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Approximate Confidence Computation in Probabilistic Databases

http://www.comlab.ox.ac.uk/projects/SPROUT/

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Uncertain and Probabilistic Data

Uncertain and probabilistic data is commonplace:

- Information extraction
- Processing manually entered data (such as census forms)
- Data cleaning, data integration
- Risk management: Decision support queries, hypothetical queries
- Social network analysis
- ...

Recent years have seen advances in developing

- representation models for uncertain/probabilistic data,
- uncertainty-aware query languages, and
- query evaluation techniques for such data.
  - scope of this work.

Contributions of this Work

- Efficient deterministic technique for confidence computation.
  - approximate confidences with error guarantees
    - positive relational algebra queries
    - U-relational databases
  - exact confidences for known tractable queries in polynomial time
    - hierarchical conjunctive queries without self-joins, max-one inequality queries
    - tuple-independent probabilistic databases
- Implementation of this technique in the SPROUT query engine.
  - extends PostgreSQL backend with confidence computation operators
  - used by MayBMS (`maybms.sourceforge.net`)
- Experimental comparison with existing techniques.
  - fastest technique so far for tractable queries (previous SPROUT)
  - Monte Carlo algorithm (MayBMS)

U-relational Probabilistic Databases

Syntax.
Probabilistic databases are relational databases where

- There is a finite set of independent random variables $\mathbf{X} = \{x_1, \ldots, x_n\}$ with finite domains $\mathrm{Dom}_{x_1}, \ldots, \mathrm{Dom}_{x_n}$.
- Each tuple is associated with a conjunction of atomic events of the form $x_i = a$ or $x_i \neq a$ where $x_i \in \mathbf{X}$ and $a \in \mathrm{Dom}_{x_i}$.
- There is a probability distribution over the assignments of each variable.

Semantics.

- *Possible worlds* defined by total assignments θ over $\mathbf{X}$.
- The world defined by assignment θ
  - consists of all tuples with condition ϕ such that $\theta(\phi) = \mathit{true}$.
  - has probability defined by the product of probabilities of each assignment in θ.

This formalism can represent any discrete probability distribution over relational databases.

Example: Probabilistic Databases

Consider a simplified TPC-H scenario with customers (Cust) and orders (Ord):

Cust

ckey	name	V_1	P_1	V_2	P_2
1	Joe	x_1	0.1	x_3	0.1
2	Dan	$\overline{x_1}$	0.9	x_4	0.5
3	Li	x_2	0.3	$\overline{x_4}$	0.5
4	Mo	$\overline{x_2}$	0.7	$\overline{x_5}$	0.2

Ord

okey	ckey	date	V_3	P_3	V_4	P_4
1	1	1995-01-10	y_1	0.1	$\overline{x_5}$	0.2
2	1	1996-01-09	y_2	0.2	$\overline{x_4}$	0.5
3	2	1994-11-11	y_3	0.3	x_3	0.1

- Variables are Boolean (wlog); write x instead of $x = 1$, $\overline{x}$ instead of $x = 0$.
- A pair (V_i, P_i) states that the variable assignment given by V_i has the probability given by P_i.
- Conditions can represent arbitrary correlations between tuples, eg,
  - (1,Joe) and (3,Li) are independent: They use disjoint sets of variables.
  - (1,Joe) and (2,Dan) are mutually exclusive: x_1 is either true or false.

Query Evaluation in Probabilistic Databases

- Semantically, the query is evaluated in each world.
  - Too expensive for any practical purpose!

- Common approach:
  1. Evaluate the query directly on the representation.
     - Done with relational query plans for our probabilistic data formalism.
     - In addition to standard evaluation, copy the input conditions to the output.
  2. Compute the confidence of each answer tuple.
     - Reducible to probability computation of Boolean formulas over random variables
     - Known to be #P-hard already for positive bipartite DNF formulas!

Example: Query Evaluation

Query asking for the probability that customer 'Joe' has placed orders:

$$Q = \pi_{\emptyset}(\sigma_{name='Joe'}(\text{Cust}) \bowtie_{ckey} \text{Ord})$$

	V_1	P_1	V_2	P_2	V_3	P_3	V_4	P_4
	x_1	0.1	x_3	0.1	y_1	0.1	$\overline{x_5}$	0.2
	x_1	0.1	x_3	0.1	y_2	0.2	$\overline{x_4}$	0.5

- Probability of the answer tuple is the probability of the associated DNF $x_1 x_3 y_1 \overline{x_5} + x_1 x_3 y_2 \overline{x_4}$.

Difficulty:

- The sets of satisfying assigments of any two clauses in the DNF may overlap.
- It may require to iterate over its (exponentially many) satisfying assignments.

Approximate computation, if done quickly enough, may suffice in most applications.

Approximate Confidence Computation in SPROUT

Basic algorithm:

- decompose the DNF into an equivalent form that allows for efficient probability computation.
- after each decomposition step, compute lower and upper bounds on the probabilities of the DNFs obtained by decomposition and of the initial DNF.
- stop when the desired approximation is obtained or on timeout.
- otherwise, continue with a new decomposition step.

In practice, good approximations can be obtained after a few *well-chosen* decomposition steps.

Types of Decompositions

Given DNF formula Φ. Apply the following steps in the given order.

1. Independent-or $\otimes$: Partition Φ into independent DNFs $\Phi_1, \Phi_2 \subset \Phi$ such that $\Phi \equiv \Phi_1 \vee \Phi_2$.

2. Independent-and $\odot$: Partition Φ into independent DNFs Φ_1 and Φ_2 such that $\Phi \equiv \Phi_1 \wedge \Phi_2$.

3. Exclusive-or $\oplus$: Choose a variable x in Φ. Then,

$$\Phi \equiv \bigoplus_{a \in \mathrm{Dom}_x, \Phi|_{x=a} \neq \emptyset} \left((x = a) \odot \Phi \,|_{x=a} \right).$$

For DNFs of query answers, the decompositions

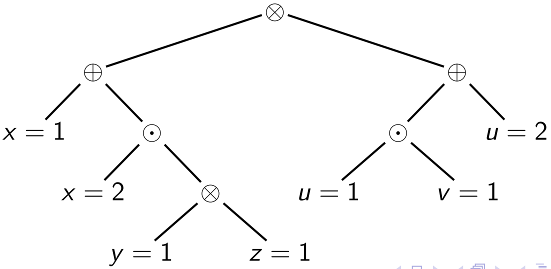
- preserve equivalence,
- are efficiently computable, and
- allow for efficient probability computation.

D-trees: Decomposition Trees

A *d-tree* is a formula constructed from $\otimes$, $\oplus$, $\odot$ and nonempty DNFs (as "leaves"). If each leaf holds one clause, the d-tree is *complete*.

Example: Complete d-tree for

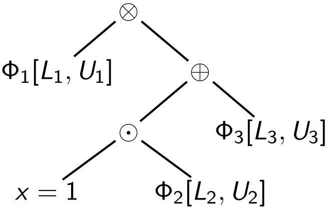
$$
\begin{aligned}
& x = 1 \vee \\
& x = 2 \wedge y = 1 \vee \\
& x = 2 \wedge z = 1 \vee \\
& u = 1 \wedge v = 1 \vee \\
& u = 2.
\end{aligned}
$$



Lower and Upper Bounds for D-trees

Bounds $[L, U]$ on the probability of a d-tree can be computed efficiently if each leaf of a d-tree has lower L_i and upper U_i bounds of its probability.

Example: D-tree for $\Phi = \Phi_1 \otimes \{[(x=1) \odot \Phi_2] \oplus \Phi_3\}$:



Then,

$$L(\Phi) = L_1 \otimes [Pr(x=1) \odot L_2 \oplus L_3]$$
$$U(\Phi) = U_1 \otimes [Pr(x=1) \odot U_2 \oplus U_3]$$

How to Efficiently Compute Probability Bounds for Leaves?

Many possible approaches. We used the following simple approach:

- Given a leaf (that is, a DNF) Ψ.
- Choose a maximal subset S of pairwise independent clauses in Ψ.
- Then, $P(S)$ is a lower bound for $P(\Psi)$, and
- $\min(1, P(S) + \sum_{c \in (\Psi - S)} (P(c)))$ is an upper bound for $P(\Psi)$.

Rationale: We want a quick solution for computing the bounds, since this operation needs to be done for each node of the d-tree.

- We compute in one scan over Ψ the lower and upper bounds for $P(\Psi)$.

Absolute and Relative Approximation Errors

- $\hat{p}$ is an <u>absolute</u> ϵ-approximation of p if $p - \epsilon \leq \hat{p} \leq p + \epsilon$.

- $\hat{p}$ is a <u>relative</u> ϵ-approximation of p if $(1 - \epsilon) \cdot p \leq \hat{p} \leq (1 + \epsilon) \cdot p$.

Given a DNF Φ, a fixed error ϵ, and a d-tree for Φ with bounds $[L, U]$.

- If $U - \epsilon \leq L + \epsilon$, then any value in $[U - \epsilon, L + \epsilon]$ is an absolute ϵ-approximation of $P(\Phi)$.

- If $(1 - \epsilon) \cdot U \leq (1 + \epsilon) \cdot L$, then any value in $[(1 - \epsilon) \cdot U, (1 + \epsilon) \cdot L]$ is a relative ϵ-approximation of $P(\Phi)$.

Memory-Efficient Version of the Algorithm

The previous algorithm keeps the entire d-tree in main memory.

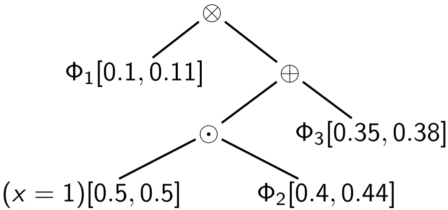
Improvement idea:

- Construct the d-tree in depth-first traversal.
- When at a leaf, decide locally whether we should further decompose it or *close* it, that is, move up the tree to the following *open* leaf.

When can we close a leaf?

- compute the bounds of the d-tree with largest difference for any possible probability each open leaf may take.
- these bounds must satisfy the condition for an ϵ-approximation.
- efficient way: bounds computed by choosing for each open leaf the bounds $[L_i, L_i]$, where L_i is a lower bound for that leaf.

Example: Memory-efficient Algorithm



Assume Φ_1 is closed, Φ_2 is current, Φ_3 is open. Let absolute error $\epsilon = 0.012$.

Test at Φ_2 whether

- we can stop with an absolute ϵ-approximation.
  - NO! Check by considering all leaves closed and compute the bounds.
  - $U - L = 0.644 - 0.595 = 0.049 \leq 2 \cdot 0.012 = 0.024$ does not hold.
- we can close Φ_2.
  - YES! Check by considering all preceding leaves closed and all following leaves open, then compute the bounds.
  - $U' - L = 0.6173 - 0.595 = 0.0223 \leq 0.024$ holds.

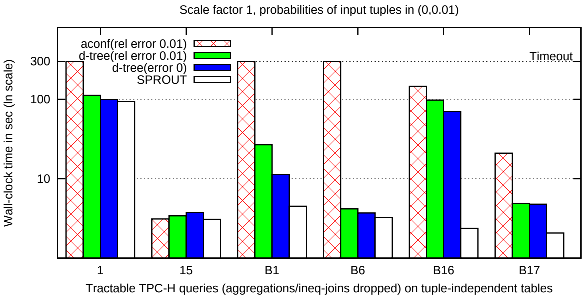
Tractable Queries on Tuple-Independent Databases

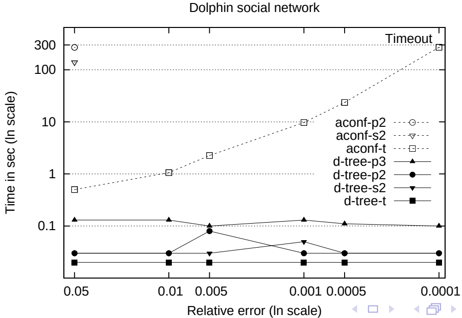
Our d-trees naturally capture DNFs for tractable queries:

- DNFs for any tractable conjunctive query without self-joins can be compiled in polynomial time into complete d-trees with nodes $\otimes$ and $\odot$.
  - In this case, the d-trees correspond to read-once functions.
- DNFs for existing (max-one) tractable inequality queries can be compiled in polynomial time into complete d-trees with nodes $\oplus$.

In both cases, the d-trees have sizes linear in the number of literals in the DNF.

Experiments





Thanks!