Provably efficient algorithms for Hybrid Systems

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Models of complex systems

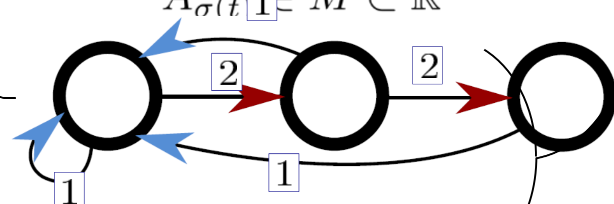
Modelling power

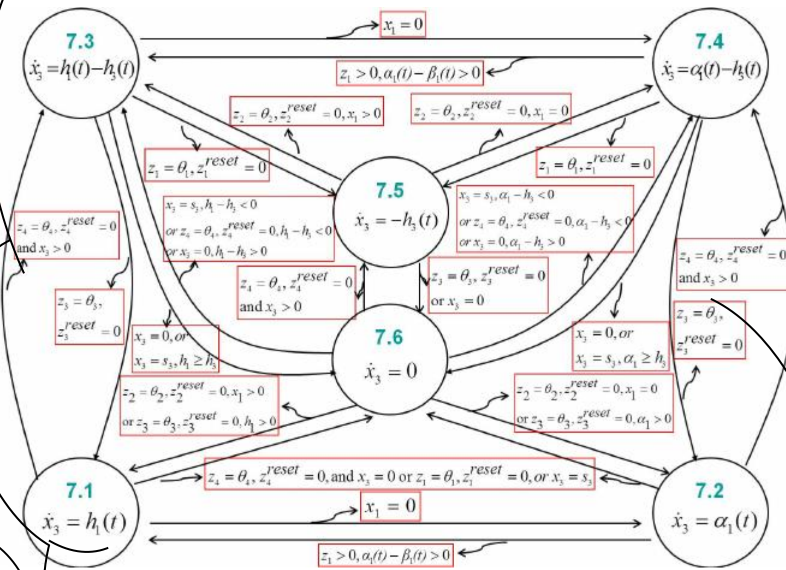
Solvability/efficiency

$$RI(t) + L\frac{dI(t)}{dt} + V(0) + \frac{1}{C}\int_0^t I(\tau)\,d\tau = V(t)$$

$$x(k+1) = \begin{cases} A_0 x(k) \\ A_1 x(k) \end{cases}$$

$$x_{t+1} = A_{\sigma(t)} x_t$$
$$x_0 \in \mathbb{R}^n$$
$$\sigma(0), \sigma(1), \cdots \in G$$
$$A_{\sigma(t)} \in M \subset \mathbb{R}^{n \times n}$$



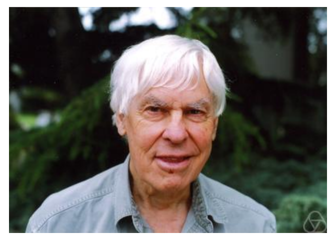


Models of complex systems

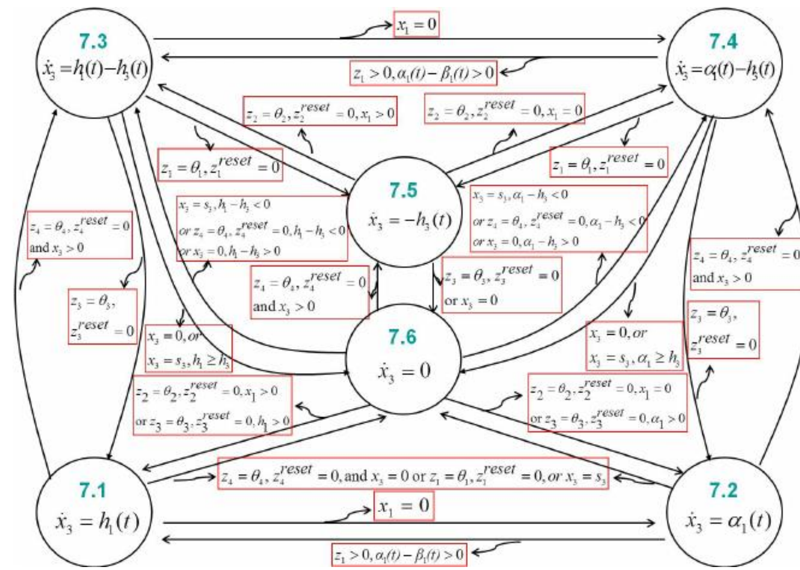
Modelling power

Solvability/efficiency

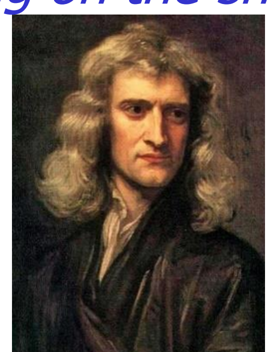
Thoralf Skolem
1887-1963



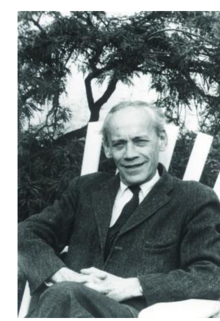
Stephen Smale
1930-
Fields Medal 1966
Wolf prize 2007



If I have seen further it is by standing on the shoulders of Giants.



Giancarlo Rota
1932-1999



Emil Artin
1898-1962

Outline

- Two motivations
  - Consensus
  - Wireless control networks
- Three techniques
  - Observability/controllability of hybrid systems
  - Guaranteed accuracy for stability analysis
  - Data-driven/blackbox control
- Discussion

Outline

- **Two motivations**
  - **Consensus**
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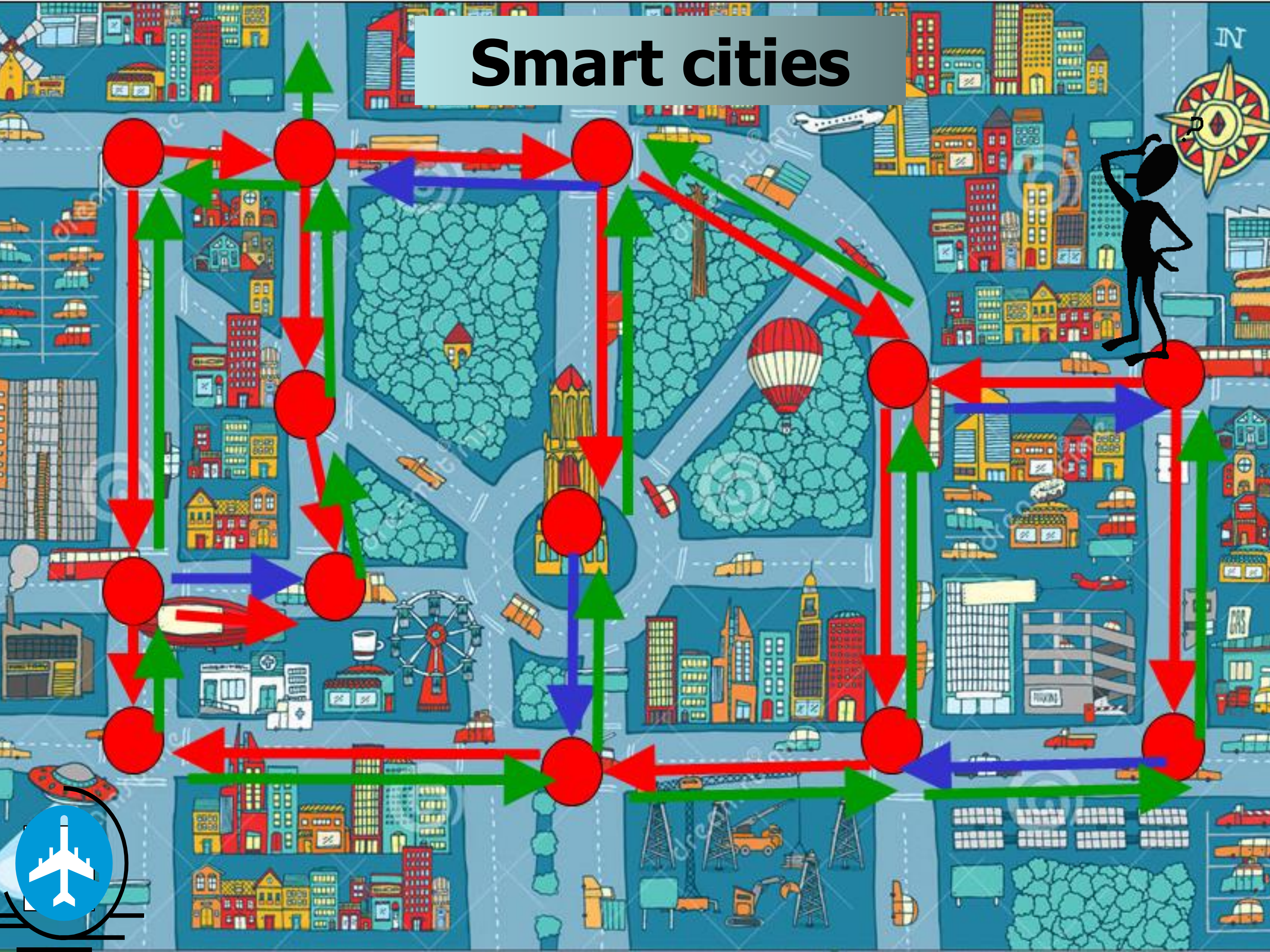
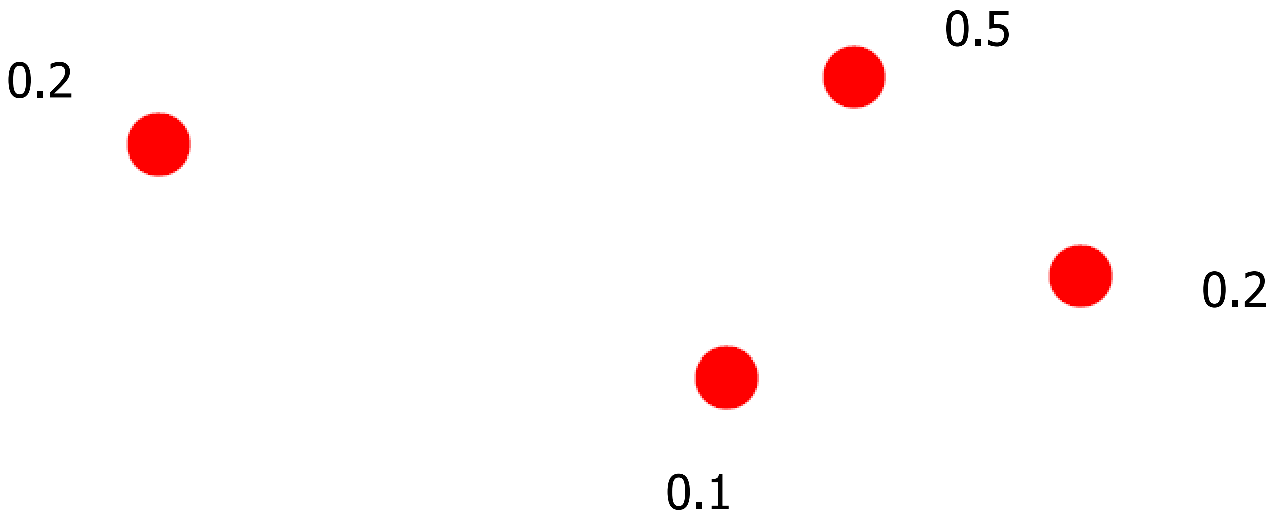
Smart cities



Autonomous robots

Gossip algorithms



S. Boyd, A. Ghosh, B. Prabhakar, D. Shah
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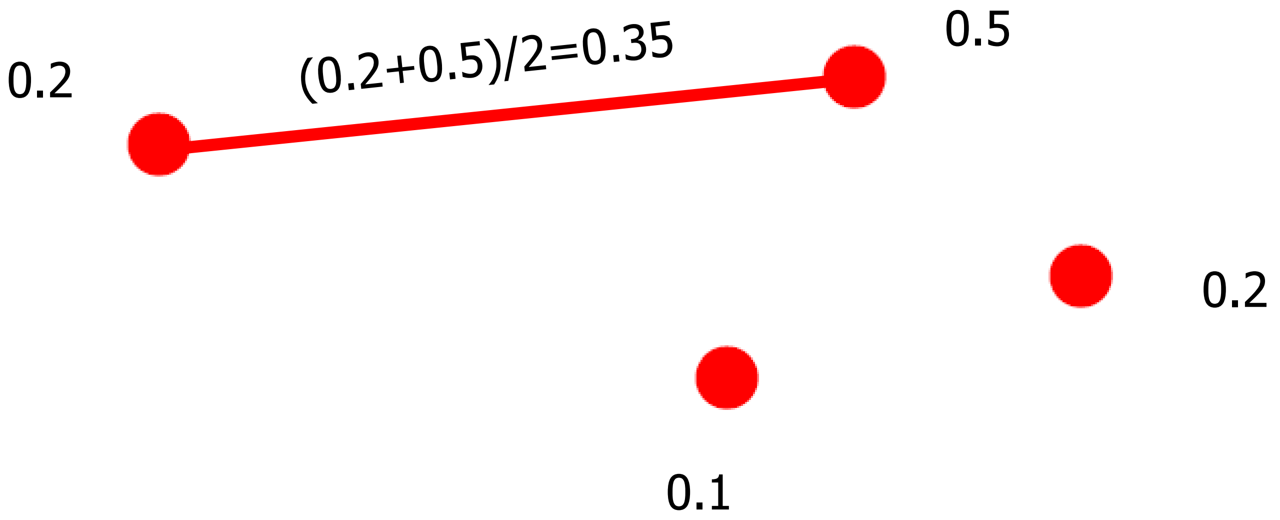
Gossip algorithms



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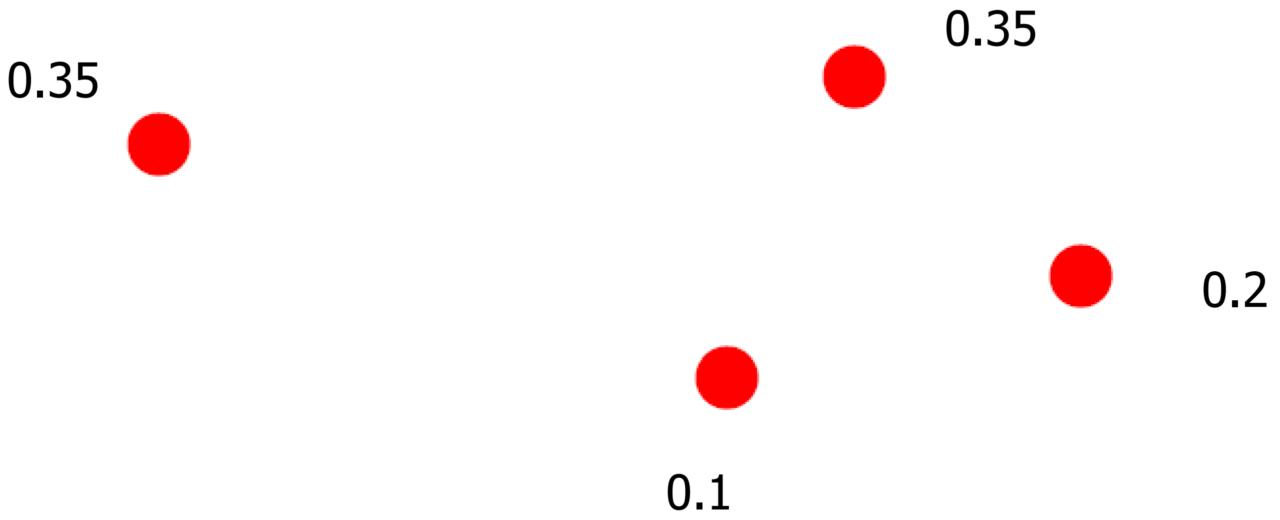

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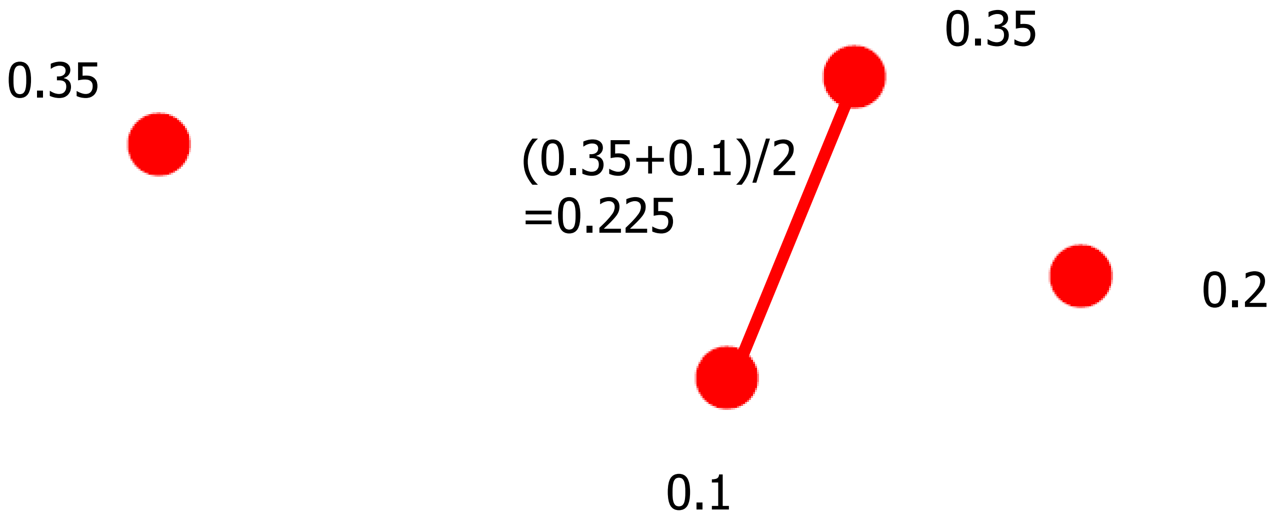

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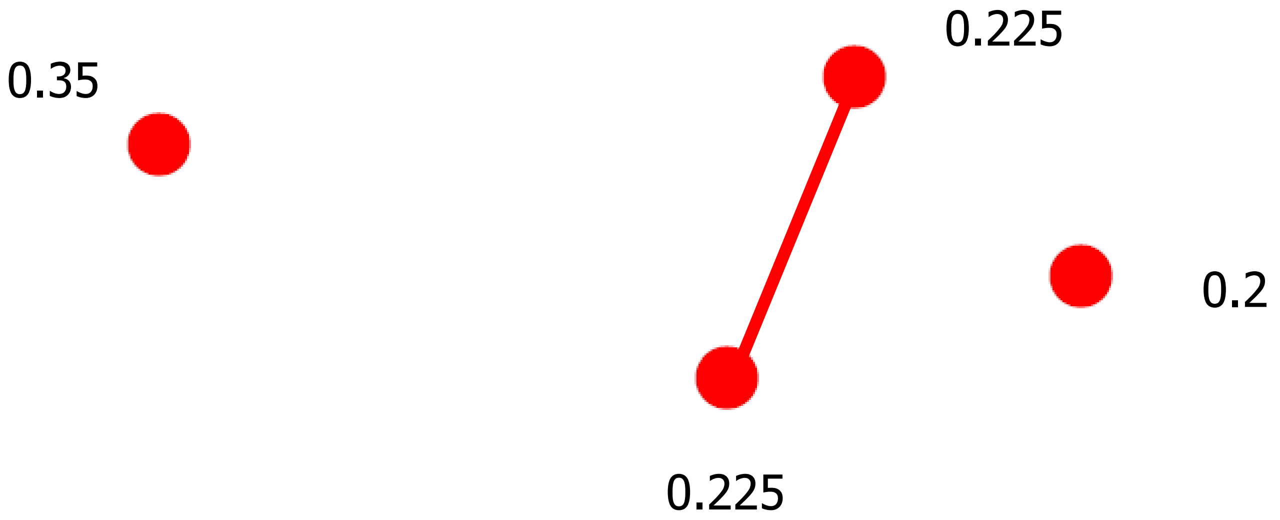

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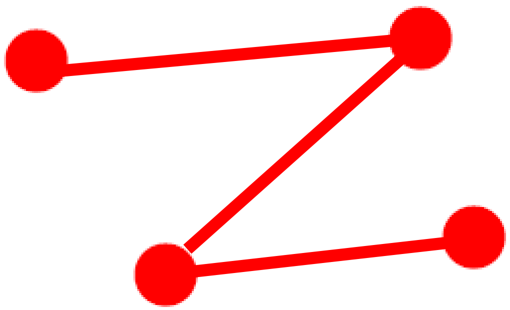
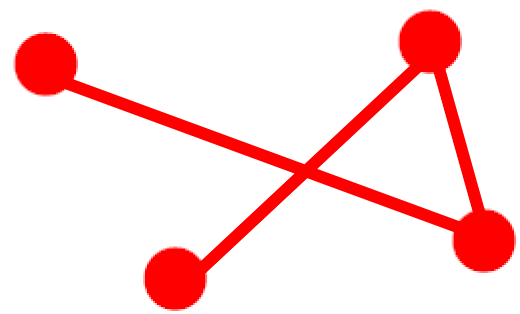
Consensus of multi-agent systems

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \leftarrow \begin{pmatrix} 0.5 & 0.5 & . & 0 \\ 0.5 & 0.5 & . & 0 \\ . & . & . & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

Our setting: We are given a set of stochastic matrices, representing different connectivity topologies



$$\boxed{x(k+1) = \begin{cases} A_0 x(k) \\ A_1 x(k) \end{cases}}$$



$$A_0 = \begin{pmatrix} 0.5 & 0.5 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \\ 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0.5 & 0.5 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 0.5 & 0 & 0 & 0.5 \\ 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0.5 & 0.5 & 0 \\ 1/3 & 1/3 & 0 & 1/3 \end{pmatrix}$$

Problem: Do all products of these matrices converge to consensus?
(that is, all rows are equal)

This is actually a stability problem

- Property: $A\mathbf{1} = \mathbf{1}$ with $\mathbf{1} = (1, 1, \ldots, 1)$

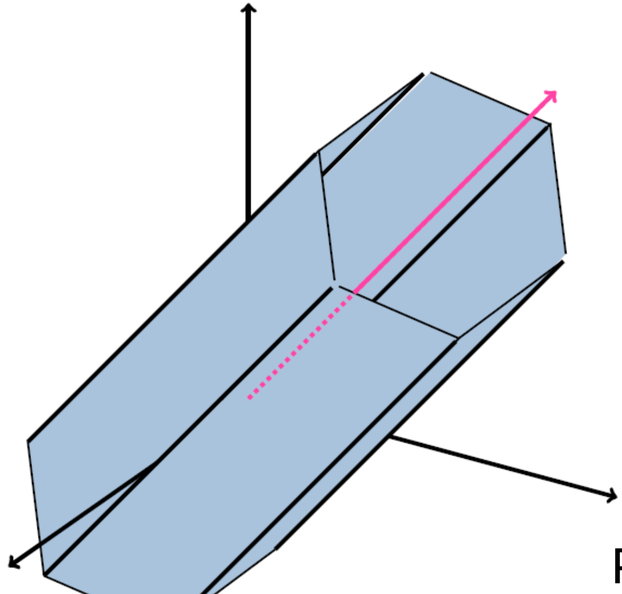
➔ Any consensus state is an equilibrium

- Proposition [Jadbabaie 03] : After projection along the (1,1,…,1) vector, convergence to consensus becomes convergence to zero

$$x(k+1) = \begin{cases} A_0 x(k) \\ A_1 x(k) \end{cases}$$

Do all the products converge to zero?

Paz, Wolfovitz, Blondel-Olshevsky, Chevalier,..



P-Y Chevalier

Switching systems

$$x(k+1) = \begin{cases} A_0 x(k) \\ A_1 x(k) \end{cases}$$

Point-to-point Given x_0 and x_*, is there a product (say, A_0 A_0 A_1 A_0 … A_1) for which $x_*=A_0$ A_0 A_1 A_0 … A_1 x_0?

Mortality Is there a product that gives the zero matrix?

Boundedness Is the set of all products $\{A_0, A_1, A_0A_0, A_0A_1, \ldots\}$ bounded?

Switching systems

$$x(k+1) = \begin{cases} A_0 x(k) \\ A_1 x(k) \end{cases}$$

Global convergence to the origin Do all products of the type $A_0\ A_0\ A_1\ A_0 \ldots A_1$ converge to zero? (GUAS)

The spectral radius of a matrix A controls the growth or decay of powers of A

$$\rho(A) = \lim_{t\to\infty} ||A^t||^{1/t}$$

The powers of A converge to zero iff $\rho(A) < 1$

The joint spectral radius of a set of matrices Σ is given by

$$\rho(\Sigma) = \lim_{t\to\infty} \max_{A_i \in \Sigma} ||A_1 A_2 \ldots A_t||^{1/t}$$

All products of matrices in Σ converge to zero iff $\rho(\Sigma) < 1$

[Rota, Strang, 1960]

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  - **Wireless control networks**
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Wireless Control Networks

Industrial automation

Physical Security and Control

Supply Chain and Asset Management

Environmental Monitoring, Disaster Recovery and Preventive Conservation

Wireless Control Networks

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 55, NO. 8, AUGUST 2010 1781

Networked Control Systems With Communication Constraints: Tradeoffs Between Transmission Intervals, Delays and Performance

W. P. Maurice H. Heemels, *Member, IEEE*, Andrew R. Teel, *Fellow, IEEE*, Nathan van de Wouw, *Member, IEEE*, and Dragan Nešić, *Fellow, IEEE*

Roughly speaking, the network-induced imperfections and constraints can be categorized in five types:

- (i) Quantization errors in the signals transmitted over the network due to the finite word length of the packets;
- (ii) Packet dropouts caused by the unreliability of the network;
- (iii) Variable sampling/transmission intervals;
- (iv) Variable communication delays;
- (v) Communication constraints caused by the sharing of the network by multiple nodes and the fact that only one node is allowed to transmit its packet per transmission.

Wireless Control Networks

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[Jungers D'Innocenzo Di Benedetto, TAC 2015]

Wireless Control Networks

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[Jungers Kundu Heemels, 2016]

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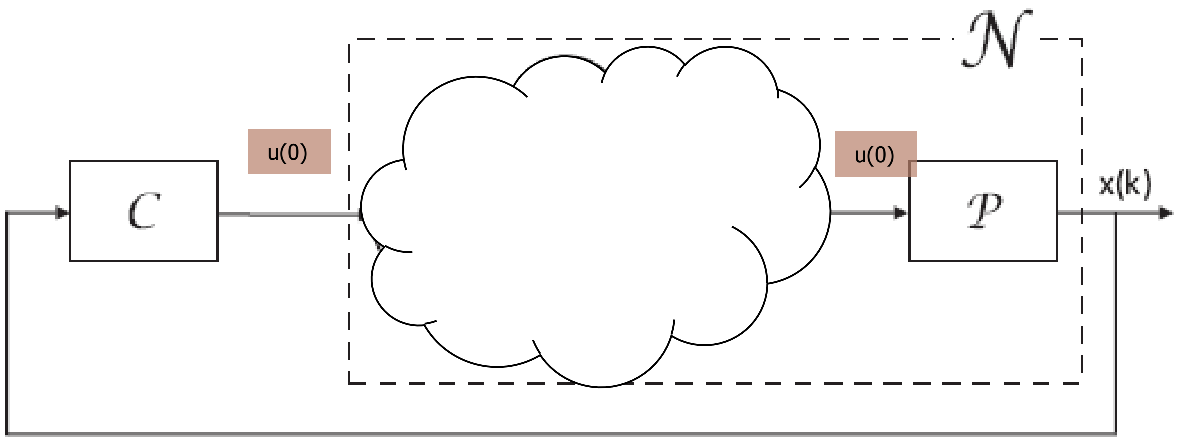
Controllability with packet dropouts

Wireless Control Networks are subject to packet dropouts

$$x(1) = Ax(0) + Bu(0)$$

$\boxed{\sigma = 1001\ldots}$

$\sigma(0) = 1$



A data loss signal determines the packet dropouts $\sigma(t) =$ 1 or 0

...this is a switching system!

$$x(t+1) = \begin{cases} Ax(t) + bu(t), & \text{if } \sigma(t) = 1, \\ Ax(t), & \text{if } \sigma(t) = 0 \end{cases}$$

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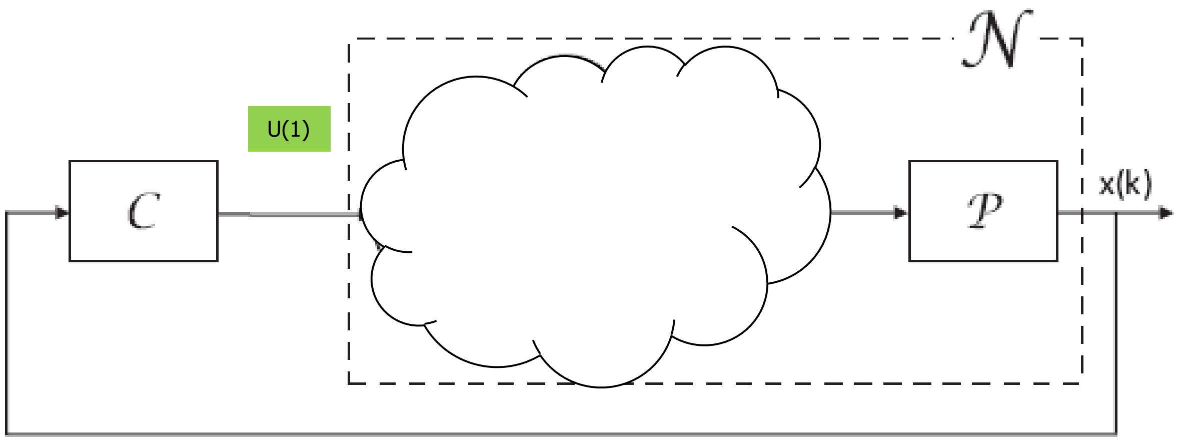

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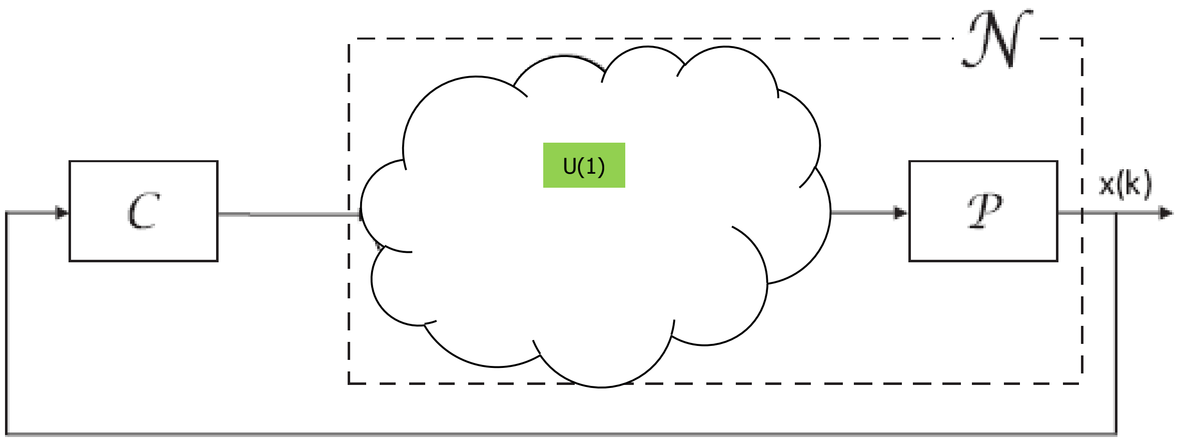
$$x(2) = A^2x(0) + ABu(0)$$



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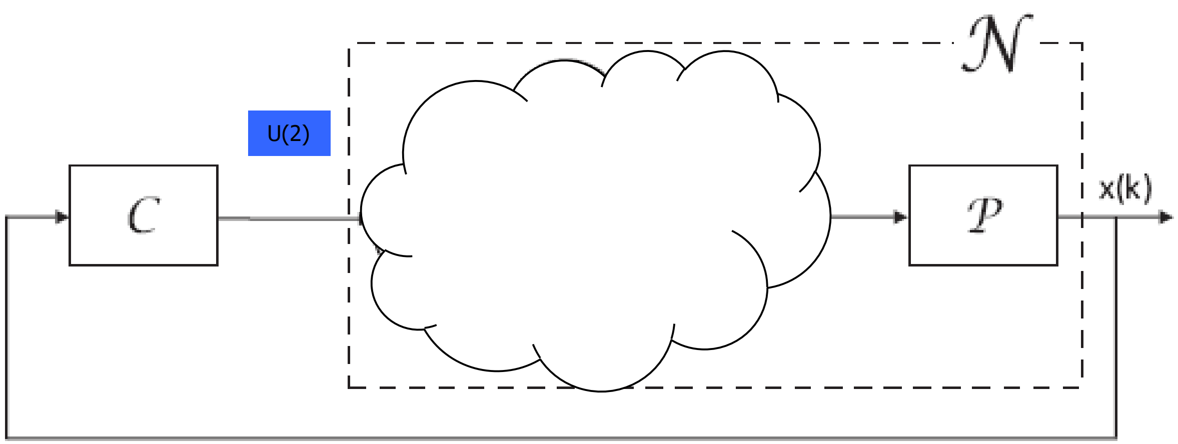

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Controllability with packet dropouts

Wireless Control Networks are subject to packet dropouts

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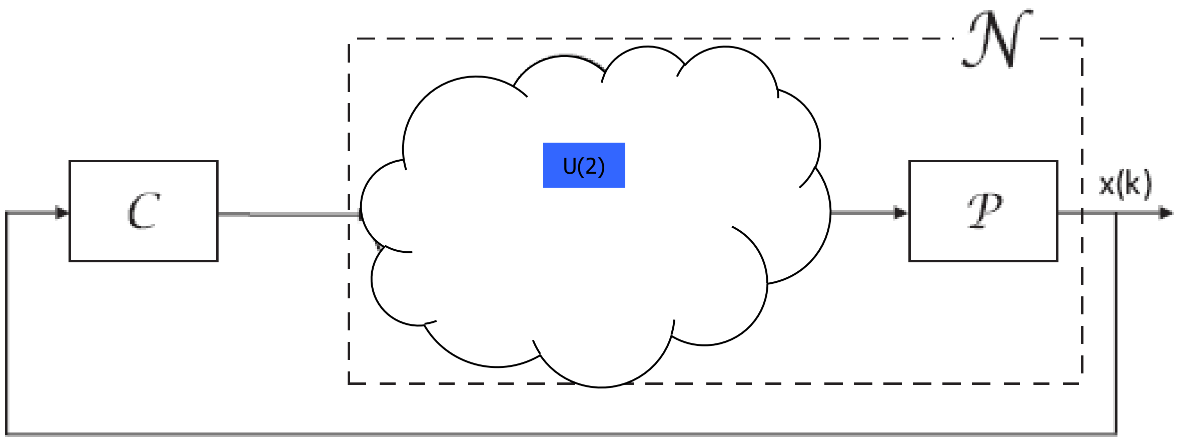

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Controllability with packet dropouts

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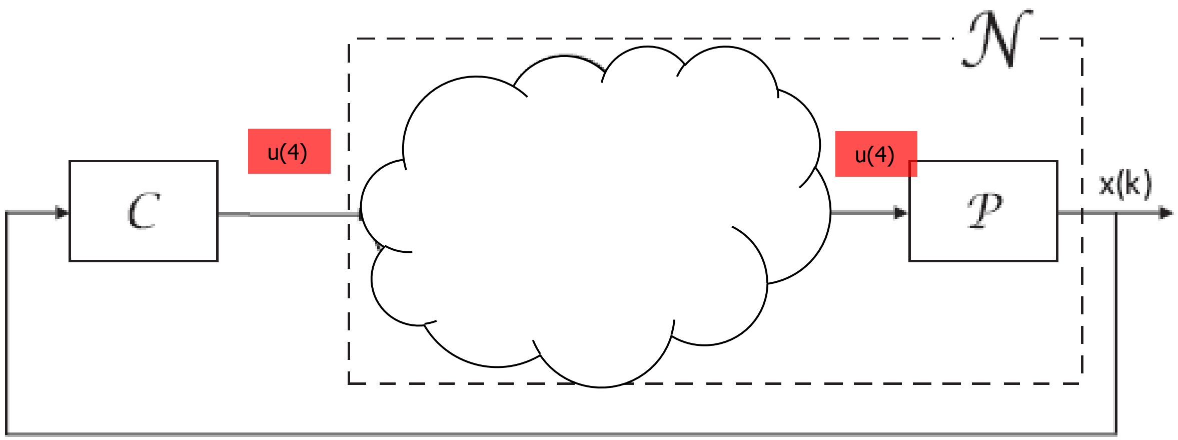

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The switching signal

We are interested in the controllability of such a system

$$\boxed{\sigma = 1001\ldots}$$

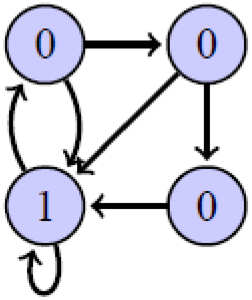
$$\sigma(0) = 1 \quad \sigma(1) = 0 \quad \sigma(2) = 0$$

$$\begin{aligned}
x(1) &= Ax(0) + Bu(0) \\
x(2) &= A^2x(0) + ABu(0) \\
x(3) &= A^3x(0) + A^2Bu(0) \\
x(4) &= A^4x(0) + A^3Bu(0) + Bu(3)
\end{aligned}$$

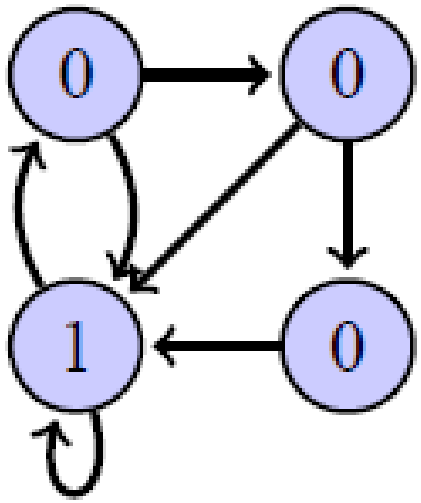
Of course we need an assumption on the switching signal

The switching signal is constrained by an automaton

Example:



Constrained switching systems
A more general model



The **dynamics** is subject to **switchings**:

$$x_{t+1} = A_{\sigma(t)} x_t + B_{\sigma(t)} u(t)$$
$$x_0 \in \mathbb{R}^n$$
$$\sigma(0), \sigma(1), \cdots \in \mathcal{L}(\theta)$$
$$A_{\sigma(t)} \in M \subset \mathbb{R}^{n \times n}$$

The **switching sequence** is **constrained** by a **graph** (AKA an automaton)

Many natural applications

-Communication networks
-Markov Chains
-Supervisory control

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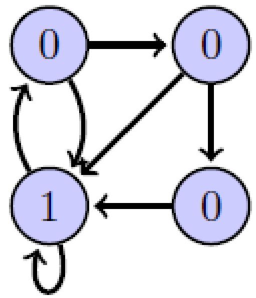
Joint work with
A. Kundu, M. Heemels

Controllability with Packet Dropouts



We are given a pair (A,b) and an automaton

$$\sigma(0) = 1$$
$$\sigma(1) = 0$$
$$\sigma(2) = 0$$

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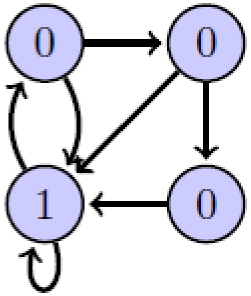
$$\boxed{\sigma = 1001\ldots}$$

The controllability problem: for any starting point x(0), and any target x*, does there exist, for any admissible switching signal, a control signal u(.) and a time T such that x(T)=x* ?

Theorem: Deciding controllability of switching systems is undecidable in general (consequence of [Blondel Tsitsiklis, 97])

Controllability with Packet Dropouts



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Proposition: The system is controllable iff the **generalized controllability matrix**

$$C_\sigma(t) = [A^{(t-1)}b\sigma(0)|A^{(t-2)}b\sigma(1)|\ldots|Ab\sigma(t-2)|b\sigma(t-1)]$$

is bound to **become full rank** at some time t

Standing on Giants shoulders…

Thoralf Skolem
1887-1963
Axioms, set theory, lattices, first order logic

$$C_\sigma(t) = [A^{(t-1)} b\sigma(0) | A^{(t-2)} b\sigma(1) | \ldots | Ab\sigma(t-2) | b\sigma(t-1)]$$

Theorem ([Skolem 34]): Given a matrix A and two vectors b,c, the set of values n such that $c^\top A^n b = 0$

is eventually periodic.

Example: 1 0 0 1 0 1 1 1 0 0 1 0 0 1 0 0 1 0 0 …

➔ Algorithm for deciding controllability

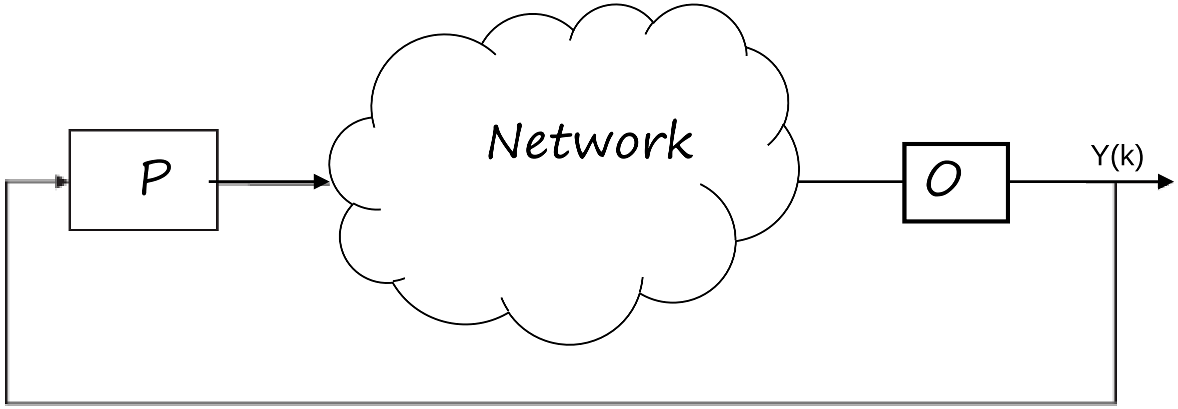

Now, how to optimally chose the control signal, if one does not know the switching signal in advance?

Jungers, Kundu, Heemels, TAC 2017

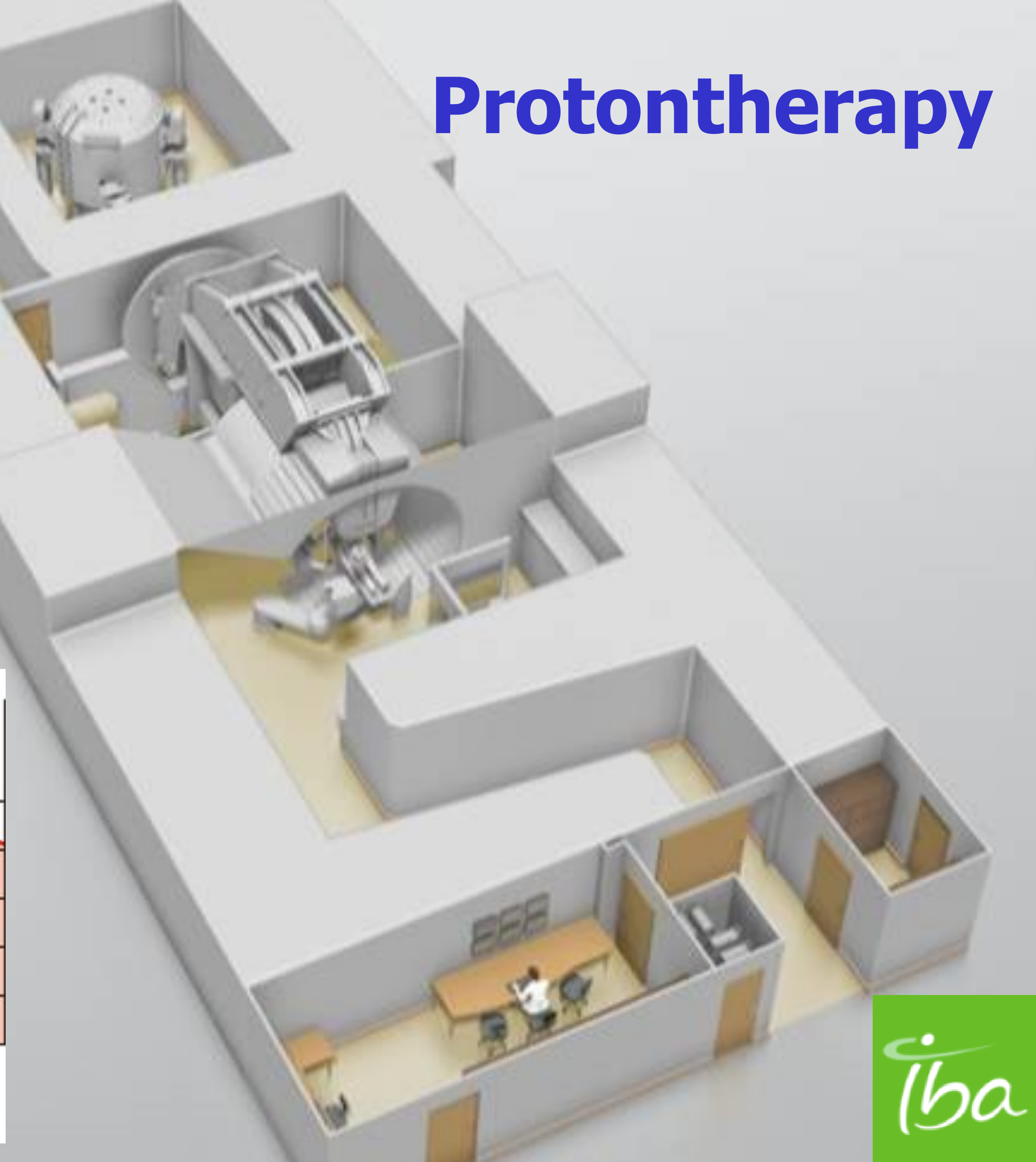
The dual observability problem

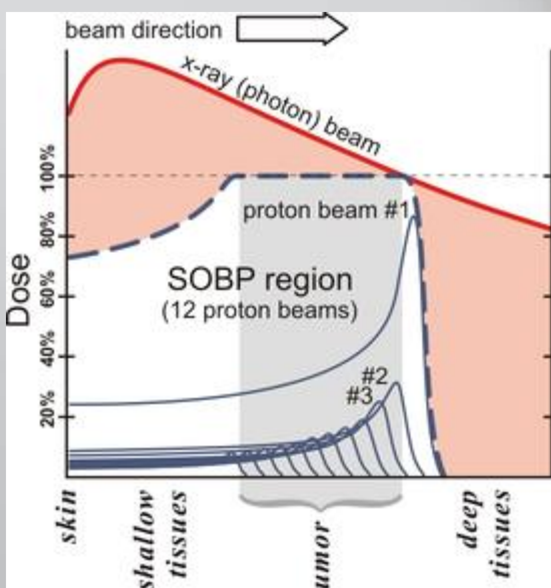
Observability under intermittent outputs is **algebraically equivalent**

$$\boxed{\begin{aligned} x(t+1) &= Ax(t), \\ y(t) &= \sigma(t)Cx(t) \end{aligned}}$$



Protontherapy





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Switching systems stability criteria

$$x(k+1) = \begin{cases} A_0 x(k) \\ A_1 x(k) \end{cases}$$

AKA computing the Joint Spectral Radius

$$\begin{array}{ll} \min_{r \in \mathbb{R}^+} & r \\ \text{s.t.} & \end{array}$$

$$\begin{array}{l} \inf_{r \in \mathbb{R}^+} \\ \text{s.t.} \\ A^T P A \\ P \end{array}$$

$$\begin{array}{lcl} \min_{r \in \mathbb{R}^+} & & r \\ \text{s.t.} & & \\ A_1^T P A_1 & \preceq & r^2 P, \\ (A_2 A_1)^T P (A_2 A_1) & \preceq & r^4 P, \\ (A_2^2)^T P (A_2^2) & \preceq & r^4 P, \\ P & \succeq & 0. \end{array}$$

[Goebel, Hu, Teel 06]

[Daafouz Bernussou 01]

[Bliman Ferrari-Trecate 03]

[Lee and Dullerud 06] …

The joint spectral radius of a set of matrices Σ is given by

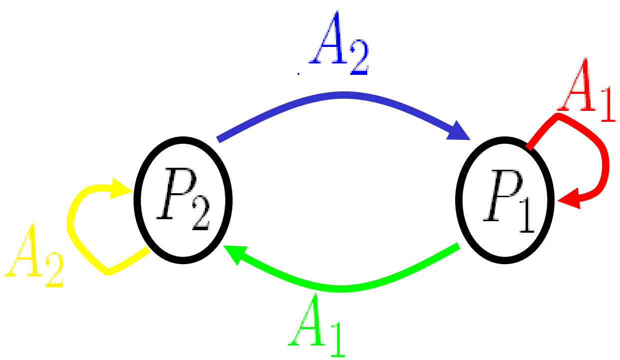
$$\rho(\Sigma) = \lim_{t \to \infty} \max_{A_i \in \Sigma} ||A_1 A_2 \ldots A_t||^{1/t}$$

All products of matrices in Σ converge to zero iff $\rho(\Sigma) < 1$

[Rota, Strang, 1960]

Path-complete stability criteria

$$
\begin{array}{lll}
\min_{r \in \mathbb{R}^+} & & r \\
\text{s.t.} & & \\
A_1^T P_1 A_1 & \preceq & r^2 P_1, \\
A_2^T P_1 A_2 & \preceq & r^2 P_2, \\
A_1^T P_2 A_1 & \preceq & r^2 P_1, \\
A_2^T P_2 A_2 & \preceq & r^2 P_2, \\
P_i & \succeq & 0.
\end{array}
$$



Sufficient condition for stability

Path complete (generates all the possible words)

Theorem: The LMIs are a sufficient condition for stability IFF their representation G is path-complete .

Results valid beyond the LMI framework

[Ahmadi J. Parrilo Roozbehani 14]

[J. Ahmadi Parrilo Roozbehani 17]

Path-complete stability criteria

$\min_{r\in\mathbb{R}^+} \quad r$

s.t.

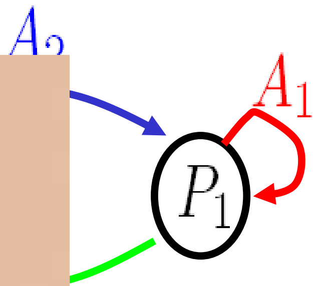
$A_1^T P_1 A$

$A_2^T P_1 A$

$A_1^T P_2 A$

$A_2^T P_2 A$

P_i



Suffici

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Theorem: We provide a **hierarchy of criteria** that reaches **arbitrary accuracy**

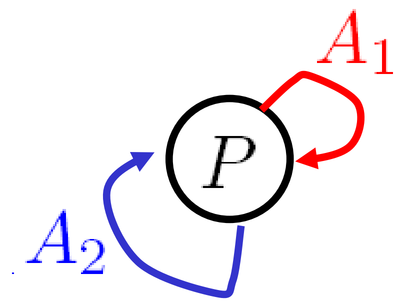
Results valid beyond the LMI framework

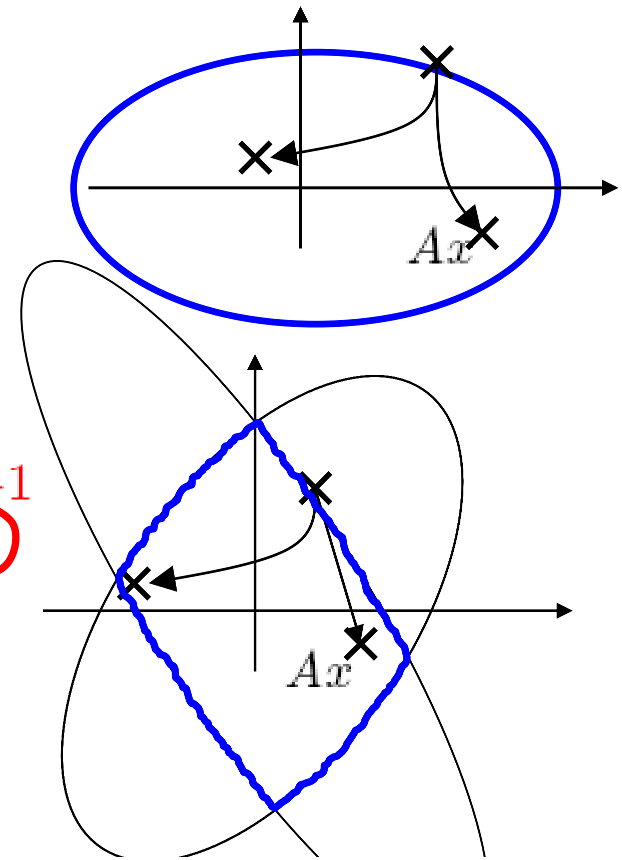
[Ahmadi J. Parrilo Roozbehani 14]

[J. Ahmadi Parrilo Roozbehani 17]

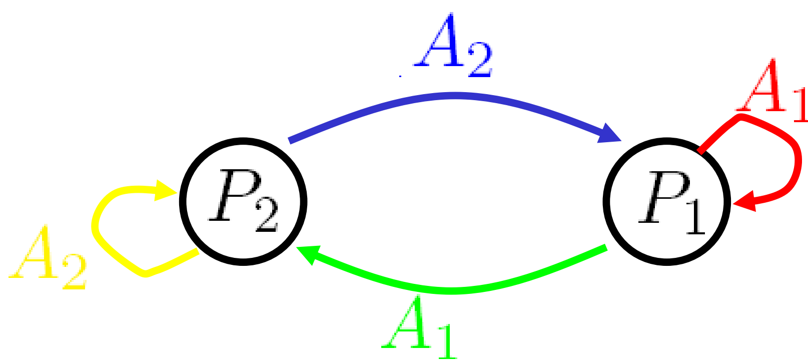
Some examples

- Examples:
  - CQLF



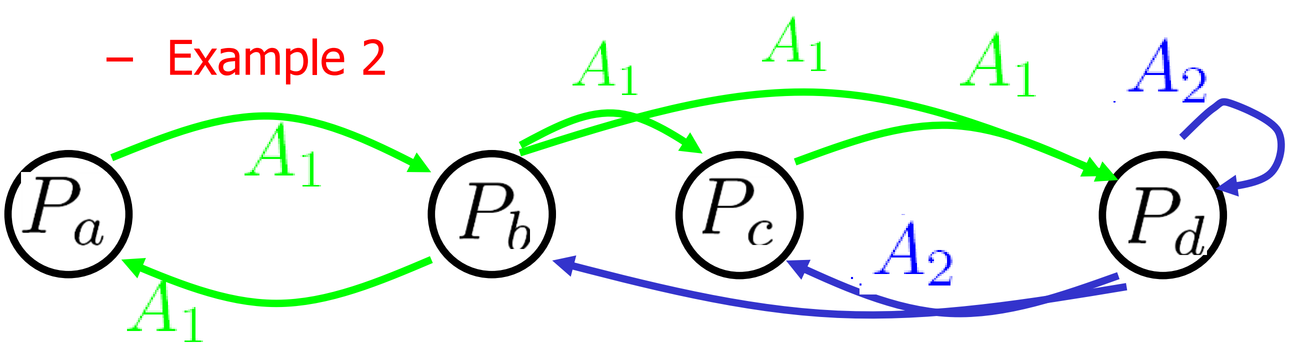


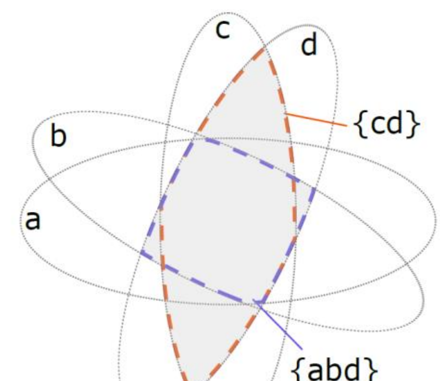
  - Example 1



This type of graph gives a max-of-quadratics Lyapunov function (i.e. intersection of ellipsoids)

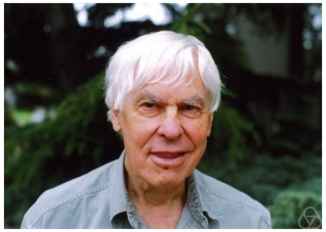
  - Example 2



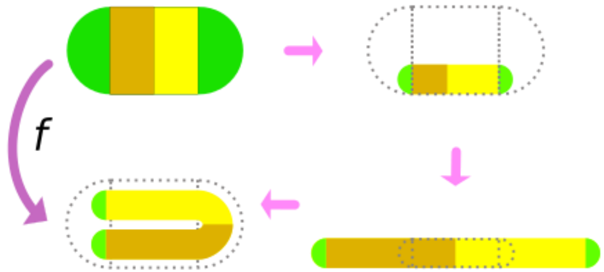


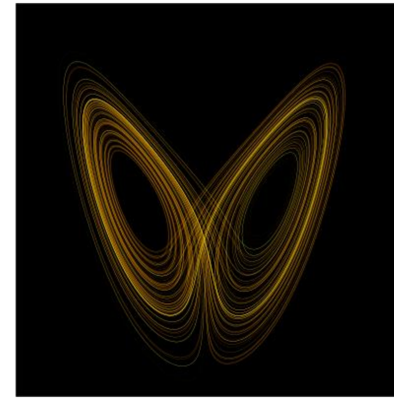
Standing on Giants shoulders

Symbolic dynamics



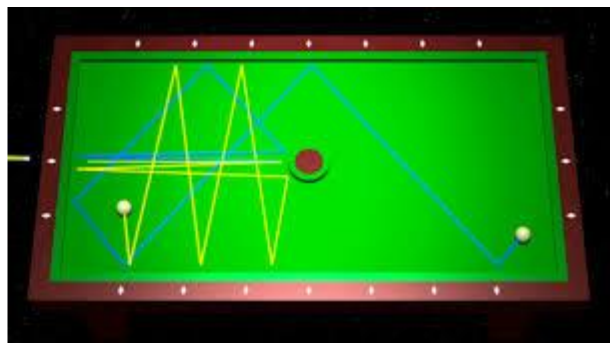
Stephen Smale
1930-
Fields Medal 1966
Wolf prize 2007



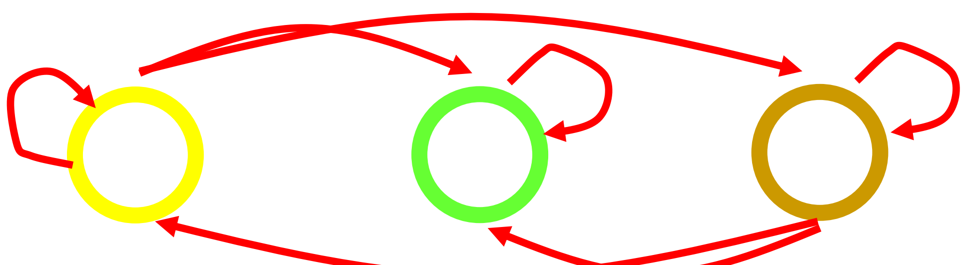


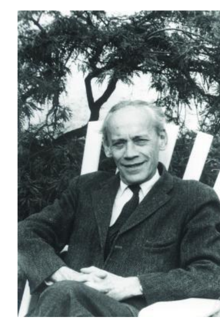
Gustav Hedlund
1904-1993

Marston Morse
1892-1977



Symbolic dynamics
M **Morse**, GA **Hedlund** - American Journal of Mathematics, 1938 - JSTOR





Emil Artin
1898-1962

Path-complete techniques

A. Ahmadi (Princeton),
P. Parrilo, M. Roozbehani (MIT)

SICON'14, TAC'16
Path-complete characterization




Geir Dullerud
And Ray Essick (UIUC)

Automatica'16
Generalization to
Constrained switching systems

Matthew Philippe

David Angeli (Imperial)

Matthew Philippe,
Nikos Athanasopoulos

TAC'18
Equivalent Common Lyapunov Function


F. Forni and R. Sepulchre (Cambridge)


IFAC WC'16
Generalization to
delta-ISS certificates

Guillaume Berger

Paulo Tabuada (UCLA)

ADHS'18
Generalization to
Invariant sets computation

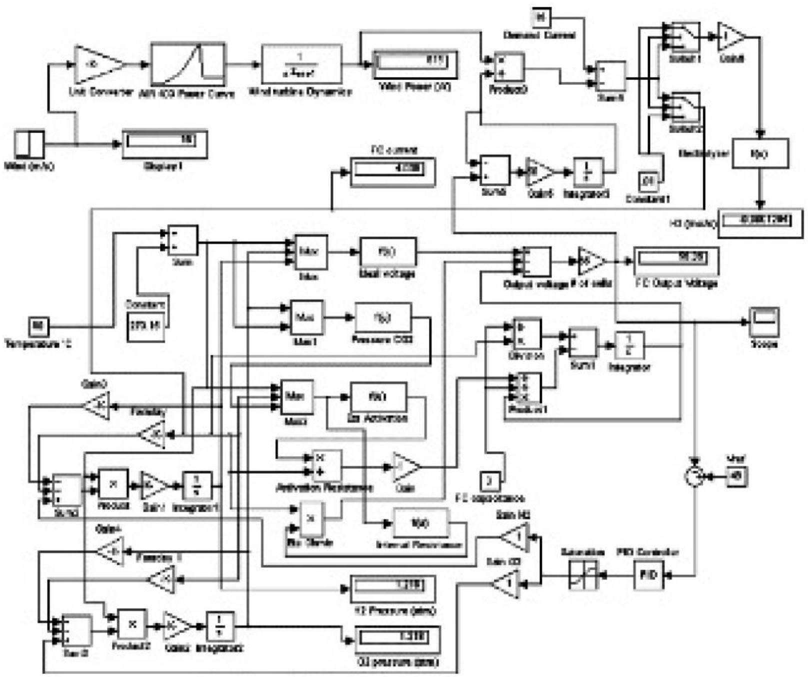
Benoit Legat

Outline

- Two motivations
  - Consensus
  - Wireless control networks
- **Three techniques**
  - Observability/controllability of hybrid systems
  - Guaranteed accuracy for stability analysis
  - **Data-driven/blackbox control**
- Discussion

Joint work with
J. Kenanian, A. Balkan,
P. Tabuada

Control in the industry



Often, models can be **nonlinear, complex, hybrid, heterogeneous**, with **look-up tables**, **pieces of code**, **proprietary softwares, old legacy** components…

Termination of Computer programs

while $(Bx > 0)$ **do**

$\{x := Ax\};$

end while

Control in the industry

Data-driven stability analysis of SS

$$\mathbf{x}_{t+1} = \begin{matrix} \mathbf{A}_0\, \mathbf{x}_t \\ \mathbf{A}_1\, \mathbf{x}_t \end{matrix}$$

Global convergence to the origin Do all products of the type $A_0\ A_0\ A_1\ A_0\ \ldots\ A_1$ converge to zero? (GUAS)

The joint spectral radius of a set of matrices Σ is given by

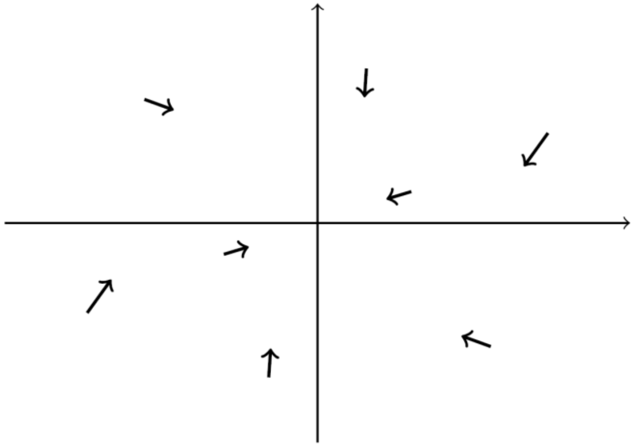
$$\rho(\Sigma) = \lim_{t \to \infty} \max_{A_i \in \Sigma} ||A_1 A_2 \ldots A_t||^{1/t}$$

All products of matrices in Σ converge to zero iff $\rho(\Sigma) < 1$

[Rota, Strang, 1960]

Data-driven stability analysis of SS

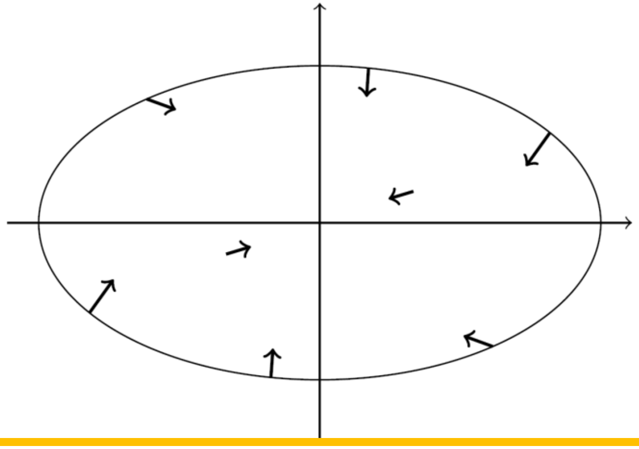
Setting: we sample N points at random in the state space, and observe their image by one (unknown) of the modes



Question: Is the system stable?

Data-driven stability analysis of SS

Observation: One thing we can to is to check for the existence of a Common Lyapunov function for the N sampled points



$$\begin{aligned} \min_P \quad & \lambda_{\max}(P) \\ \text{s.t.} \quad & (A_j x)^T P (A_j x) \leq \gamma^2 x^T P x, \ \forall (x, j) \in \omega_N \subset Z \\ & P \succeq I \end{aligned}$$

Standing on giants shoulders…

Theorem [adapted from Campi, Calafiore]: Consider the optimization problem below, where ω_N is a random homogeneous N-sampling of the infinite number of constraints **Z.** For any desired correctness level ε one can guarantee

$$\mu^N\{\omega_N \in Z^N : \mu(V(\omega_N)) \leq \varepsilon\} \geq 1 - \sum_{j=0}^{d} \binom{N}{j} \varepsilon^j (1-\varepsilon)^{N-j}$$

Measure of 'good' N-Samplings

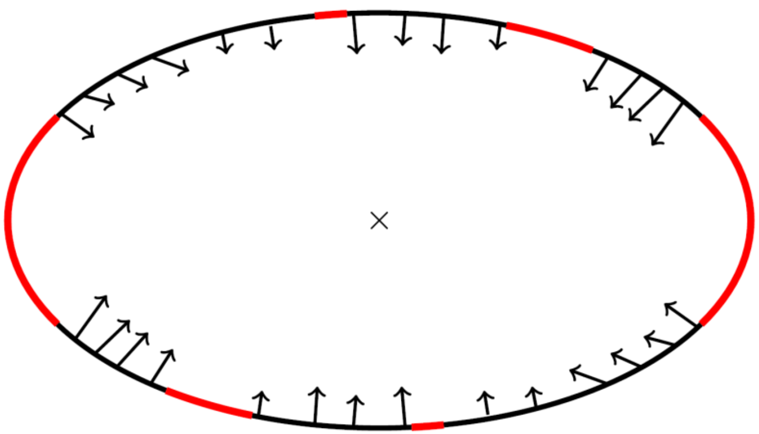
Measure of violated constraints

Lower bound on the measure of good samplings (Tends to 1 when N grows)

$$\begin{array}{ll} \min_P & \lambda_{\max}(P) \\ \text{s.t.} & (A_j x)^T P (A_j x) \leq \gamma^2 x^T P x, \ \forall (x, j) \in \omega_N \subset Z \\ & P \succeq I \end{array}$$

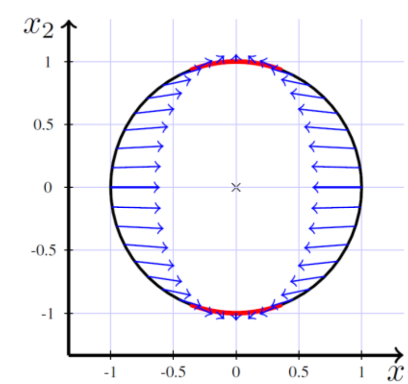
From a partial guarantee to a formal upper bound

With some level of confidence, we have:



What does this result tell us?

Not much!
Already for a linear system, this does not imply much:



$$x^+ = \begin{bmatrix} 0.14 & 0 \\ 0 & 1.35 \end{bmatrix} x$$

The challenge: translate this guarantee on a subset of the statespace into a global guarantee (on the whole statespace)

From a partial guarantee to a formal upper bound

Theorem:

Consider a n-dimensional linear switched system and a uniform random sampling $\omega_N \subset Z$, where $N \geq \frac{n(n+1)}{2} + 1$. Let $\gamma^*(\omega_N)$ be the optimal solution to Opt(ω_N). Given a desired level of certainty $\beta \in (0,1)$, given a size of sample N and $\alpha > 1$, we can compute $\delta(\beta, \omega_N)$, such that with probability at least β we have:

$$\rho \leq \frac{\alpha \gamma^*(\omega_N)}{\delta(\beta, \omega_N)},$$

where $\lim_{N \to \infty} \delta(\beta, \omega_N) = 1$ *with probability* 1.

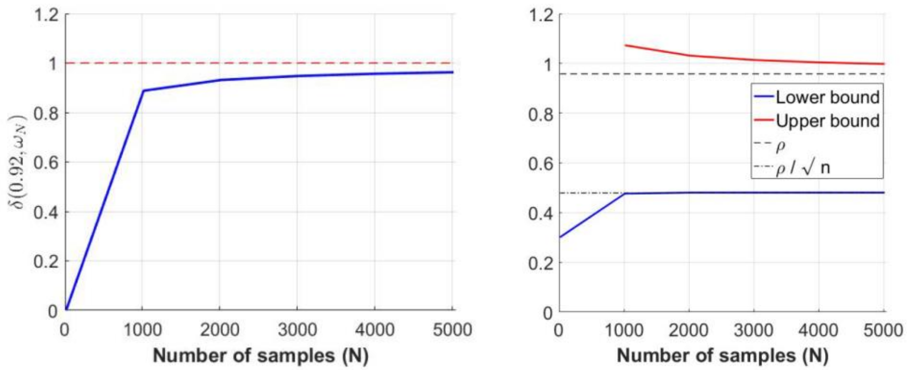
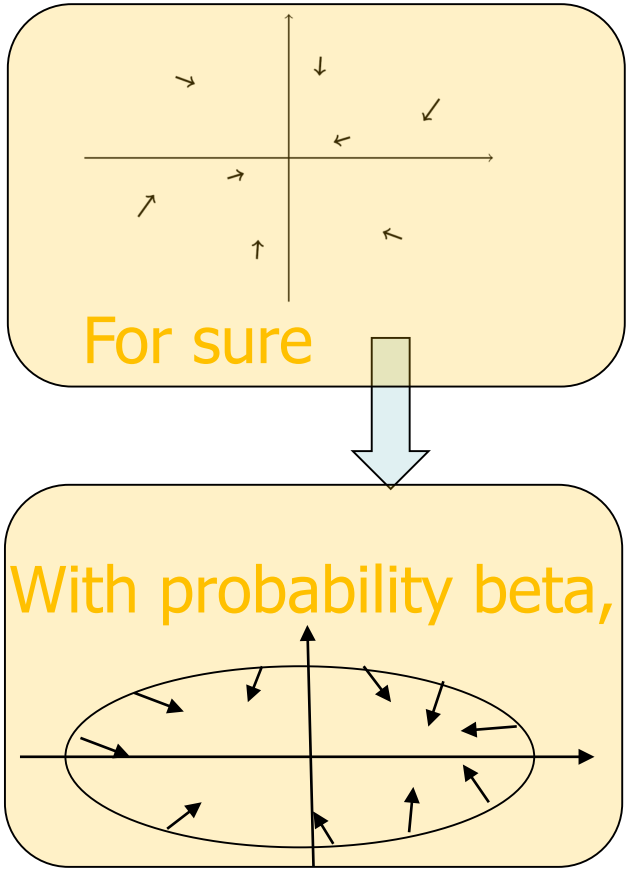


Figure: Evolution of δ (left) and of the upper and lower bounds on the JSR (right) with increasing N, for $\beta = 0.92$.



Outline

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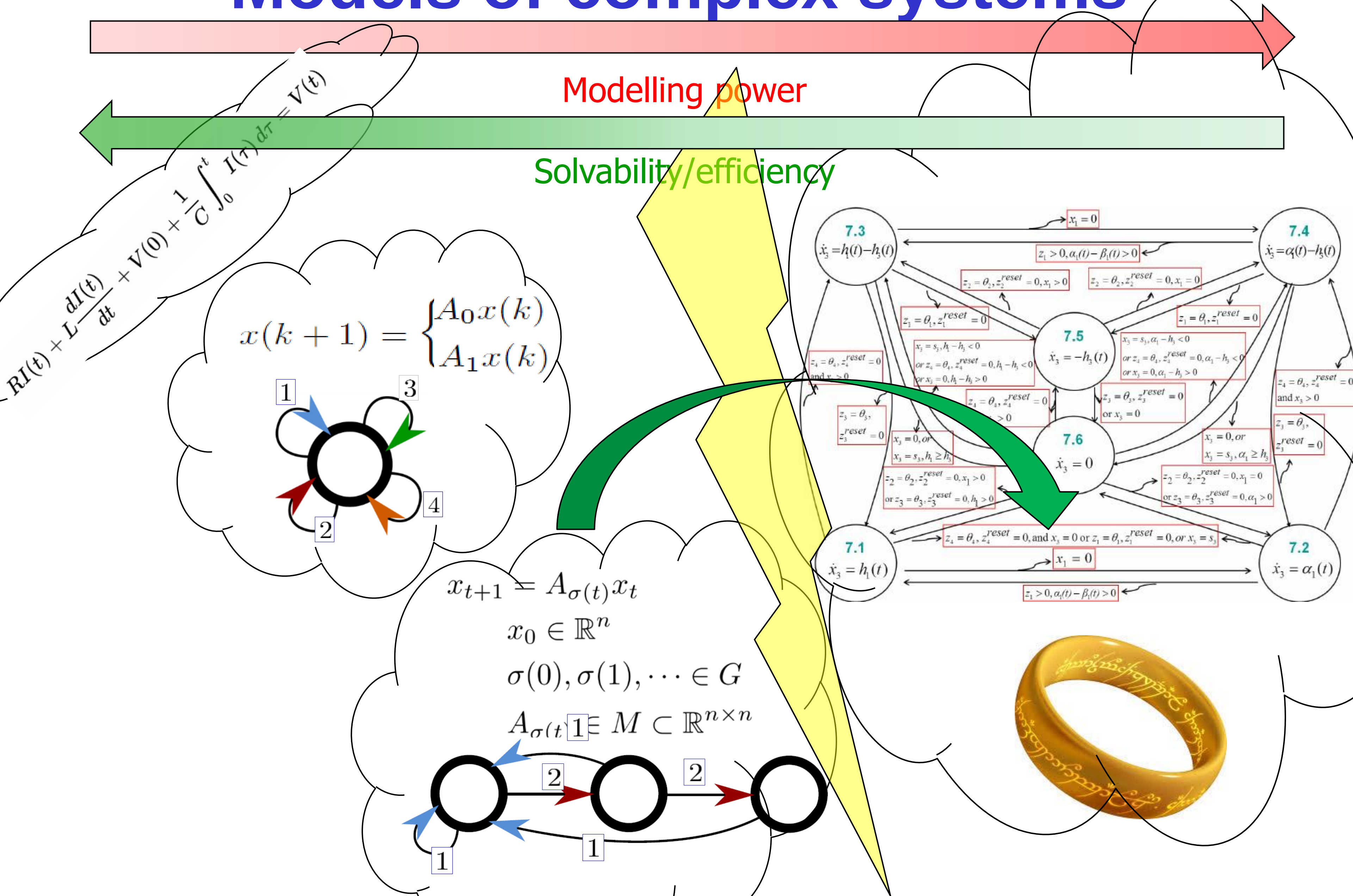
Models of complex systems



Thanks!

Questions?

Ads

The JSR Toolbox:
http://www.mathworks.com/matlabcentral/fileexchange/33202-the-jsr-toolbox
[Van Keerberghen, Hendrickx, J. HSCC 2014]
The CSS toolbox, 2015

Several open positions:
raphael.jungers@uclouvain.be

References:
http://perso.uclouvain.be/raphael.jungers/

Joint work with

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