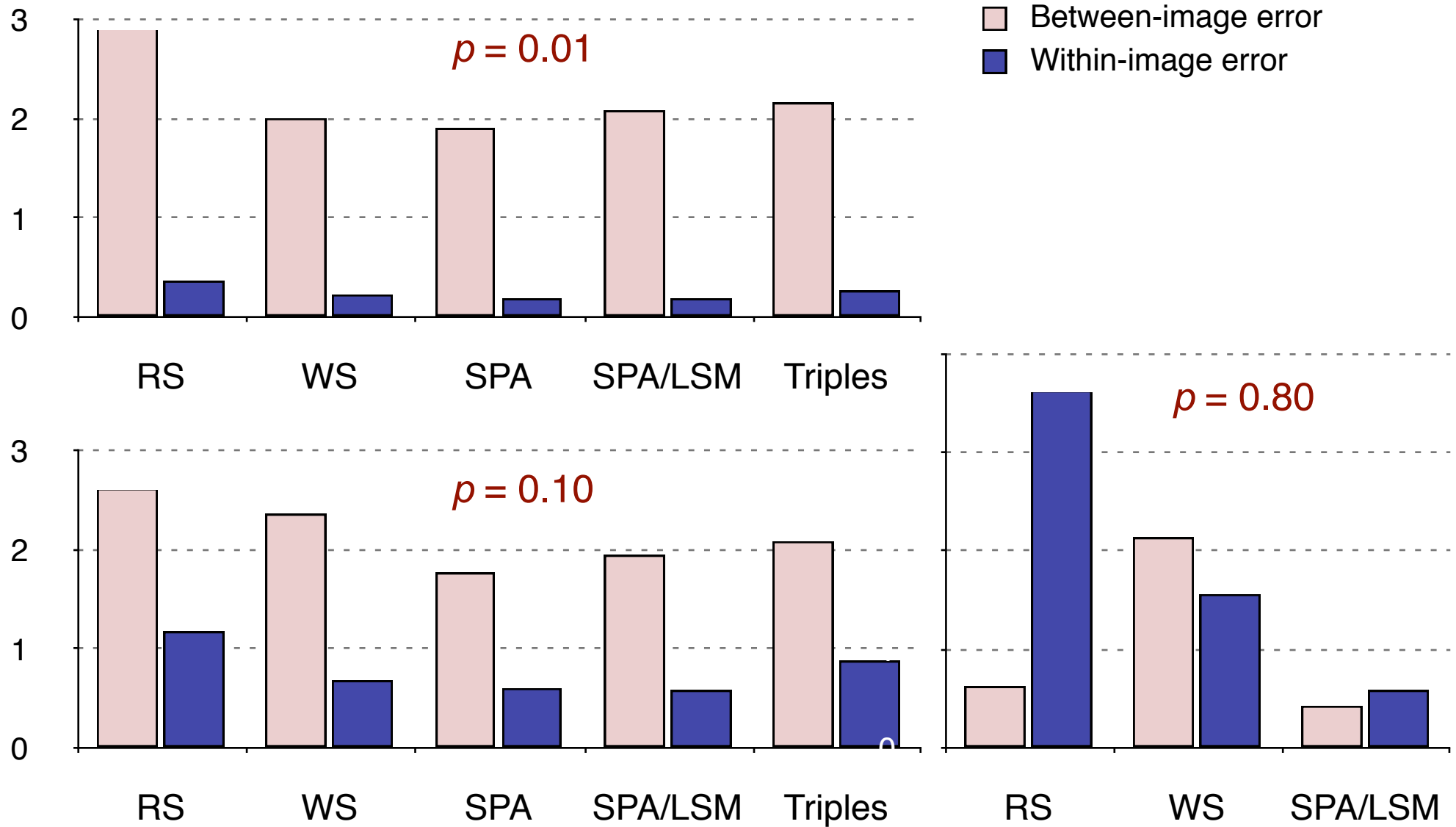
$$\text{IQR} = Q_{75} - Q_{25} = 1.633$$

Is it inappropriate to compare short- and heavy-tailed distributions with moment statistics.

# Comparison of Error Magnitudes

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

Inter-quartile ranges  $Q_{75} - Q_{25}$

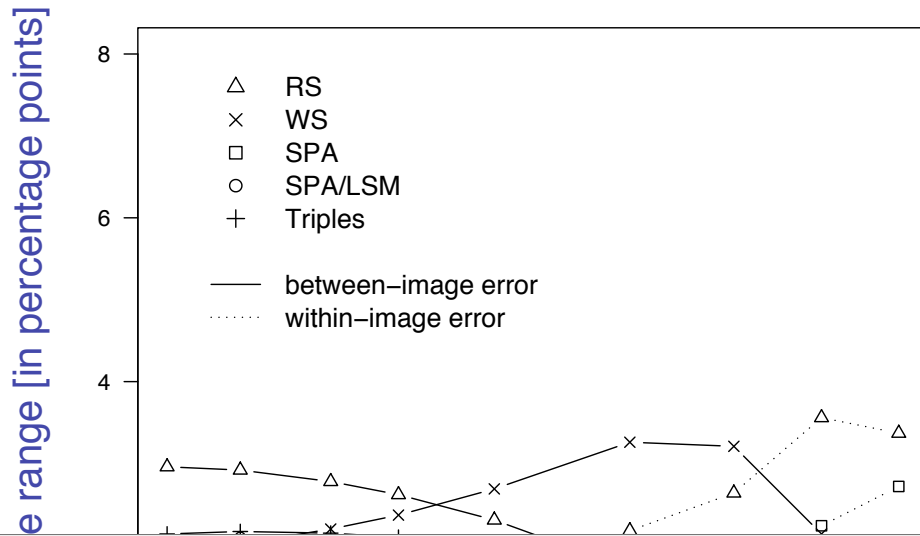


Data from 2 M attacks on 800 never-compressed grayscale images.

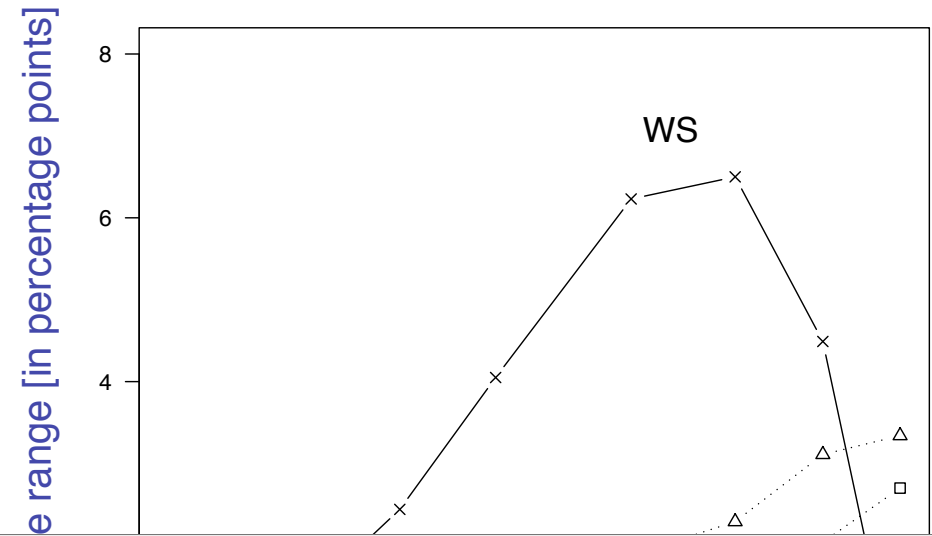
# Comparison of Error Magnitudes (cont'd)

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

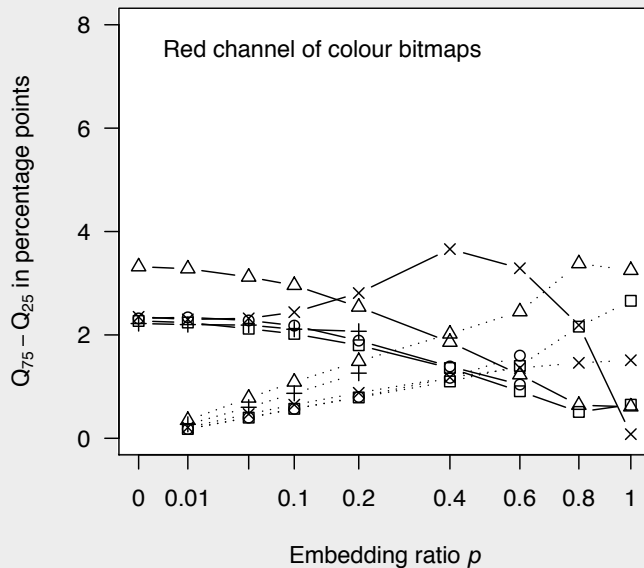
## Never-compressed grayscale images



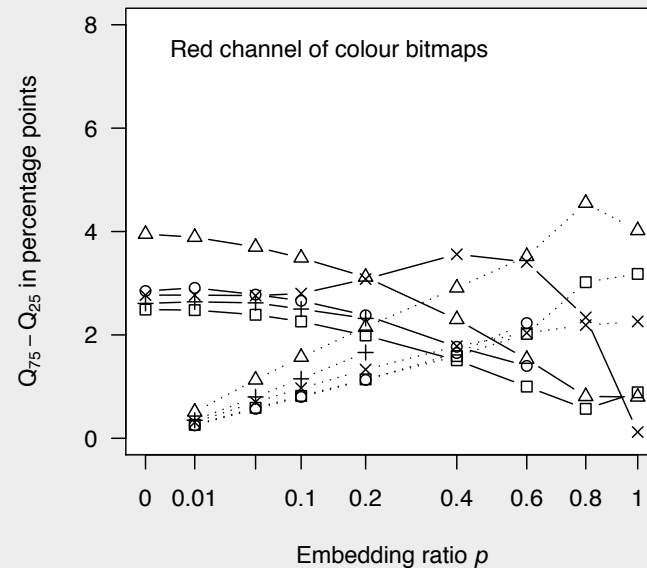
## JPEG compressed grayscale images



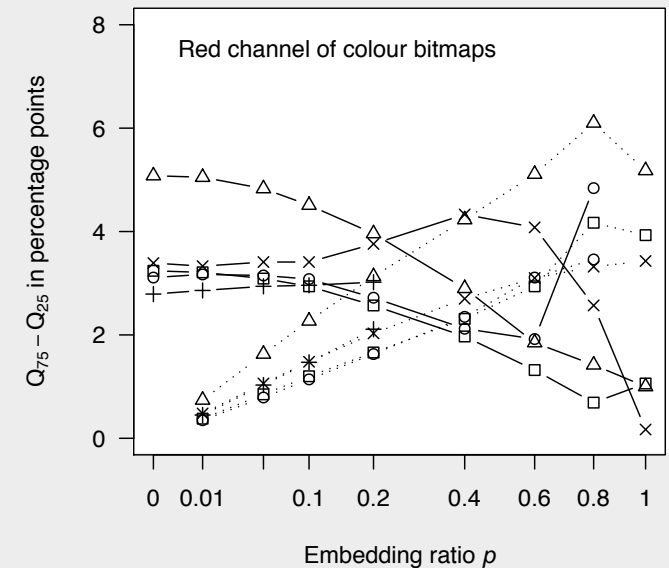
### Large images



### Medium images



### Small images





# Regression Models for Between-Image Bias

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

## Default model

Estimate detector specific constant and scale bias for between-image errors.

$$\bar{p}_i = a_0 + a_1 \cdot p + e \quad \text{with} \quad \bar{p}_i = \frac{1}{200} \sum_j \hat{p}_{i,j}$$

## Absolute bias model

Assume that an image-specific bias can be approximated from the detector result if nothing is embedded.

$$\bar{p}_i = a_1 \cdot p + a_2 \cdot \hat{p}_i^{(0)} + e$$

## Relative bias model

Test hypothesis that  $(\hat{p}_i - p) = \hat{p}_i^{(0)} + p \cdot (1 - \hat{p}_i^{(0)})$

$$\bar{p}_i = \underset{\substack{\downarrow \\ 1}}{a_1} \cdot p + \underset{\substack{\downarrow \\ 1}}{a_2} \cdot (\hat{p}_i^{(0)} - p \cdot \hat{p}_i^{(0)}) + e$$

Assumed residual distribution:  $e \sim t_2(0, \lambda)$

# Fitted Coefficients for Between-Image Bias

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

Dependent variable  $\bar{p}_i$  with predictors  $p$  and  $\hat{p}_i^{(0)}$ .

Model	Parameter	Detector				
		RS	WS	SPA	SPA/LSM	Triples
<b>Default</b>	$a_0$	0.01	0.01	0.00	0.00	0.00
	$a_1$	0.98	1.00	0.99	0.99	0.99
	Residual IQR	2.98	2.30	1.99	1.65	2.03
<b>Absolute bias</b>	$a_1$	1.01	1.01	1.00	1.00	0.99
	$a_2$	0.79	0.83	0.82	0.89	0.95
	Residual IQR	1.00	1.65	0.48	0.25	0.14
<b>Relative bias</b>	$a_1$	1.01	1.01	1.00	1.00	0.99
	$a_2$	0.99	0.99	1.00	1.00	1.00
	Residual IQR	0.49	1.55	0.06	0.06	0.08

All coefficients significant on the  $t$ -test. **84%** **33%** **97%** **96%** **96%**  
 Data from 27 k attacks on never-compressed grayscale images.

Data fitted with heteroscedastic  $t$  regression methods. (see Taylor & Verbyla, 2005)

# Correlation of Image-Specific Bias

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

Correlation coefficients of  $\hat{\rho}_i^{(0)}$  between detectors over images.

	Detector				
	RS	WS	SPA	SPA/LSM	Triples
RS	1.00				
WS	0.64	1.00			
SPA	0.76	0.86	1.00		
SPA/LSM	0.60	0.86	0.89	1.00	
Triples	0.45	0.66	0.64	0.70	1.00

Data from 4000 attacks on never-compressed grayscale images.

Correlation coefficients for JPEG images are somewhat lower.

All correlation matrices have only one eigenvalue larger than 1.

Correlation coefficients estimated with the  $t$ -copula method.

(see Dematra & McNeil, 2005)

# Regression Models for Influencing Factors

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

## Magnitude of within-image error

Ordinary least squares regression to data with  $p > 0 = \text{const}$

$$\log \hat{\sigma}_i = b_0 + b_1 \cdot x + e \quad e \sim N(0, \sigma_e)$$

## Magnitude of between-image error

Maximum likelihood fit for heteroscedastic  $t$  regression:

$$\hat{\rho}_i^{(0)} = a_0 + \quad \sim t_2(0, \lambda)$$

$$\log \lambda^2 = b_0 + b_1 \cdot x + e \quad e \sim t_2(0, \cdot)$$

## Image-specific bias


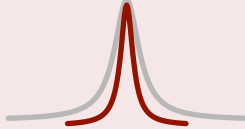
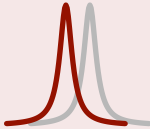
Maximum likelihood fit for heteroscedastic  $t$  regression:

$$\hat{\rho}_i^{(0)} = a_0 + a_1 \cdot x + \quad \sim t_2(0, \lambda)$$

$$\log \lambda^2 = b_0 + e \quad e \sim t_2(0, \cdot)$$

# Result Summary: Factors Influencing Detection Accuracy

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

	Within-image error	Between-image error	
Predictor	 dispersion	 dispersion	 bias
<b>Embedding ratio</b>	increases for all detectors	reduces for all detectors but WS	reduces for all detectors
<b>Local variance</b>	increases for all detectors R <sup>2</sup> : 12 (Triples) – 39% (RS)	increases for all detectors R <sup>2</sup> : 1 (WS) – 3% (RS)	no direct effects
<b>Saturation</b> .. % at hist. ends	reduces for all detectors R <sup>2</sup> : 4 (WS) – 19% (Triples)	increases all but Triples R <sup>2</sup> : 1 (RS) – 2% (SPA)	under-estimation for all detectors R <sup>2</sup> : 1 (Triples) – 6% (RS)
.. % at hist. mode	reduces for all detectors R <sup>2</sup> : 15 (WS) - 36% (Triples)	increases for all detectors R <sup>2</sup> : 3 (RS) – 8% (WS)	under-estimation for all detectors R <sup>2</sup> : 4 (Triples) – 7% (RS)

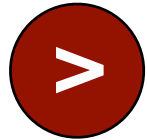
Results for local variance and saturation are estimated on data with  $p = 0.05$  bpp.

# Concluding Remarks

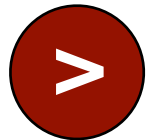
## A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

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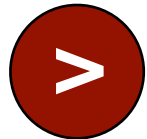
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Rationale and evidence for (at least) two error components in quantitative LSB steganalysis



Separation of components allows for more prudent analysis of the sources for estimation errors



Don't ignore within-image errors, and don't benchmark stego-estimators with moment statistics.



*Thanks for your attention!*

# Q&A



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# Towards Tailored Steganalysis

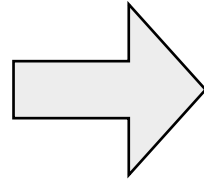
A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

## Steganalysis

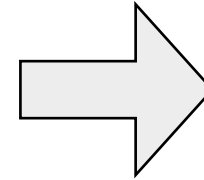
Suspect object



Extract parameters



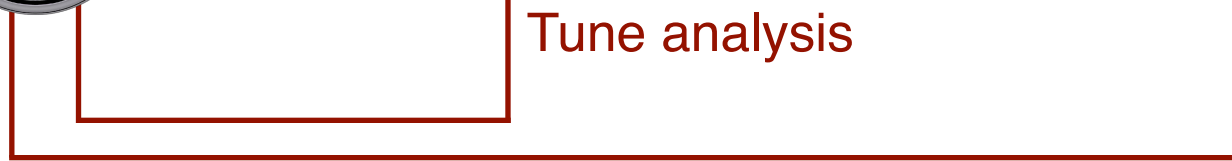
Tune analysis



Decision



Adjust criteria

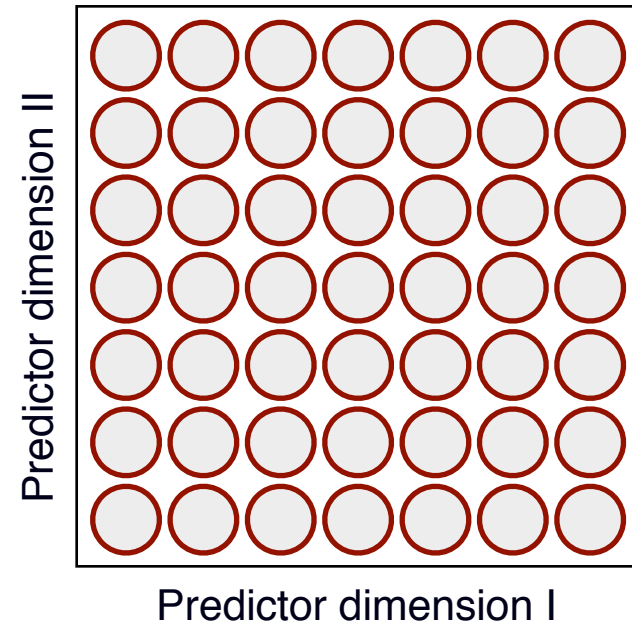




# Appendix: Notes on Test Data Structure

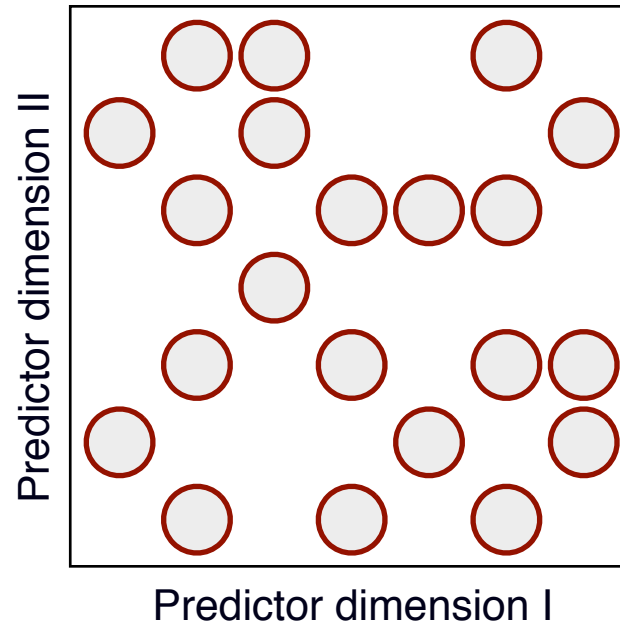
A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

## Exhaustive tests



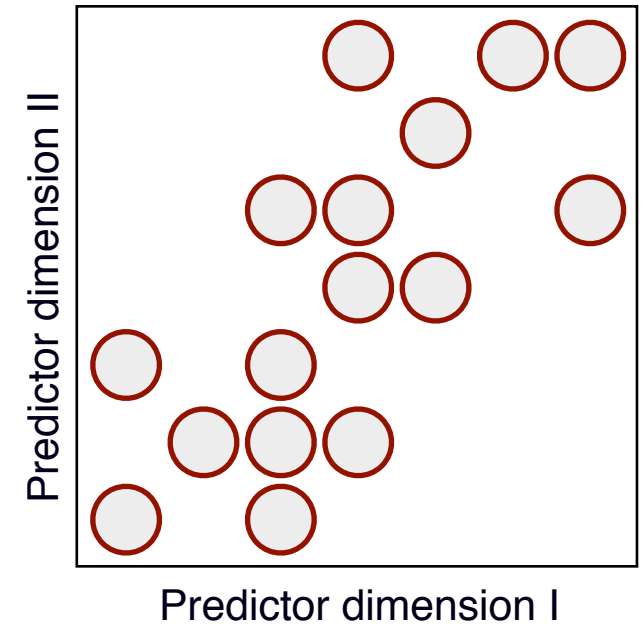
No model required since all breakdowns can be tabulated

## Typical case



Full range of all predictor dimensions covered

## Confounded predictors



Identification of individual influence from dependent predictors is error-prone

# Structure of the Talk

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

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1

## Quantitative Steganalysis

RS Analysis · WS Analysis · Error Distribution

2

## Methodology

Linear and Nonlinear Regression · Example Models

3

## Influence of Image Properties

Image Size · Macro Characteristics

4

## Concluding Remarks

Outlook · Limitations · Summary

# 1

## **Quantitative Steganalysis**

RS Analysis · WS Analysis · Error Distribution

# Recall: Regular-Singular Analysis (RS)

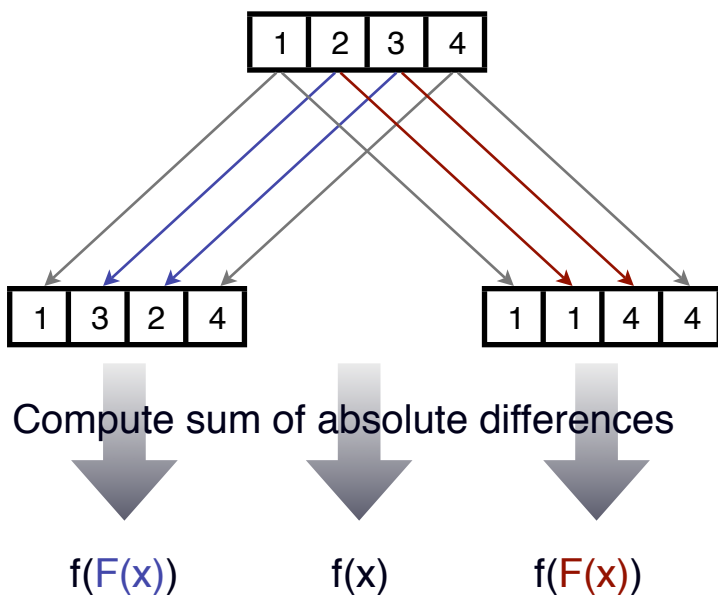
A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS

Define two flipping functions

$$F_{+1}: (0 \leftrightarrow 1, 2 \leftrightarrow 3, \dots)$$

$$F_{-1}: (1 \leftrightarrow 2, 3 \leftrightarrow 4, \dots)$$

Cut image in groups of pixels, compute dual statistics

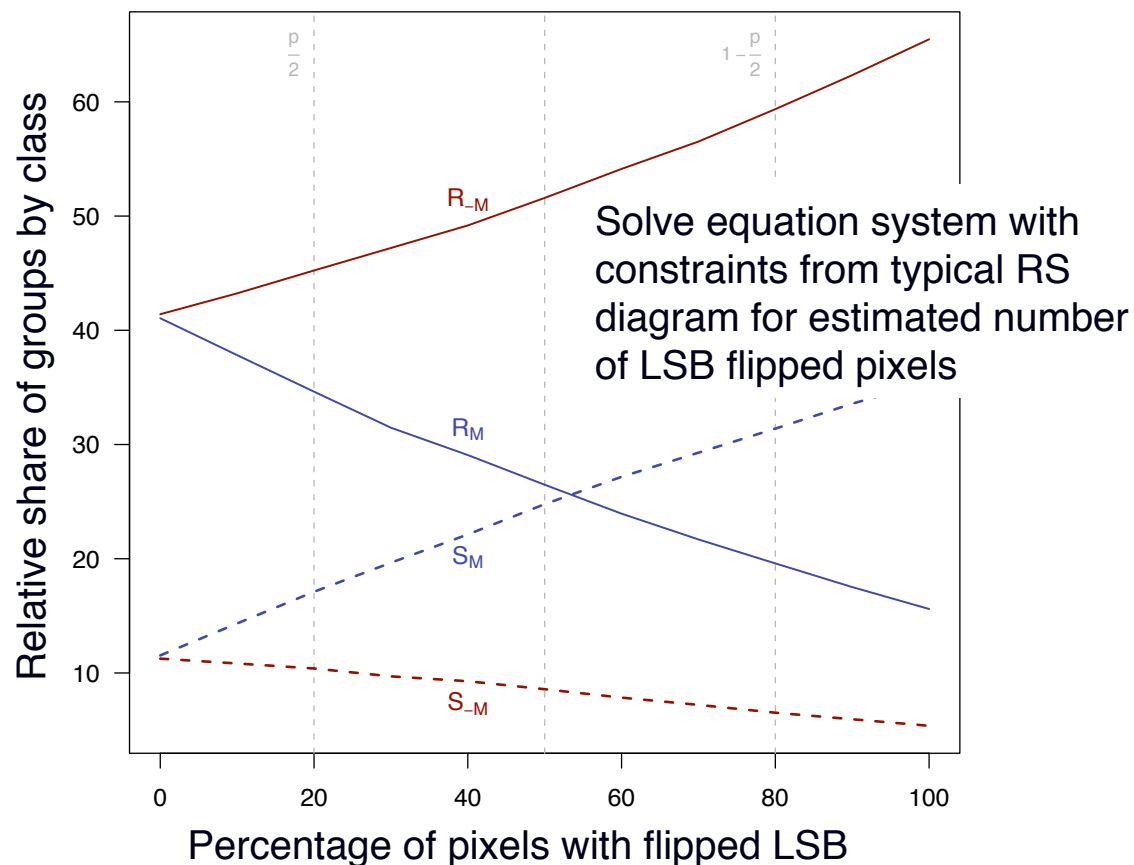


Classify groups per inequation

- $\vdots > \vdots < \vdots \Rightarrow$  Count as **regular group**  $R_M$  and  $R_{-M}$ , respectively.
- $\vdots < \vdots > \vdots \Rightarrow$  Count as **singular group**  $S_M$  and  $S_{-M}$ , respectively.

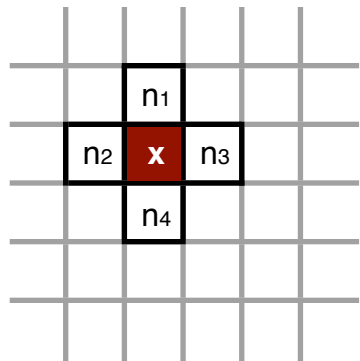
Empirical RS diagram:

relation between embedding ratio and share of groups



# Recall: Weighted Stego Image Analysis (WS)

A TWO-FACTOR ERROR MODEL FOR QUANTITATIVE STEGANALYSIS



## Estimate cover image

as arithmetic average  $\mu(x)$  of the four closest neighbors of pixel  $x$  in the stego image.

## Weight influence of pixels

since the accuracy of cover image estimation varies with local variance  $v(x)$ .

## Infer secret message length

from the difference of the observed stego image and an the estimated cover image

$$\hat{q} = - \frac{2}{\sum v(x)} \sum_{x \in X} \frac{(F(x) - x) \cdot (x - \mu(x))}{1 + v(x)^\alpha}$$

$q$  : estimated embedding ratio

$\alpha$  : a constant used to control the influence of weighting

$F$  : LSB flipping function