# Probabilistic Databases and Reasoning 

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## Probabilistic Databases and Reasoning

This 3-hour tutorial has two main parts:

1. Dan Olteanu: Probabilistic Databases

Now: 8.30am - 10am.
2. Thomas Lukasiewicz: Probabilistic Reasoning

Next: 10am-10.30am, then a break, then 11am-12pm.

Further 1-hour lectures on advanced topics in probabilistic databases:

1. DL invited talk today at 12.10 pm

Dan Suciu: Lifted Inference in Probabilistic Database
2. KR invited lecture tomorrow at 9.30am

Dan Suciu: Query compilation: the View from the Database Side

KR features several more papers on probabilistic data and knowledge bases!

## Probabilistic Databases

For the purpose of the first half of this tutorial:

Probabilistic data $=$

- Relational data $+$
- Probabilities that measure the degree of uncertainty in the data.

Long-term key challenges:

- Models for probabilistic data to capture data and its uncertainty.
- Query evaluation = Probabilistic inference Query answers are annotated with output probabilities.


## Outline

Why Probabilistic Databases?



Probabilistic Databases

Dan Sacie
Dan Olteanu
Christopher Rê
Christoph Koch


## Probabilistic Data Models

The Query Evaluation Problem
Dichotomies for Query Evaluation

- The Hard Queries
- The Tractable Queries

Ranking Queries
Next Steps
References

## Research Development Map

We can unify logic and probability by defining distributions over possible worlds that are first-order model structures (objects and relations).

Gaifman'64

Early work (80s and 90s):

- Basic data models and query processing

Wong'82, Shoshani'82, Cavallo \& Pittarelli'87, Barbará'92, Lakshmanan'97,'01, Fuhr\& Röllke'97, Zimányi'97, ..

Recent wave (2004-now):

- Computational complexity of query evaluation
- Probabilistic database systems

Stanford (Trio), UW (MystiQ), Cornell \& Oxford (MayBMS/SPROUT), IBM Almaden \& Rice (MCDB), LogicBlox \& Technion \& Oxford (PPDL), Florida, Maryland, Purdue, Waterloo, Wisconsin, ..

## Why This Interest in Probabilistic Databases?

Probabilistic relational data is commonplace. It accommodates several possible interpretations of the data weighted by probabilities.

■ Information extraction: Probabilistic data inferred from unstructured data (e.g., web) text using statistical models

Google Knowledge Vault, DeepDive, NELL

- Manually entered data

Represent several possible readings with MayBMS
Infer missing data with meta-rule semi-lattices
Manage OCR data with Staccato/Google OCRopus
[Antova'07]
[Stoyanovich'11]
[Kumar'11]

- Data cleaning

Represent several possible data repairs
[Beskales'09]

- Data integration

Google Squared and SPROUT ${ }^{2}$
[Fink'11]
■ Risk management (Decision support queries, hypothetical queries); ...

## Information Extraction

Possible segmentations of unstructured text
[Sarawagi'06]
52-A Goregaon West Mumbai 400076

| $\underline{I D}$ | HouseNo | Area | City | PinCode | $\mathbf{P}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 52 | Goregaon West | Mumbai | 400062 | $\mathbf{0 . 1}$ |
| 1 | $52-A$ | Goregaon | West Mumbai | 400062 | $\mathbf{0 . 2}$ |
| 1 | $52-A$ | Goregaon West | Mumbai | 400062 | $\mathbf{0 . 4}$ |
| 1 | 52 | Goregaon | West Mumbai | 400062 | $\mathbf{0 . 2}$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |  |

- Probabilities obtained using probabilistic extraction models (e.g., CRF) The probabilities correlate with the precision of the extraction.
- The output is a ranked list of possible extractions
- Several segmentations are required to cover most of the probability mass and improve recall
Avoid empty answer to queries such as Find areas in 'West Mumbai'


## Continuously－Improving Information Extraction

Never－Ending Language Learner（NELL）database

## Recently－Learned Facts twitter

instance
biscutate swift is an animal
pedigree animals is a mammal
poppy seed holiday bread is a baked good
manuel criado de val is a South American person
dillon county airport is an airport
the sports team toronto blue jays was the winner of n 1993 world series mozart is a person who died at the age of 35
peoria and arizona are proxies for eachother
wutv tv is a TV affiliate of the network fox
white stripes collaborates with jack white
iteration date learned confidence
211 18－feb－2011 100.0 \％\％
210 17－feb－2011 99.5 图
212 20－feb－2011 100.0 क्ष
210 17－feb－2011 99．5 \％\％
210 17－feb－2011 93.8 禺
212 20－feb－2011 96.9 \％
210 17－feb－2011 96.9 图
210 17－feb－2011 99.9 领
210 17－feb－2011 96.9 额
210 17－feb－2011 93.8 \＆्रि

## Manu ${ }^{e}$ l?y-enter d census data

MayBMS manages $10^{10^{6}}$ possible readings of census data


We want to enter the information from forms like these into a database.
■ What is the marital status of the first resp. the second person?
■ What are the social security numbers? 185? 186? 785?

## Manu ${ }^{\text {I }}$ ? y-enter d census data



Much of the available information cannot be represented and is lost, e.g.

- Smith's SSN is either 185 or 785; Brown's SSN is either 185 or 186.
- Data cleaning: No two distinct persons can have the same SSN.


## OCR on manually-entered data

Staccato


■ Stochastic automaton constructed from text using Google OCRopus.

- String FO rd has the highest probability (0.21).
- String Ford has lower probability (0.12).

Staccato accommodates several possible readings of the text to increase recall.

## Web Data Integration with Google Squared

－Tables instead of page links as answers to Google queries
［Fink＇11］
－Integration of data sources with contradicting information or different schemas，degrees of trust，and degrees of completion
－Confidence values mapped to $[0,1]$


| comedy movies |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Item Nam | er $\quad \square$ | Language $\quad$ X | Director | 7 Х | Release Date |
| 区 | The Mask |  | English | Chuck Russel |  | 29 July 1994 |
| X | Scary M | －English <br> language for the mask www．infibeam．com－all 9 sources» <br> Other possible values |  | © Chuck Russell <br> directed by for The Mask www．infibeam．com－all 9 sources » |  |  |
| 区 | Superba | English Language Low confidence language for Mask www．freebase．com |  | Other possible valuesJohn R．Dilworth Low confidence director for The Mask www．freebase．com |  |  |
| 区 | Music | english，french Low confidence languages for the mask www．dvdreview．com |  | Fiorella Infascelli Low confidence directed by for The Mask www．freebase．com－all 2 sources» |  |  |
| 区 | Knocked | Italian Language Low confidence language for The Mask www．freebase．com | om <br> ues＂ | Charles Russell Low confidence directed by for The Mask www．freebase．com－all 2 sources » |  |  |

## Outline



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## Revisiting the Census Data Example



NULL values are too uninformative.
We could instead incorporate all available possibilities:

- Smith's SSN is either 185 or 785; Brown's SSN is either 185 or 186.

■ Smith's M is either 1 or 2 ; Brown's M is either 1,2 , 3 , or 4 .

## Revisiting the Census Data Example

There are $2 \times 2 \times 2 \times 4=32$ possible readings of our two census entries.

|  | SSN | N | M | SSN | N | M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 185 | Smith | 1 | 185 | Smith | 1 |
|  | 185 | Brown | 1 | 185 | Brown | 2 |
| Social Security Number: $\quad 38$ |  |  |  |  |  |  |
| Smoth | SSN | N | M | SSN | N | M |
| Name: SWMuth | 185 | Smith | 1 | 185 | Smith | 1 |
| Marital Status: (1) single 区 (2) married ¢ | 185 | Brown | 3 | 185 | Brown | 4 |
| (3) divorced $\square$ (4) widowed $\square$ |  |  |  |  |  |  |
| Social Security Number: $\frac{18}{\text { Name: }}$ | SSN | N | M | SSN | N | M |
|  | 185 | Smith | 1 | 185 | Smith | 1 |
|  | 186 | Brown | 1 | 186 | Brown | 2 |
| Marital Status: (1) single $\square$ (2) married $\square$ <br>  (3) divorced $\square$ | SSN | N | M | SSN | N | M |
|  | 185 | Smith | 1 | 185 | Smith | 1 |
|  | 186 | Brown | 3 | 186 | Brown | 4 |

## Incomplete Databases

An Incomplete Database is a finite set of database instances $\mathbf{W}=\left(W_{1}, \ldots, W_{n}\right)$.

| $W_{1}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 1 |


| $W_{2}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 2 |

Each $W_{i}$ is a possible world.

| $W_{3}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 3 |


| $W_{4}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 4 |


| $W_{5}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 186 | Brown | $\mathbf{1}$ |


| $W_{6}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 186 | Brown | 2 |

## Incomplete Databases

An Incomplete Database is a finite set of database instances $\mathbf{W}=\left(W_{1}, \ldots, W_{n}\right)$.

| $W_{1}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 1 |


| $W_{2}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 2 |

Each $W_{i}$ is a possible world.
Typical scenario: 200M people

| $W_{3}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 3 |


| $W_{4}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 4 | ( $2 / 3$ US census), 50 questions, 1 in 10000 ambiguous (2 options)

- $2^{10^{6}}$ possible worlds
- A world is a table with 50 columns and 200M rows!
[Antova'07]
$\rightarrow$ Key challenge: How to succinctly represent incomplete databases?


## Probabilistic Databases

A Probabilistic Database is $(\mathbf{W}, P)$, where $\mathbf{W}$ is an incomplete database and $P: \mathbf{W} \rightarrow[0,1]$ is a probability distribution: $\sum_{W_{i} \in \mathbf{W}} P\left(W_{i}\right)=1$.

| $W_{1}: P\left(W_{1}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 1 |


| $W_{2}: P\left(W_{2}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 2 |


| $W_{3}: P\left(W_{3}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 3 |


| $W_{4}: P\left(W_{4}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 4 |

$$
\begin{aligned}
& \text { For } \mathbf{W}=\left\{W_{1}, \ldots, W_{6}\right\} \\
& \sum_{W_{i} \in \mathbf{W}} P\left(W_{i}\right)=1
\end{aligned}
$$

| $W_{5}: P\left(W_{5}\right)=0.3$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 186 | Brown | $\mathbf{1}$ |


| $W_{6}: P\left(W_{6}\right)=0.3$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 186 | Brown | 2 |

## Succinct Representations of Incomplete/Probabilistic Data



Succinct or-set representation:
[Imielinski'91]

| SSN | N | M |
| :---: | :---: | :---: |
| $\{185,785\}$ | Smith | $\{1,2\}$ |
| $\{185,186\}$ | Brown | $\{1,2,3,4\}$ |

It exploits independence of possible values for different fields:
■ Choice for Smith's SSN independent of choice of for Brown's SSN.

- Likewise, the probability distributions associated with these choices are independent (not shown).


## BID: Alternative Representation of Our Or-Set

| RID | SSN | P |
| :---: | :---: | :---: |
| $t_{1}$ | 185 | 0.7 |
| $t_{1}$ | 785 | 0.3 |
| $t_{2}$ | 185 | 0.8 |
| $t_{2}$ | 186 | 0.2 |


| $\underline{\text { RID }}$ | N | P |
| :---: | :---: | :---: |
| $t_{1}$ | Smith | 1 |
| $t_{2}$ | Brown | 1 |


| $\underline{\mathrm{RID}}$ | M | P |
| :---: | :---: | :---: |
| $t_{1}$ | 1 | 0.9 |
| $t_{1}$ | 2 | 0.1 |
| $t_{2}$ | 1 | 0.25 |
| $t_{2}$ | 2 | 0.25 |
| $t_{2}$ | 3 | 0.25 |
| $t_{2}$ | 4 | 0.25 |

## BID: Alternative Representation of Our Or-Set

| RID | SSN | P |
| :---: | :---: | :---: |
| $t_{1}$ | 185 | 0.7 |
| $t_{1}$ | 785 | 0.3 |
| $t_{2}$ | 185 | 0.8 |
| $t_{2}$ | 186 | 0.2 |


|  |  |  |
| :---: | :---: | :---: |
| RID | N | P |
| $t_{1}$ | Smith | 1 |
| $t_{2}$ | Brown | 1 | | $\underline{\mathrm{RID}}$ | M | P |
| :---: | :---: | :---: |
| $t_{1}$ | 1 | 0.9 |
| $t_{1}$ | 2 | 0.1 |
| $t_{2}$ | 1 | 0.25 |
| $t_{2}$ | 2 | 0.25 |
| $t_{2}$ | 3 | 0.25 |
| $t_{2}$ | 4 | 0.25 |

Interpretation:
■ The tuples within each block with the same key RID are disjoint
Each world contains one tuple per block, so the tuples within a block are mutually exclusive.

## BID: Alternative Representation of Our Or-Set

| RID | SSN | P |
| :---: | :---: | :---: |
| $t_{1}$ | 185 | 0.7 |
| $t_{1}$ | 785 | 0.3 |
| $t_{2}$ | 185 | 0.8 |
| $t_{2}$ | 186 | 0.2 |


| RID | N | P |
| :---: | :---: | :---: |
| $t_{1}$ | Smith | 1 |
| $t_{2}$ | Brown | 1 |


| $\underline{\mathrm{RID}}$ | M | P |
| :---: | :---: | :---: |
| $t_{1}$ | 1 | 0.9 |
| $t_{1}$ | 2 | 0.1 |
| $t_{2}$ | 1 | 0.25 |
| $t_{2}$ | 2 | 0.25 |
| $t_{2}$ | 3 | 0.25 |
| $t_{2}$ | 4 | 0.25 |

Interpretation:

- The tuples within each block with the same key RID are disjoint

Each world contains one tuple per block, so the tuples within a block are mutually exclusive.

- Blocks are independent of each other.

The choices of tuples within different blocks are independent.
The aggregated probability of the worlds taking the first tuple of the first block in each relation is $0.7 \times 1 \times 0.9=0.63$.

These block-independent disjoint (BID) relations are sometimes called x-relations or x-tables. Google squares are prime examples.

## More on BID Databases

BIDs also allow blocks with probabilities less than 1 :

| $\underline{\text { RID }}$ | SSN | P |
| :---: | :---: | :---: |
| $t_{1}$ | 185 | 0.6 |
| $t_{1}$ | 785 | 0.3 |
| $t_{2}$ | 185 | 0.8 |
| $t_{2}$ | 186 | 0.2 |


| $\underline{\mathrm{RID}}$ | M | P |
| :---: | :---: | :---: |
| $t_{1}$ | 1 | 0.8 |
| $t_{1}$ | 2 | 0.1 |
| $t_{2}$ | 1 | 0.25 |
| $t_{2}$ | 2 | 0.25 |
| $t_{2}$ | 3 | 0.25 |
| $t_{2}$ | 4 | 0.25 |

Interpretation:

- There are worlds where the first block of each of the three relations is empty, e.g., the following world:

| $\underline{\text { RID }}$ | SSN | P |
| :---: | :---: | :---: |
| $t_{2}$ | 186 | 0.2 |


| $\underline{\text { RID }}$ | N | P |
| :---: | :---: | :---: |
| $t_{2}$ | Brown | 1 |


| $\underline{\text { RID }}$ | M | P |
| :---: | :---: | :---: |
| $t_{2}$ | 4 | 0.25 |

The probability of this world is

$$
0.2 \times 1 \times 0.25 \times(\mathbf{1}-\mathbf{0 . 6}-\mathbf{0 . 3}) \times(\mathbf{1}-\mathbf{0 . 9}) \times(\mathbf{1}-\mathbf{0 . 8}-\mathbf{0 . 1})=5 \times 10^{-5}
$$

Clarification notes to come with the previous slide and to answer questions posed during the tutorial:

* The two BIDs from the previous two slides are not equivalent since they do not represent the same probabilistic database! Furthermore, by allowing groups with empty instances, some tuples are only partially defined in the column-oriented representation.
* See [Antova'08] for column-oriented representation of relations with attribute-level uncertainty.


## TI: Tuple-Independent Databases

TI databases are BID databases where each block has exactly one tuple.

TI databases are the simplest and most common probabilistic data model.

| RID | SSN | P |
| :---: | :---: | :---: |
| $t_{1}$ | 185 | 0.7 |
| $t_{2}$ | 185 | 0.8 |


| RID | N | P |
| :---: | :---: | :---: |
| $t_{1}$ | Smith | 1 |
| $t_{2}$ | Brown | 1 |


| RID | M | P |
| :---: | :---: | :---: |
| $t_{1}$ | 1 | 0.9 |
| $t_{2}$ | 2 | 0.25 |

Interpretation:

- Each tuple $t$ is in a random world with its probability $p(t)$.
- A relation with $n$ tuples, whose probabilities are less than 1 , has $2^{n}$ possible worlds, since each tuple may be in or out.
■ Our TI example has $2^{4}$ worlds: Any subset of the first and third relation and the entire second relation.


## Are BID Databases Enough?

BIDs (and TIs) are good at capturing independence and local choice.
What about correlations across blocks?

- Enforce the key dependency on SSN in each world.

That is: Discard the worlds where both $t_{1}$ and $t_{2}$ have $\mathrm{SSN}=185$.

| $\underline{\text { RID }}$ | SSN | $\mathbf{P}$ |
| :---: | :---: | :---: |
| $t_{1}$ | 185 | 0.6 |
| $t_{1}$ | 785 | 0.3 |
| $t_{2}$ | 185 | 0.8 |
| $t_{2}$ | 186 | 0.2 |

## Are BID Databases Enough?

BIDs (and Tls) are good at capturing independence and local choice.
What about correlations across blocks?

- Enforce the key dependency on SSN in each world.

That is: Discard the worlds where both $t_{1}$ and $t_{2}$ have $\mathrm{SSN}=185$.

| $\underline{\mathrm{RID}}$ | SSN | P |
| :---: | :---: | :---: |
| $t_{1}$ | 185 | 0.6 |
| $t_{1}$ | 785 | 0.3 |
| $t_{2}$ | 185 | 0.8 |
| $t_{2}$ | 186 | 0.2 |$\quad \Rightarrow \quad$| $\underline{\mathrm{RID}}$ | SSN | $\Phi$ |
| :---: | :---: | :--- |
| $t_{1}$ | 185 | $X=1$ |
| $t_{1}$ | 785 | $X=2$ |
| $t_{2}$ | 185 | $Y=1 \wedge X \neq 1$ |
| $t_{2}$ | 186 | $Y=2$ |

This constraint is supported by a probabilistic version of conditional databases.
[Imielinski'84]
Idea: Use random variables to encode correlations between tuples.

- Exclude the world where $t_{1}$ and $t_{2}$ have the same SSN 185 by using contradicting assignments for variable $X$.
- Transfer probabilities of tuples to probability distributions of variables.


## PC: Probabilistic Conditional Databases

A $P C$ database is $(\mathbf{D}, \mathbf{X}, \Phi)$, where $\mathbf{D}$ is a relational database, $\mathbf{X}$ is a set of independent random variables, and $\Phi$ is a function mapping each tuple in $\mathbf{D}$ to a propositional formula over $\mathbf{X}$.

| $\underline{\mathrm{RID}}$ | SSN | $\Phi$ |
| :---: | :---: | :--- |
| $t_{1}$ | 185 | $X=1$ |
| $t_{1}$ | 785 | $X=2$ |
| $t_{2}$ | 185 | $Y=1 \wedge X \neq 1$ |
| $t_{2}$ | 186 | $Y=2$ |


| VAR | Dom | P |
| :---: | :---: | :---: |
| $X$ | 1 | 0.6 |
| $X$ | 2 | 0.3 |
| $Y$ | 1 | 0.8 |
| $Y$ | 2 | 0.2 |

Interpretation:

- The world table (right) lists the probability distribution for each independent random variable in $\mathbf{X}$.
- Each total valuation of variables in $\mathbf{X}$ defines a world whose probability is the product of probabilities of the variable assignments.
- Each tuple $t$ is conditional on the satisfiability of the formula $\Phi(t)$ and is contained in those worlds defined by valuations that satisfy $\Phi(t)$.

Clarification notes to come with the previous slide and to answer questions posed during the tutorial:

* The PC table from the previous slide is not equivalent to the BID table from two slides ago: While the PC table captures the key dependency on SSN, the BID table does not.
* However, the PC table is not the BID table where the key dependency is enforced: This is because we did not adjust the probabilities of the remaining worlds that satisfy the key dependency.
* The mechanism for this adjustment is called conditioning, see [Koch'08].


## TIs and BIDs are Special Cases of PCs

Recall our previous TI database example:

| RID | SSN | $\mathbf{P}$ |
| :---: | :---: | :---: |
| $t_{1}$ | 185 | 0.7 |
| $t_{2}$ | 185 | 0.8 |


| RID | N | $\mathbf{P}$ |
| :---: | :---: | :---: |
| $t_{1}$ | Smith | 1 |
| $t_{2}$ | Brown | 1 |


| RID | M | P |
| :---: | :---: | :---: |
| $t_{1}$ | 1 | 0.9 |
| $t_{2}$ | 2 | 0.25 |

Here is a PC encoding of the above TI database:

| RID | SSN | $\Phi$ | $\mathbf{P}$ |
| :---: | :---: | :---: | :---: |
| $t_{1}$ | 185 | $s_{1}$ | 0.7 |
| $t_{2}$ | 185 | $s_{2}$ | 0.8 |


| $\underline{\text { RID }}$ | $\mathbf{N}$ | $\Phi$ | $\mathbf{P}$ |
| :---: | :---: | :---: | :---: |
| $t_{1}$ | Smith | $n_{1}$ | 1 |
| $t_{2}$ | Brown | $n_{2}$ | 1 |


| RID | M | $\Phi$ | $\mathbf{P}$ |
| :---: | :---: | :---: | :---: |
| $t_{1}$ | 1 | $m_{1}$ | 0.9 |
| $t_{2}$ | 2 | $m_{2}$ | 0.25 |

Idea:

- Consider a set of Boolean random variables
- Associate each tuple in the TI database with exactly one of them
- For instance, $s_{1}$ annotates $\left(t_{1}, 185\right)$ and $P\left(s_{1}\right)=0.7$

■ World table with variable assignments may be stored explicitly

## Takeaways

Various representations for probabilistic databases of increasing expressiveness.

- Most complex: probabilistic conditioned databases.
[Imielinski'84]
- Trio's ULDBs [Benjelloun'06] and MayBMS' U-relations [Antova'07].
- Completeness: They can represent any probabilistic database.

■ Mid-level: block-independent disjoint databases.
[Barbará'92]

- MystiQ, Trio, MayBMS, SPROUT ${ }^{2}$.
- Prime examples of BIDs: Google squares.
- Not complete, but achieve completeness via conjunctive queries over BIDs.
[Poole'93]
- Simplest: tuple-independent databases.
- The norm in real-world repositories like Google's, DeepDive, and NELL.
- Most theoretical work on complexity of query evaluation done for them.
- Not complete even via unions of conjunctive queries.
- However, inference in Markov Logic Networks is captured by relational queries on TI databases! See Dan Suciu's invited DL'16 talk. Also work by Guy van den Broeck.
[Jha'12]


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Dichotomies for Query Evaluation- The Hard Queries- The Tractable Queries
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## Possible Worlds Semantics

The underlying semantics of query evaluation in probabilistic databases:
Possible worlds semantics: Given a database $\mathbf{W}=\left\{W_{1}, \ldots, W_{n}\right\}$ and a query $Q$, the query answer is $Q(\mathbf{W})=\left\{Q\left(W_{1}\right), \ldots, Q\left(W_{n}\right)\right\}$.

## Possible Worlds Semantics

The underlying semantics of query evaluation in probabilistic databases:
Possible worlds semantics: Given a database $\mathbf{W}=\left\{W_{1}, \ldots, W_{n}\right\}$ and a query $Q$, the query answer is $Q(\mathbf{W})=\left\{Q\left(W_{1}\right), \ldots, Q\left(W_{n}\right)\right\}$.

Investigations so far followed three main directions:

1. Possible and certain query answers for incomplete databases.
2. Probabilities of query answers for probabilistic databases.
3. Succinct representation of $Q(\mathbf{W})$ for query languages and data models.

Approaches $1 \& 2$ close the possible worlds semantics: They compute one relation with answer tuples and possibly their probabilities.

## Queries on Incomplete Databases

Given query $Q$ and incomplete database $\mathbf{W}$ :

- An answer $t$ is certain, if $\forall: W_{i} \in \mathbf{W}, t \in Q\left(W_{i}\right)$
- An answer $t$ is possible if $\exists W_{i} \in \mathbf{W}, t \in Q\left(W_{i}\right)$

| $W_{1}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | $\mathbf{1}$ |


| $W_{2}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 2 |


| $W_{3}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 3 |


| $W_{4}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 4 |


| $W_{5}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 186 | Brown | $\mathbf{1}$ |


| $W_{6}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 186 | Brown | 2 |

## Queries on Incomplete Databases

Given query $Q$ and incomplete database $\mathbf{W}$ :
■ An answer $t$ is certain, if $\forall: W_{i} \in \mathbf{W}, t \in Q\left(W_{i}\right)$
■ An answer $t$ is possible if $\exists W_{i} \in \mathbf{W}, t \in Q\left(W_{i}\right)$

| $W_{1}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | $\mathbf{1}$ |


| $W_{3}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 3 |


| $W_{2}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 2 |

Let $\mathbf{W}=\left\{W_{1}, \ldots, W_{6}\right\}$.
Query
$\exists_{N} \exists_{M} \operatorname{Census}(S, N, M)$ has certain answer (185) and possible answers (185) and (186).

| $W_{5}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 186 | Brown | $\mathbf{1}$ |


| $W_{6}$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 186 | Brown | 2 |

- Query
$\exists_{s} \exists_{M} \operatorname{Census}(S, N, M)$ has the same possible and certain answers (Smith) and (Brown).


## Queries on Incomplete Databases

Several studies on this started back in the 90s for various models, in particular conditional databases.
[Abiteboul'91, O.'08a]
Hard tasks already for positive relational algebra:

- Tuple possibility is NP-complete
- Tuple certainty is coNP-complete

We next focus on probabilistic databases.

## Queries on Probabilistic Databases

Given query $Q$ and probabilistic database $(\mathbf{W}, P)$ : The Marginal Probability of an answer $t$ is: $P(t)=\sum\left\{P\left(W_{i}\right) \mid W_{i} \in \mathbf{W}, t \in Q\left(W_{i}\right)\right\}$.

| $W_{1}: P\left(W_{1}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 1 |


| $W_{2}: P\left(W_{2}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 2 |


| $W_{3}: P\left(W_{3}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 3 |


| $W_{4}: P\left(W_{4}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 4 |


| $W_{5}: P\left(W_{5}\right)=0.3$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 186 | Brown | $\mathbf{1}$ |


| $W_{6}: P\left(W_{6}\right)=0.3$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 186 | Brown | 2 |

## Queries on Probabilistic Databases

Given query $Q$ and probabilistic database $(\mathbf{W}, P)$ : The Marginal Probability of an answer $t$ is: $P(t)=\sum\left\{P\left(W_{i}\right) \mid W_{i} \in \mathbf{W}, t \in Q\left(W_{i}\right)\right\}$.

| $W_{1}: P\left(W_{1}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 1 |


| $W_{2}: P\left(W_{2}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 2 |

$$
\begin{aligned}
& \text { Let } \mathbf{W}=\left\{W_{1}, \ldots, W_{6}\right\} \\
& \square \exists_{N} \exists_{M} \operatorname{Census}(S, N, M) \text { : } \\
& P(185)=1 \text { and } \\
& P(186)=0.6 \\
& \square \exists_{s} \exists_{M} \operatorname{Census}(S, N, M) \text { : } \\
& P(\text { Smith })=P(\text { Brown })=1
\end{aligned}
$$

| $W_{3}: P\left(W_{3}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 3 |


| $W_{4}: P\left(W_{4}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 4 |


| $W_{5}: P\left(W_{5}\right)=0.3$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 186 | Brown | $\mathbf{1}$ |


| $W_{6}: P\left(W_{6}\right)=0.3$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 186 | Brown | 2 |

These are trivial queries! Computing the marginal probability is hard in general!

## Queries on Probabilistic Databases

Given query $Q$ and probabilistic database $(\mathbf{W}, P)$ : The Marginal Probability of an answer $t$ is: $P(t)=\sum\left\{P\left(W_{i}\right) \mid W_{i} \in \mathbf{W}, t \in Q\left(W_{i}\right)\right\}$.

| $W_{1}: P\left(W_{1}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 1 |


| $W_{2}: P\left(W_{2}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 2 |

$$
\begin{aligned}
& \text { Let } \mathbf{W}=\left\{W_{1}, \ldots, W_{6}\right\} \\
& \square \exists_{N} \exists_{M} \operatorname{Census}(S, N, M): \\
& P(185)=1 \text { and } \\
& P(186)=0.6 \\
& \exists_{S} \exists_{M} \operatorname{Census}(S, N, M): \\
& P(\text { Smith })=P(\text { Brown })=1 .
\end{aligned}
$$

| $W_{3}: P\left(W_{3}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 3 |


| $W_{4}: P\left(W_{4}\right)=0.1$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 185 | Brown | 4 |


| $W_{5}: P\left(W_{5}\right)=0.3$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 186 | Brown | $\mathbf{1}$ |


| $W_{6}: P\left(W_{6}\right)=0.3$ |  |  |
| :---: | :---: | :---: |
| SSN | N | M |
| 185 | Smith | 1 |
| 186 | Brown | 2 |

These are trivial queries! Computing the marginal probability is hard in general!
$\rightarrow$ Key challenge: Which queries admit efficient (polynomial time) computation of marginal probabilities for their answers?

## Representability of Query Answers

For a given query language $\mathcal{Q}$ and data model $\mathcal{W}$ :
For any query $Q \in \mathcal{Q}$ and database $\mathbf{W} \in \mathcal{W}$, is there $\bar{Q} \in \mathcal{Q}$ such that $\bar{Q}(\mathbf{W})=$ $\left\{Q\left(W_{i}\right) \mid W_{i} \in \mathbf{W}\right\}$ and can be represented in $\mathcal{W}$ ?


- This holds for relational algebra and PC databases:
[Imielinski'84] $\bar{Q}(T)$ is an extension of $Q$ to also compute the query lineage.
- This does not hold for BIDs and Tls, but query lineage still useful for computing marginal probabilities of query answers on BIDs and TIs.
- This idea is also used by Trio and MayBMS. [Das Sarma'06, Antova'08]


## Query Lineage by Example

| Customer |  |  | Orders |  |  |  | Lineitem |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | okey | disc | ckey | $\Phi$ |
| ckey | name | $\Phi$ |  |  |  |  | okey | ckey | date | $\Phi$ | 1 | 0.1 | 1 | $z_{1}$ |
| 1 | Joe | $x_{1}$ |  |  | 1995-01-10 |  | 1 | 0.2 | 1 | $z_{2}$ |
| 2 | Dan | $x_{2}$ | 2 | 1 | 1996-01-09 | $y_{2}$ | 3 | 0.4 | 2 | $z_{3}$ |
|  |  |  | 3 | 2 | 1994-11-11 | $y_{3}$ | 3 | 0.1 | 2 | $z_{4}$ |

Query asking for the dates of discounted orders shipped to customer 'Joe': $\exists_{C} \exists_{O} \exists_{D}$ Customer ( $C$, Joe $)$, Orders ( $O, C, D$ ), Lineitem ( $O, S, C$ ), $S>0$

| Query answer and lineage |  |
| :---: | :---: |
| odate | $\Phi$ |
| $1995-01-10$ | $x_{1} y_{1} z_{1}+x_{1} y_{1} z_{2}$ |

$\bar{Q}$ does $Q$ and propagates the input conditions $\Phi$ to the answers:

- join of tuples leads to conjunction of their conditions
- union/disjunction of tuples leads to disjunction of their conditions.

Query lineage traces the computation of an answer back to its input.

## Marginal Probabilities via Query Lineage

The marginal probability of a query answer is the probability of its lineage.
How to compute the lineage probability?

| $x_{1}$ | $y_{1}$ | $z_{1}$ | $z_{2}$ | $x_{1} y_{1} z_{1}+x_{1} y_{1} z_{2}$ | Probability |
| :---: | :---: | :---: | :---: | :---: | :--- |
| 0 | $*$ | $*$ | $*$ | 0 | 0 |
| 1 | 0 | $*$ | $*$ | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | $P\left(x_{1}\right) \cdot P\left(y_{1}\right) \cdot\left(1-P\left(z_{1}\right)\right) \cdot P\left(z_{2}\right)$ |
| 1 | 1 | 1 | 0 | 1 | $P\left(x_{1}\right) \cdot P\left(y_{1}\right) \cdot P\left(z_{1}\right) \cdot\left(1-P\left(z_{2}\right)\right)$ |
| 1 | 1 | 1 | 1 | 1 | $P\left(x_{1}\right) \cdot P\left(y_{1}\right) \cdot P\left(z_{1}\right) \cdot P\left(z_{2}\right)$ |

$P\left(x_{1} y_{1} z_{1}+x_{1} y_{1} z_{2}\right)=P\left(x_{1}\right) \cdot P\left(y_{1}\right) \cdot\left[1-\left(1-P\left(z_{1}\right)\left(1-P\left(z_{2}\right)\right)\right]\right.$.
■ Going over its truth table is exponential in the number of variables.
Two ideas:
[O.'08b]

- Read-once lineage factorization
$x_{1} y_{1} z_{1}+x_{1} y_{1} z_{2}=x_{1} y_{1}\left(z_{1}+z_{2}\right)$
■ Lineage compilation into polysize decision diagrams.


## Where Are We Now?

- We know how to compute the query answers using a simple query extension that also computes the query lineage.
- We do not know yet how to compute the marginal probabilities of query answers efficiently.

Next part of the tutorial:

- Analyze the complexity of computing marginal probabilities as a function of database size and query structure.


## Outline


Why Probabilistic Databases?Probabilistic Data ModelsThe Query Evaluation Problem
Dichotomies for Query Evaluation

- The Hard Queries
- The Tractable Queries
Ranking Queries
Next Steps
References


## Short Recap on Complexity Class \#P (Sharp P)

\# $\mathrm{P}=$ Class of functions $f(x)$ for which there exists a PTIME non-deterministic Turing machine $M$ such that $f(x)=$ number of accepting computations of $M$ on input $x$.

Class of counting problems associated with decision problems in NP:

- SAT (given formula $\phi$, is $\phi$ satisfiable?) is NP-complete

■ \#SAT (given formula $\phi$, count \# of satisfying assignments) is \#P-complete

A PTIME machine with a \#P oracle can solve any problem in polynomial hierarchy with one \#P query.
[Toda'91]
\#SAT is \#P-complete already for bipartite positive DNFs!
[Provan'83]

- .. yet SAT is trivially PTIME for DNFs.


## Dichotomies for Queries on Probabilistic Databases

The following property has been observed for several classes $\mathcal{Q}$ of relational queries on TI databases:

The data complexity of every query in $\mathcal{Q}$ is either polynomial time or \#P-hard.

## Dichotomies for Queries on Probabilistic Databases

The following property has been observed for several classes $\mathcal{Q}$ of relational queries on Tl databases:

The data complexity of every query in $\mathcal{Q}$ is either polynomial time or \#P-hard.

Examples of such classes $\mathcal{Q}$ of relational queries:

- NCQ: non-repeating conjunctive queries
[Dalvi'07]
■ NCQs under functional dependencies
- Quantified queries (division, set comparisons)
[Fink'11]
- UCQ: unions of conjunctive queries
[Dalvi'12]
- RNCQ: ranking NCQ
- $1 \mathrm{RA}^{-}$: NCQ's relational algebra counterpart extended with negation
[Fink'16]


## Syntactic Characterizations of Tractable Queries

The tractable queries in (R)NCQ and 1RA ${ }^{-}$admit an efficient syntactic characterization via the hierarchical property.

A (Boolean) NCQ or $1 \mathrm{RA}^{-}$query $Q$ is hierarchical if:
For every pair of distinct variables $A$ and $B$ in $Q$, there is no triple of relation symbols $R, S$, and $T$ in $Q$ such that:

- $R^{A \neg B}$ has query variable $A$ and not $B$,
- $S^{A B}$ has both query variables $A$ and $B$, and
- $T^{\neg A B}$ has query variable $B$ and not in $A$.


## Examples

Non-hierarchical queries:

- $\exists_{A} \exists_{B}[R(A) \wedge S(A, B) \wedge T(B)]$
- $\exists_{B}\left[\exists_{A}(R(A) \wedge S(A, B)) \wedge \neg T(B)\right]$

■ $\exists_{B}\left[T(B) \wedge \neg \exists_{A}(R(A) \wedge S(A, B))\right]$

## Examples

Non-hierarchical queries:

- $\exists_{A} \exists_{B}[R(A) \wedge S(A, B) \wedge T(B)]$
- $\exists_{B}\left[\exists_{A}(R(A) \wedge S(A, B)) \wedge \neg T(B)\right]$
$\square \exists_{B}\left[T(B) \wedge \neg \exists_{A}(R(A) \wedge S(A, B))\right]$



## Examples

Hierarchical queries:

- $\exists_{A} \exists_{B}[(R(A) \wedge S(A, B)) \wedge \neg T(A, B)]$
- $\exists_{A} \exists_{B}[(R(A) \wedge T(B)) \wedge \neg(U(A) \wedge V(B))]$
- $\exists_{A} \exists_{B}[(M(A) \wedge N(B)) \wedge \neg[(R(A) \wedge T(B)) \wedge \neg(U(A) \wedge V(B))]]$


## Examples

Hierarchical queries:

- $\exists_{A} \exists_{B}[(R(A) \wedge S(A, B)) \wedge \neg T(A, B)]$
- $\exists_{A} \exists_{B}[(R(A) \wedge T(B)) \wedge \neg(U(A) \wedge V(B))]$
- $\exists_{A} \exists_{B}[(M(A) \wedge N(B)) \wedge \neg[(R(A) \wedge T(B)) \wedge \neg(U(A) \wedge V(B))]]$



## Outline

## Why Probabilistic Databases?



## Probabilistic Data Models

The Query Evaluation Problem
Dichotomies for Query Evaluation

- The Hard Queries
- The Tractable Queries

Ranking Queries
Next Steps
References

## Hardness Proof Idea

Reduction from \#P-hard model counting problem for positive bipartite DNF:
■ Given a non-hierarchical $1 \mathrm{RA}^{-}$query $Q$ and

- Any positive bipartite DNF formula $\Psi$ over disjoint sets $\mathbf{X}$ and $\mathbf{Y}$ of random variables.

■ \# $\Psi$ can be computed using linearly (in most cases constantly) many calls to an oracle for $P(Q)$, where $Q$ is evaluated on tuple-independent databases with sizes polynomial in the size of $\Psi$.

## Simplest Example of Hardness Reduction

[Grädel'98, Dalvi'07]
Input formula and query:
■ $\Psi=x_{1} y_{1} \vee x_{1} y_{2} \vee x_{2} y_{1}$ over sets $\mathbf{X}=\left\{x_{1}, x_{2}\right\}, \mathbf{Y}=\left\{y_{1}, y_{2}\right\}$

- $Q=\exists_{A} \exists_{B}[R(A) \wedge S(A, B) \wedge T(B)]$

Construct a TI database $\mathbf{D}$ such that $\Psi$ annotates $Q(\mathbf{D})$ :

- Column $\Phi$ holds random variables in $\Psi$.
- Notation: T (true)
- Variables also used as constants for $A$ and $B$.

■ $S\left(x_{i}, y_{j}, \top\right): x_{i} y_{j}$ is a clause in $\Psi$.
■ $R\left(x_{i}, \mathbf{x}_{\mathbf{i}}\right)$ and $T\left(y_{j}, \mathbf{y}_{\mathbf{j}}\right): x_{i}$ is a variable in $\mathbf{X}$ and $y_{j}$ is a variable in $\mathbf{Y}$.

| $R$ | $T$ | $S$ | $R \wedge S \wedge T$ | $Q$ |
| :---: | :---: | :---: | :---: | :---: |
| A ${ }^{\text {d }}$ | $B$ ¢ | $A B \Phi$ | A B $\quad$ ¢ | Ф |
| ${ }_{1} \mathbf{x}_{1}$ | $y_{1} \mathbf{y}_{1}$ | $x_{1} y_{1} \top$ | $x_{1} y_{1} \mathrm{x}_{1} \mathrm{y}_{1}$ | () $\Psi$ |
| $x_{2} \mathbf{x}_{2}$ | $y_{2} \mathbf{y}_{2}$ | $x_{1} y_{2} \top$ | $x_{1} y_{2} x_{1} y_{2}$ |  |
|  |  | $x_{2} y_{1} \top$ | $x_{2} y_{1} \mathbf{x}_{2} \mathbf{y}_{1}$ |  |

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| $R$ | $T$ | $S$ | $R \wedge S \wedge T$ | $Q$ |
| :---: | :---: | :---: | :---: | :---: |
| A ${ }^{\text {d }}$ | $B$ ¢ | A $B$ ¢ | A B $\Phi^{\text {d }}$ | $\Phi$ |
| $x_{1} \mathrm{x}_{1}$ | $y_{1} \mathbf{y}_{1}$ | $x_{1} y_{1} \top$ | $x_{1} y_{1} \mathbf{x}_{1} \mathrm{y}_{1}$ | () $\Psi$ |
| $x_{2} \mathbf{x}_{2}$ | $y_{2} \mathbf{y}_{2}$ | $x_{1} y_{2} \top$ | $x_{1} y_{2} \mathrm{x}_{1} \mathrm{y}_{2}$ |  |
|  |  | $x_{2} y_{1} \top$ | $x_{2} y_{1} \mathbf{x}_{2} y_{1}$ |  |

Query $Q$ is the only minimal hard pattern in case of queries without negation!

## A Surprising Example of Hardness Reduction

Input formula and query:
[Fink'16]
■ $\Psi=x_{1} y_{1} \vee x_{1} y_{2}$ over sets $\mathbf{X}=\left\{x_{1}\right\}, \mathbf{Y}=\left\{y_{1}, y_{2}\right\}$

- $Q=\exists_{A}\left[R(A) \wedge \neg \exists_{B}(T(B) \wedge S(A, B))\right]$

Construct a TI database $\mathbf{D}$ such that $\Psi$ annotates $Q(\mathbf{D})$ :
$\square S(i, b, \top)$ : Clause $i$ in $\Psi$ has variable $b$.
■ $R(i, \top)$ and $T(b, \neg b): i$ is a clause and $b$ is a variable in $\Psi$.

| $R$ | $T$ | $S$ | $T \wedge S$ | $\exists_{B}(T \wedge S)$ | $R \wedge \neg \exists_{B}(T \wedge S)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A \Phi$ | $B$ ¢ | $A B$ ¢ | $A B \quad \Phi$ | $A \quad \Phi$ | A | Ф |
| 1 T | $x_{1} \neg \mathrm{x}_{1}$ | $1 x_{1}{ }^{\top}$ | $1 \mathrm{x}_{1} \neg \mathrm{x}_{1}$ | $1 \neg \mathrm{x}_{1} \vee \neg \mathrm{y}_{1}$ | 1 | $\mathrm{x}_{1} \mathrm{y}_{1}$ |
| 2 T | $y_{1} \neg \mathrm{y}_{1}$ | $1 y_{1} \top$ | $1 y_{1} \neg \mathrm{y}_{1}$ | $2 \neg \mathrm{x}_{1} \vee \neg \mathrm{y}_{2}$ | 2 | $\mathrm{x}_{1} \mathrm{y}_{2}$ |
|  | $y_{2} \neg \mathbf{y}_{2}$ | $2 x_{1} \top$ | $2 \mathrm{x}_{1} \neg \mathrm{x}_{1}$ |  |  |  |
|  |  | $2 y_{2}$ T | $2 y_{2} \neg \mathrm{y}_{2}$ |  |  |  |

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Input formula and query:
[Fink'16]
■ $\Psi=x_{1} y_{1} \vee x_{1} y_{2}$ over sets $\mathbf{X}=\left\{x_{1}\right\}, \mathbf{Y}=\left\{y_{1}, y_{2}\right\}$

- $Q=\exists_{A}\left[R(A) \wedge \neg \exists_{B}(T(B) \wedge S(A, B))\right]$

Construct a TI database $\mathbf{D}$ such that $\Psi$ annotates $Q(\mathbf{D})$ :

- $S(i, b, \top)$ : Clause $i$ in $\Psi$ has variable $b$.

■ $R(i, \top)$ and $T(b, \neg b): i$ is a clause and $b$ is a variable in $\Psi$.

| $R$ | $T$ | $S$ | $T \wedge S$ | $\exists_{B}(T \wedge S)$ | $R \wedge \neg \exists_{B}(T \wedge S)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ ¢ | $B \quad \Phi$ | $A B \Phi$ | $A B \quad \Phi$ | $A \quad \Phi$ | A | Ф |
| 1 T | $x_{1} \neg \mathrm{x}_{1}$ | $1 x_{1}{ }^{\top}$ | $1 \mathrm{x}_{1} \neg \mathrm{x}_{1}$ | $1 \neg \mathrm{x}_{1} \vee \neg \mathrm{y}_{1}$ | 1 | $\mathrm{x}_{1} \mathrm{y}_{1}$ |
| 2 T | $y_{1} \neg \mathbf{y}_{1}$ | $1 y_{1} \top$ | $1 y_{1} \neg \mathrm{y}_{1}$ | $2 \neg \mathrm{x}_{1} \vee \neg \mathrm{y}_{2}$ | 2 | $\mathrm{x}_{1} \mathrm{y}_{2}$ |
|  | $y_{2} \neg \mathbf{y}_{2}$ | $2 x_{1} \top$ | $2 \mathrm{x}_{1} \neg \mathrm{x}_{1}$ |  |  |  |
|  |  | $2 y_{2} \top$ | $2 y_{2} \neg \mathrm{y}_{2}$ |  |  |  |

Query $Q$ is already hard when $T$ is the only uncertain input relation!

## Outline



Why Probabilistic Databases?<br>Probabilistic Data Models<br>The Query Evaluation Problem<br>Dichotomies for Query Evaluation<br>- The Hard Queries

- The Tractable Queries


## Ranking Queries

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## Evaluation of Hierarchical 1RA- Queries

Approach based on knowledge compilation

- For any TI database $\mathbf{D}$, the probability $P_{Q(\mathbf{D})}$ of a 1 RA ${ }^{-}$query $Q$ is the probability $P_{\psi}$ of the query lineage $\Psi$.
- Compile $\Psi$ into poly-size $\operatorname{OBDD}(\Psi)$.
- Compute probability of $\operatorname{OBDD}(\Psi)$ in time linear in its size.


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- Compile $\Psi$ into poly-size $\operatorname{OBDD}(\Psi)$.

■ Compute probability of $\operatorname{OBDD}(\Psi)$ in time linear in its size.

Lineage of tractable $1 \mathrm{RA}^{-}$queries:

- Read-once for queries without negation (so NCQ) It admits linear-size OBBDs.
- Not read-once for queries with negation
[Fink'16]
- It admits OBBDs of size linear in the database size but exponential in the query size.


## The Inner Workings

From hierarchical $1 \mathrm{RA}^{-}$to RC-hierarchical $\exists$-consistent $\mathrm{RC}^{\exists}$ :

- Translate query $Q$ into an equivalent disjunction of disjunction-free existential relational calculus queries $Q_{1} \vee \cdots \vee Q_{k}$.
- RC-hierarchical:

For each $\exists_{X}\left(Q^{\prime}\right)$, every relation symbol in $Q^{\prime}$ has variable $X$.

- Each of the disjuncts gives rise to a poly-size OBDD.
- $\exists$-consistent:

The nesting order of the quantifiers is the same in $Q_{1}, \cdots, Q_{k}$.

- All OBDDs have compatible variable orders and their disjunction is a poly-size OBDD.
- The OBDD width grows exponentially with $k$, its height stays linear in the size of the database.
- Width = maximum number of edges crossing the section between any two consecutive levels.

Similar ideas used for the evaluation of inversion-free UCQs.

## Query Evaluation Example (1/3)

Consider the following query and TI database:

$$
\begin{aligned}
& Q=\exists_{A} \exists_{B}[(R(A) \wedge T(B)) \wedge \neg(U(A) \wedge V(B))]
\end{aligned}
$$

## Query Evaluation Example (1/3)

Consider the following query and TI database:

$$
Q=\exists_{A} \exists_{B}[(R(A) \wedge T(B)) \wedge \neg(U(A) \wedge V(B))]
$$

The lineage of $Q$ is:

$$
\Psi=r_{1}\left[t_{1}\left(\neg u_{1} \vee \neg v_{1}\right) \vee t_{2}\left(\neg u_{1} \vee \neg v_{2}\right)\right] \vee r_{2}\left[t_{1}\left(\neg u_{2} \vee \neg v_{1}\right) \vee t_{2}\left(\neg u_{2} \vee \neg v_{2}\right)\right] .
$$

- Variables entangle in $\Psi$ beyond read-once factorization.
- This is the pivotal intricacy introduced by negation.


## Query Evaluation Example (2/3)

Translate $Q=\exists_{A} \exists_{B}[(R(A) \wedge T(B)) \wedge \neg(U(A) \wedge V(B))]$ into $\mathrm{RC}^{\exists}:$

$$
Q_{R C}=\underbrace{\exists_{A}(R(A) \wedge \neg U(A)) \wedge \exists_{B} T(B)}_{Q_{1}} \vee \underbrace{\exists_{A} R(A) \wedge \exists_{B}(T(B) \wedge \neg V(B))}_{Q_{2}} .
$$

- Both $Q_{1}$ and $Q_{2}$ are RC-hierarchical.
- $Q_{1} \vee Q_{2}$ is $\exists$-consistent: Same order $\exists_{A} \exists_{B}$ for $Q_{1}$ and $Q_{2}$.

Query annotation:

$$
\Psi=\underbrace{\left(r_{1} \neg u_{1} \vee r_{2} \neg u_{2}\right) \wedge\left(t_{1} \vee t_{2}\right)}_{\psi_{1}} \vee \underbrace{\left(r_{1} \vee r_{2}\right) \wedge\left(t_{1} \neg v_{1} \vee t_{2} \neg v_{2}\right)}_{\psi_{2}} .
$$

■ Both $\Psi_{1}$ and $\Psi_{2}$ admit linear-size OBDDs.

- The two OBDDs have compatible orders and their disjunction is an OBDD whose width is the product of the widths of the two OBDDs.


## Query Evaluation Example (3/3)

Compile query annotation into OBDD:

$$
\Psi=\underbrace{\left(r_{1} \neg u_{1} \vee r_{2} \neg u_{2}\right) \wedge\left(t_{1} \vee t_{2}\right)}_{\Psi_{1}} \vee \underbrace{\left(r_{1} \vee r_{2}\right) \wedge\left(t_{1} \neg v_{1} \vee t_{2} \neg v_{2}\right)}_{\Psi_{2}} .
$$



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## Ranking Answers in Probabilistic Databases

Given a NCQ query $Q$, a TI database $\mathbf{D}$, and any two answers $t_{1}, t_{2} \in Q(\mathbf{D})$, does $P\left(t_{1}\right) \leq P\left(t_{2}\right)$ hold?

Motivation:

- Probabilities are mere degrees of uncertainty in the data and are not otherwise meaningful to the user.
- Users mostly care about the ranking of answers in decreasing order of their probabilities or about a few most likely answers.


## Ranking versus Query Evaluation

Two complementary observations

1. Probability computation for distinct answers may share a common factor

- That can be computed only once
- Save computation time for both query evaluation and ranking!
- Or that can be uniformly ignored for all answers.
- For ranking purposes, we may ignore computationally hard tasks!

Ranking is computationally easier than query evaluation.

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Ranking is computationally easier than query evaluation.
2. To compute the exact ranking of query answers, approximate probabilities of the individual answers may suffice.

- Compute lower and upper bounds on these probabilities.
- Incrementally refine the bounds to the extent needed to rank the answers.


## Share Query Plans and Anytime Approximation

Approach with two main ingredients

1. Share query plans to detect factors common to query answers

- Static analysis on the query structure to identify subqueries whose computation can be shared across distinct query answers.
- Equivalently, they identify factors shared by lineage of query answers.

2. Ranking based on anytime deterministic approximate inference

- Incremental compilation of lineage with shared factors into BDDs
- Each compilation step refines lower and upper bounds on lineage probabilities


## Share Query Plans and Anytime Approximation

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[O.'12]

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Alternative approach using FPRAS-based Monte Carlo

- Ranking with probabilistic guarantee only
- Not truly incremental

■ Black box approach, structure and common factors of query lineage not exploited.

## Example

List topics posted by users who have mentioned their followers: $Q(X)=\exists_{Y} \exists_{Z} \exists u \operatorname{Trends}(X, Y)$, Follows $(Y, Z)$, Mentions $(U, Y, Z)$, Tweets $(U, Y)$. (User $Y$ contributed to trendy topic $X$, user $Y$ follows user $Z$, user $Y$ mentions user $Z$ in tweet $U$, tweet $U$ of user $Y$.)

A share plan for $Q$ is as follows

and corresponds to the following rewriting:

$$
\begin{aligned}
Q(X) & =\operatorname{Trends}(X, Y), Q^{\prime}(Y) \\
Q^{\prime}(Y) & =\operatorname{Follows}(Y, Z), \operatorname{Mentions}(U, Y, Z), \operatorname{Tweets}(U, Y)
\end{aligned}
$$

- Several answers ( $X$-values) can be paired with the same value $y$ of the variable $Y$ and thus share the lineage $Q^{\prime}(y)$.

■ For any value $y$, the query $Q^{\prime}(y)$ is non-hierarchical and thus \#P-hard!

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## Next Steps

- Declarative Probabilistic Programming with Datalog: Probabilities become first-class citizens in the query language.
- PPDL, semantics given by a notion of probabilistic chase
[Bárány'16]
- Incorporating ontologies
- Vast literature (including MLNs) but missing the declarativity aspect!


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■ Further push the barrier on complexity

- See Dan Suciu's advanced lecture on lifted inference!
- Understand tractability for probabilistic programs at large


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- Build open-source systems, provide benchmarks


## Outline

Why Probabilistic Databases?
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Thank you!

