On Factorisation of Provenance Polynomials

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Provenance Polynomials

Unifying framework (Green et al.) that captures the semantics of
- incomplete information and uncertain databases,
- query evaluation under set/bag semantics,
- annotation propagation for why- and how-provenance.

In provenance polynomials, we denote provenance of
- input tuples by **variables**,
- a join of tuples by a **product** of their provenance,
- a union of tuples by a **sum** of their provenance.

Example Database

Order

	id	item
o_1	01	Printer
o_2	02	Plotter
o_3	03	Ink
o_4	04	Printer
o_5	05	Ink

Store

	location	item
s_1	Depot1	Printer
s_2	Depot1	Plotter
s_3	Depot2	Printer
s_4	StoreA	Ink

Emp

	operator	location
e_1	Joe	Depot1
e_2	Bob	Depot1
e_3	Dan	Depot2
e_4	Dan	StoreA

Example Query

Order $\bowtie_{\text{item}}$ Store $\bowtie_{\text{location}}$ Emp

	id	item	location	operator
$o_1 s_1 e_1$	01	Printer	Depot1	Joe
$o_1 s_1 e_2$	01	Printer	Depot1	Bob
$o_1 s_3 e_3$	01	Printer	Depot2	Dan
$o_2 s_2 e_1$	02	Plotter	Depot1	Joe
$\dots$			$\dots$	

Provenance Polynomial of the Query Result

$$\begin{aligned}\Phi_1 =\ & o_1 s_1 e_1 + o_1 s_1 e_2 + o_1 s_3 e_3 + o_2 s_2 e_1 + o_2 s_2 e_2 + \\ & o_3 s_4 e_4 + o_4 s_1 e_1 + o_4 s_1 e_2 + o_4 s_3 e_3 + o_5 s_4 e_4.\end{aligned}$$

Special cases:
- Boolean semiring $(\mathbb{B}, \vee, \wedge)$
  - Each variable encodes the presence of its input tuple.
  - Used in incomplete information and probabilistic databases.
- Semiring over natural numbers $(\mathbb{N}, +, \bullet)$
  - Each variable encodes tuple multiplicity.
  - Used in bag semantics of positive queries.
- If the variables encode the tuples themselves, the provenance polynomial encodes the whole query result.

Factorisation of Provenance Polynomials

Algebraic factorisation of Φ_1:

$$\Phi_2 = (o_1 + o_4)(s_1(e_1 + e_2) + s_3 e_3) + o_2 s_2 (e_1 + e_2) + (o_3 + o_5) s_4 e_4.$$

expresses explicitly how groups of input tuples combine and thus shows the nested structure of the query result and its provenance.

- Factorisations can be **more informative** and **exponentially more succinct** than flat representations.
- The monomials can be extracted from the factorisation **with polynomial delay**.

Challenge: Queries with Factorised Polynomials of Bounded Size

Classification of queries based on
- the minimal size of the factorised polynomials of query results for any input database
- result polynomials with factorisations of **bounded readability** for any input database
  - Polynomial Φ is **read-k** if each variable occurs at most k times in Φ.
  - Polynomial Φ has **readability** k if k is the smallest number such that there is a read-k polynomial equivalent to Φ.

Examples: The readability of
- the query [Store $\bowtie_{\text{location}}$ Emp] is one for any database.
  - In our example, the factorised polynomial is $(s_1 + s_2)(e_1 + e_2) + s_3 e_3 + s_4 e_4$.
  - For each location, we get a product of sums of distinct variables.
- the query [Order $\bowtie_{\text{item}}$ Store $\bowtie_{\text{location}}$ Emp] is dependent on the input database size.

Challenge: Efficient Computation of Factorised Polynomials

- Compute factorisations of low/minimal readability for any polynomial.
  - Minimality may be with respect to a restricted class of factorisations.
- For a query and a database, compute the factorised polynomial of the query result
  - **without** first computing the flat polynomial of the query result.

Challenge: Querying Factorised Relations and Polynomials

- Assume that variables in polynomials carry the input tuples.
- Evaluate queries **directly** on factorised polynomials.

Example: Equivalent factorisations of the result of [Order $\bowtie_{\text{item}}$ Store $\bowtie_{\text{location}}$ Emp]:

$$\Phi_9 = (o_1 + o_2)(s_1(e_1 + e_2) + s_2(e_3 + e_4)) + (o_3 + o_4)(s_3(e_1 + e_2) + s_4(e_3 + e_4)),$$
$$\Phi_{10} = ((o_1 + o_2)s_1 + (o_3 + o_4)s_3)(e_1 + e_2) + ((o_1 + o_2)s_2 + (o_3 + o_4)s_4)(e_3 + e_4).$$

- Here, variables $o_i(e_j)$ are annotated with tuples from Order (Emp)
- $\Phi_9(\Phi_{10})$ is suitable for joining on Order (Emp) without unfolding

Challenge: Approximation by Factorised Polynomials

Given a polynomial Φ, find **lower** and **upper bounds** Φ_L, Φ_U with lower readability.
- Definition of lower and upper bounds depends on the semiring.
  - In the Boolean semiring: $\Phi_L \models \Phi \models \Phi_U$
  - In the semiring over natural numbers: $\Phi_L \le \Phi \le \Phi_U$
  - For all semirings: Drop (add) monomials for lower (upper) bounds

  Lower bound for Φ_1: $\Phi_L = (o_1 + o_4)(s_1(e_1 + e_2) + s_3 e_3) + (o_3 + o_5) s_4 e_4$

  Upper bound for Φ_1: $\Phi_U = (o_1 + o_2 + o_4)((s_1 + s_2)(e_1 + e_2) + s_3 e_3) + (o_3 + o_5) s_4 e_4.$
- Search for closest bounds in a given class $\mathcal{C}$ of well factorisable polynomials.
  - $\mathcal{C}$ could be the class of polynomials with readability one.

Query approximation:
- Approximate a query Q by lower and upper bound queries Q_L and Q_U.
  - For any database, the polynomials Φ_L and Φ_U of Q_L and Q_U are lower and upper bounds for the polynomial Φ of Q and have lower readability.

Results: Queries with Factorisations of Bounded Size

- We introduce **factorisation trees** which
  - are statically derived from a query Q,
  - are independent of the input database,
  - define a factorisation of the polynomial of $Q(\mathbf{D})$, for any database $\mathbf{D}$.

Characterisation of Conjunctive Queries

- For any query Q, there is a rational number $f(Q)$ such that for any database $\mathbf{D}$, $Q(\mathbf{D})$ has a factorised polynomial
  - with readability $O(|\mathbf{D}|^{f(Q)})$,
  - with size at most $|\mathbf{D}|^{f(Q)+1}$.

  Moreover, $f(Q)$ is the smallest such number when restricted to factorisations defined by factorisation trees.
- A query satisfies $f(Q) = 0$ iff it is **hierarchical**. Then the polynomial of any $Q(\mathbf{D})$ has a factorisation
  - with **bounded readability**,
  - with size linear in the sizes of input database and query.

  For hierarchical queries w/o self-joins it is also known that
  - in probabilistic databases, their exact probability can be computed in polynomial time,
  - in the finite cursor machine model, they can be evaluated in just one pass over the database.

Results: Efficient Computation of Factorisations

- For any Q and $\mathbf{D}$, we compute a factorisation of readability $O(|\mathbf{D}|^{f(Q)})$ and size at most $|\mathbf{D}|^{f(Q)+1}$ in time $O(|\mathbf{D}|^{f(Q)+1})$
- .. without computing the flat polynomial!

Results: Approximation by Factorised Polynomials

Approximation by polynomials of readability one, over the Boolean semiring.
- Equivalent syntactic and model-theoretic characterisations of lower and upper bounds.
- Algorithms to enumerate bounds with polynomial delay.

Selected Publications

- **On Factorisation of Provenance Polynomials**
  D. Olteanu and J. Závodný. In *TaPP*, 2011.
- **Factorised Representations of Query Results.**
  D. Olteanu and J. Závodný. Tech. rep., Oxford, April 2011. Also arXiv report 1104.0867.
- **On the Optimal Approximation of Queries Using Tractable Propositional Languages.**
  R. Fink and D. Olteanu. In *ICDT*, 2011.