Network as a computer: Ranking paths to find flows

Dusko Pavlovic

Oxford University and Kestrel Institute

CSR, Moscow
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Outline

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Introduction

Definition

A network is an annotated graph

$$A \quad = \quad \left(R \xleftarrow{\varphi} E \underset{\varrho}{\overset{\delta}{\rightrightarrows}} N\right)$$

where

- N is a finite set of *nodes*,
- E is a finite set of *edges* (or links),
- R is an ordered rig of *rates* (e.g. $\mathbb{R}_+$).

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Notation

- $i \overset{e}{\underset{v}{\rightarrow}} j$ denotes $e \in E$ such that

$$\delta(e) = i, \;\; \varrho(e) = j \;\text{ and }\; \varphi(e) = v$$

Introduction

Networks are used to model

- social groups
- traffic, distribution systems,
- Web, Internet
- protein interactions, gene regulation, metabolism,
- food webs, populations,
- neural nets,
- probabilistic grammars (generative, phonological)

Problem and objective

Problem

Networks get large and complex.

Problem and objective

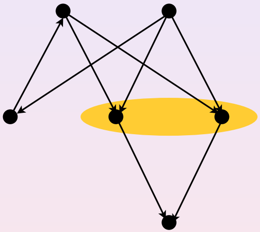
Problem

Networks get large and complex.

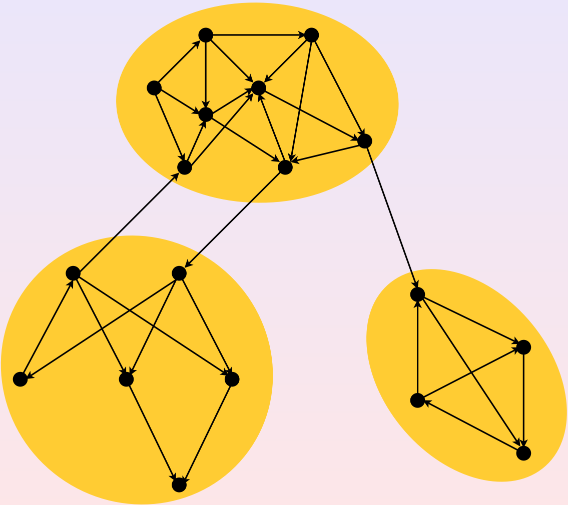
Objective

Simplify them. . .

. . . by clustering similar nodes



. . . by extracting functional modules



Problem and objective

Problem of the Web

Data structures and semantics vary from node to node.

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Solutions

- the Semantic Web
- search, latent semantics
  - extract structure from network
  - **concepts = communities = modules**

Approach

Notation

Given a network $A = \left(R \xleftarrow{\varphi} E \underset{\varrho}{\overset{\delta}{\rightrightarrows}} N\right)$, define

- *total flow* $A_{ij} = \sum_{i \xrightarrow{e} j} \varphi(e)$

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- *total flow* $A_{ij} = \sum_{i \stackrel{e}{\to} j} \varphi(e)$
- *flow distribution* $\Phi_{ij} = \frac{A_{ij}}{A_{\bullet\bullet}}$,
  where $A_{\bullet\bullet} = \sum_{ij} A_{ij}$

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- *flow distribution* $\Phi_{ij} = \frac{A_{ij}}{A_{\bullet\bullet}}$,
  where $A_{\bullet\bullet} = \sum_{ij} A_{ij}$
- *flow bias* $\Upsilon_{ij} = \Phi_{ij} - \Phi_{i\bullet}\Phi_{\bullet j}$
  where $\Phi_{i\bullet} = \sum_k \Phi_{ik}$ and $\Phi_{\bullet j} = \sum_k \Phi_{kj}$.

Cohesion and adhesion

Definition

Cohesion of $U \subseteq N$ is the total flow bias between its members

$$\mathrm{Coh}(U) \;=\; \sum_{i,j \in U} \Upsilon_{ij}$$

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Adhesion of $U \subseteq N$ is the total flow bias of its members and nonmembers

$$\mathrm{Adh}(U) = \sum_{i \in U, j \notin U} \Upsilon_{ij} + \Upsilon_{ji}$$

Modularity

Definition

Modularity of $U \subseteq N$ is the difference of its cohesion and its adhesion

$$\mathrm{Mdu}(U) = \mathrm{Coh}(U) - \mathrm{Adh}(U)$$

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Idea

Find modules by maximizing modularity.

Task

Finding network modules boils down to evaluating

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- flow distribution $\Phi : N \times N \longrightarrow [0, 1]$,

  which is induced by
- flow $\varphi : E \longrightarrow R$

Problems

- flows unknown
- network dynamics unknown
- modules disjoint and hard to compute

Problems

- flows unknown

  Step 1: use cost and paths to estimate flows

- network dynamics unknown

  Step 2: use Markovian and ranking methods

- modules disjoint and hard to compute

  Step 3: parametrize modularity

Outline

Cost instead of flow

Modified definition

A network is a labelled graph

$$A \quad = \quad \left(R \xleftarrow{\gamma} E \underset{\varrho}{\overset{\delta}{\rightrightarrows}} N\right)$$

where the **cost** γ determines the likely flow φ

$$\varphi(e) \quad = \quad 2^{-\gamma(e)}$$

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$$\varphi(e) = 2^{-\gamma(e)}$$

The estimated total flow $i \to j$ in A is now thus

$$A_{ij} = \sum_{i \xrightarrow{e} j} 2^{-\gamma(e)}$$

Path completion

Definition

Given

- network $A = \left(R \xleftarrow{\gamma} E \underset{\varrho}{\overset{\delta}{\rightrightarrows}} N\right)$,
- cutoff value $v \in R$, and
- length penalty $d \in R$,

Path completion

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- network $A^{*vd} = \left(R \xleftarrow{\gamma} E^{*vd} \underset{\varrho}{\overset{\delta}{\rightrightarrows}} N\right)$

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- links $E^{*vd} = \{a \in E^+ \mid \gamma(a) \leq v\}$
  - E^+ is the set of nonempty paths
- cost $\gamma\left(i_0 \xrightarrow{a_1} i_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} i_n\right) = (n-1)d + \gamma(a_1) + \cdots + \gamma(a_n)$.

Outline

Simple network dynamics

forward: probability that traffic at i flows to j

$$A^{\triangleright}_{ij} = \frac{A_{ij}}{A_{i\bullet}} \text{ where}$$

$$A_{i\bullet} = \sum_{k=1}^{N} A_{ik}$$

backward: probability that traffic at j flows from i

$$A^{\triangleleft}_{ij} = \frac{A_{ij}}{A_{\bullet j}} \text{ where}$$

$$A_{\bullet j} = \sum_{k=1}^{N} A_{kj}$$

Simple node ranking

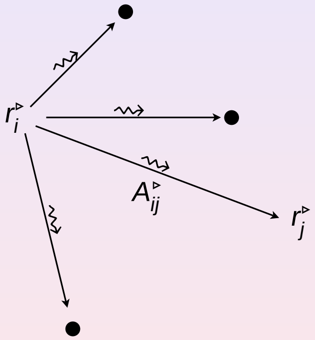
pull rank (reputation): probability that the traffic arrives at j

$$r^{\triangleright}_j = \sum_{k=1}^{N} r^{\triangleright}_k A^{\triangleright}_{kj}$$

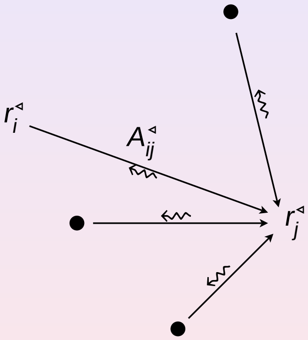
push rank (promotion): probability that the traffic departs from i

$$r^{\triangleleft}_i = \sum_{k=1}^{N} A^{\triangleleft}_{ik} r^{\triangleleft}_k$$

Simple pull rank: Reputation



Simple push rank: Promotion



One-hop dynamics

forward out: probability that traffic at i flows to a hub j

$$A^{\blacktriangleright}_{ij} = A^{\triangleright}_{ij} \cdot \Phi_{j\bullet} \text{ where}$$

$$\Phi_{j\bullet} = \frac{\sum_k A_{jk}}{\sum_{\ell k} A_{\ell k}}$$

backward in: probability that traffic at j flows from an authority i

$$A^{\blacktriangleleft}_{ij} = \Phi_{\bullet i} \cdot A^{\triangleleft}_{ij} \text{ where}$$

$$\Phi_{\bullet i} = \frac{\sum_k A_{ki}}{\sum_{k\ell} A_{k\ell}}$$

One-hop ranking

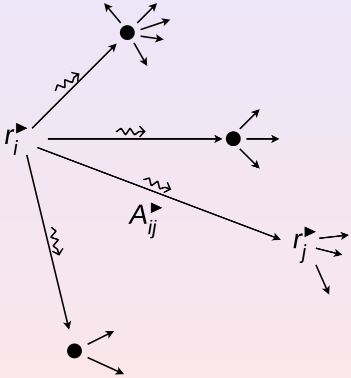
pull-out rank: probability that traffic arrives to a hub j

$$r^{\blacktriangleright}_j = \sum_{k=1}^{N} r^{\blacktriangleright}_k A^{\blacktriangleright}_{kj}$$

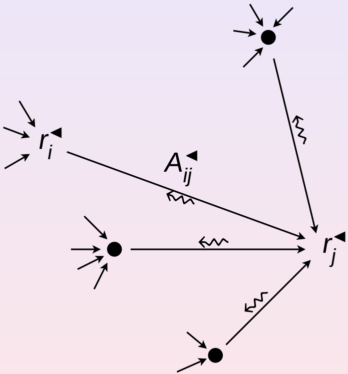
push-in rank: probability that traffic departs from an authority i

$$r^{\blacktriangleleft}_i = \sum_{k=1}^{N} A^{\blacktriangleleft}_{ik} r^{\blacktriangleleft}_k$$

Pull-out rank



Push-in rank



$r_i^\blacktriangleleft$
$A_{ij}^\blacktriangleleft$
$r_j^\blacktriangleleft$

Unibased flow

If the traffic from j to k is only driven by

- j's push $r_j^{\blacktriangleright}$, and by
- k's pull $r_k^{\blacktriangleleft}$,

which are assumed to be mutually independent

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If the traffic from j to k is only driven by

- j's push $r_j^{\blacktriangleright}$, and by
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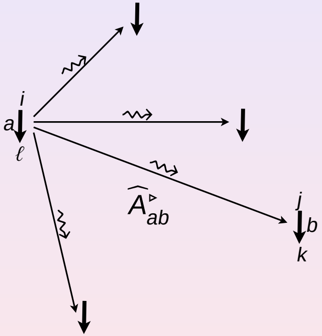
which are assumed to be mutually independent , then

expected unbiased flow from j to k is

$$
\begin{aligned}
r_{jk}^{\bowtie} &= r_j^{\blacktriangleright} r_k^{\blacktriangleleft} \\
&= \sum_{i\ell} A_{ij}^{\blacktriangleright} r_{i\ell}^{\bowtie} A_{k\ell}^{\blacktriangleleft} \\
&= \sum_{i\ell} \frac{A_{ij} A_{j\bullet} A_{\bullet k} A_{k\ell}}{A_{i\bullet} A_{\bullet\bullet}^2 A_{\bullet\ell}} r_{i\ell}^{\bowtie}
\end{aligned}
$$

Idea: capture path dynamics



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Path network

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Given

- path complete network $A = \left(R \stackrel{\gamma}{\leftarrow} E \underset{\varrho}{\overset{\delta}{\rightrightarrows}} N\right)$

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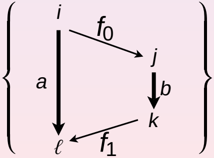
- links $\widehat{E} = \sum_{a,b\in E} \widehat{E}_{ab}$, where $\widehat{E}_{ab} =$



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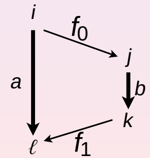
- links $\widehat{E} = \sum_{a,b\in E} \widehat{E}_{ab}$, where $\widehat{E}_{ab} = \left\{ \right.$  $\left. \right\}$

- cost $\widehat{\gamma}(f) = \gamma(f_0) + \gamma(b) + \gamma(f_1) - \gamma(a) + 2d \leq v$

Dynamics of path selection

attraction: probability that traffic through a will traverse b

$$\widehat{A}^{\triangleright}_{ab} = \frac{\widehat{A}_{ab}}{\widehat{A}_{a\bullet}} \text{ where}$$

$$\widehat{A}_{a\bullet} = \sum_x A_{ax}$$

repulsion: probability that traffic through b is diverted away from a

$$\widehat{A}^{\triangleleft}_{ab} = \frac{\widehat{A}_{ab}}{\widehat{A}_{\bullet b}} \text{ where}$$

$$\widehat{A}_{\bullet b} = \sum_x A_{xb}$$

Path ranking

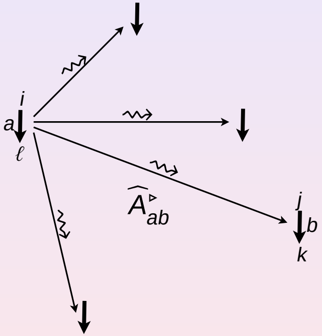
path pull: probability that traffic traverses b

$$\widehat{r}^{\triangleright}_b = \sum_a \widehat{r}^{\triangleright}_a \widehat{A}^{\triangleright}_{ab}$$

path push: probability that traffic is diverted from a

$$\widehat{r}^{\triangleleft}_a = \sum_{b \in \widehat{N}} \widehat{A}^{\triangleleft}_{ab} \widehat{r}^{\triangleleft}_b$$

Path pull rank



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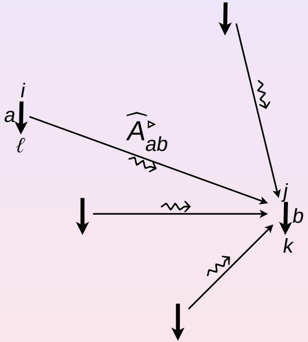

Attraction dynamics

Definition

The *node attraction* between j and k is the total attraction of all paths between them:

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Idea

Estimate the traffic bias as the difference between the node attraction and the unbiased flow

$$\Upsilon_{jk} = \widehat{r}_{jk} - r^{\bowtie}_{jk}$$

Attraction dynamics

Definition

The *attraction dynamics* of a network A is the Markov chain $\widehat{A} = \left(\widehat{A}_{(ij)(k\ell)}\right)_{N^2\times N^2}$, with the entries

$$\widehat{A}_{(ij)(k\ell)} = \frac{A_{ij}A_{jk}A_{k\ell}}{A_{i\bullet}A_{\bullet\bullet}A_{\bullet\ell}}$$

where $A_{i\bullet}A_{\bullet\bullet}A_{\bullet\ell} = \sum_{m,n\in N} A_{im}A_{mn}A_{n\ell}$.

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where $A_{i\bullet}A_{\bullet\bullet}A_{\bullet\ell} = \sum_{m,n\in N} A_{im}A_{mn}A_{n\ell}$.

Recall

$$A^{\bowtie}_{(ij)(k\ell)} = \frac{A_{ij}A_{j\bullet}A_{\bullet k}A_{k\ell}}{A_{i\bullet}A_{\bullet\bullet}A_{\bullet\bullet}A_{\bullet\ell}}$$

and

$$r^{\bowtie}_{jk} = \sum_{i,\ell\in N} A^{\bowtie}_{(ij)(k\ell)} r^{\bowtie}_{i\ell}$$

Proposition

Let a network A be path complete for a sufficiently large cutoff value v.

Then the node attraction $\widehat{r}$ is the stationary distribution of the attraction dynamics:

$$\widehat{r}_{jk} \quad = \quad \sum_{i,\ell \in N} \widehat{A}_{(ij)(k\ell)} \widehat{r}_{i\ell}$$

Corollary

The directed reputation and promotion ranks are the marginals of the node attraction

$$\sum_{k \in N} \widehat{r}_{jk} = r_j^{\blacktriangleright}$$

$$\sum_{j \in N} \widehat{r}_{jk} = r_k^{\blacktriangleleft}$$

Interpretation

$$r_j^{\blacktriangleright} = \mathrm{Prob}\left(\bullet \xrightarrow{\xi} j \mid \xi \in \widehat{A}\right)$$

$$r_k^{\blacktriangleleft} = \mathrm{Prob}\left(k \xrightarrow{\xi} \bullet \mid \xi \in \widehat{A}\right)$$

$$\widehat{r_{jk}} = \mathrm{Prob}\left(j \xrightarrow{\xi} k \mid \xi \in \widehat{A}\right)$$

Interpretation

The mutual information

$$I(r^{\blacktriangleright} \; ; \; r^{\blacktriangleleft}) \;=\; D(\widehat{r} \,\|\, r^{\bowtie}) \;=\; \sum_{j=1}^{N} \sum_{k=1}^{N} \widehat{r}_{jk} \log \frac{\widehat{r}_{jk}}{r^{\blacktriangleright}_{j} r^{\blacktriangleleft}_{k}}$$

quantifies the *non-local* information processing in A.

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quantifies the *non-local* information processing in A.

E.g, in the extremal cases,

- if $I(r^{\blacktriangleright} \; ; \; r^{\blacktriangleleft}) = 0$, i.e. $r^{\blacktriangleright}$ and $r^{\blacktriangleleft}$ are independent, all information is generated by the nodes,
- if $I(r^{\blacktriangleright} \; ; \; r^{\blacktriangleleft}) = H(r)$ for $r = r^{\blacktriangleright} = r^{\blacktriangleleft}$, all information is generated by the network.

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Communities

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- *Coherence* of $U \subseteq N$ is the minimal attraction bias of its members, in either direction

$$\Upsilon(U) = \bigwedge_{i,j \in U} (\Upsilon_{ij} \vee \Upsilon_{ji})$$

- *Communities* are coherent sets of nodes

$$\wp_\epsilon A = \{U \subseteq N \mid \Upsilon(U) \geq \epsilon\}$$

Communities

- Each $\wp_\epsilon A$, ordered by

$$U \sqsubseteq V \iff U \subseteq V \wedge \Upsilon(U) \leq \Upsilon(V)$$

is a *directed complete partial order (dcpo)*.

Communities

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  is a *directed complete partial order (dcpo)*.

- An ϵ-*concept* is a maximal element of $\wp_\epsilon(A)$.

- $\epsilon_1 \leq \epsilon_2$ implies $\wp_{\epsilon_1} A \supseteq \wp_{\epsilon_2} A$
  - $\wp_\epsilon$ is easy for large and small ϵ

Concept modules

Definition

$U \subseteq N$ is an ϵ-*concept module* if

- $\forall i, j \in U.\ \Upsilon(\{i, j\}) \geq \epsilon$, but
- $\forall k \in N \setminus U\ \exists j \in U.\ \Upsilon(\{k, j\}) < \epsilon$.

Concept modules

Definition

$U \subseteq N$ is an ϵ-*concept module* if

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- $\forall k \in N \setminus U\ \exists j \in U.\ \Upsilon\left(\{k, j\}\right) < \epsilon$.

Let $\mathcal{N}^{\epsilon}$ denote the set of ϵ-concept modules in a network A.

Association networks

Given a path complete network A, the induced concept network

$$\mathcal{A}^\epsilon = \left(R \xleftarrow{\gamma} \mathcal{E}^\epsilon \underset{\varrho}{\overset{\delta}{\rightrightarrows}} \mathcal{N}^\epsilon\right)$$

consists of

$$\begin{aligned}
\mathcal{N}^\epsilon &= \epsilon\text{-concept modules in } A \\
\mathcal{E}^\epsilon &= \sum_{U,V\in\mathcal{N}^\epsilon} \mathcal{E}^\epsilon_{UV} \text{ where} \\
\mathcal{E}^\epsilon_{UV} &= \sum_{U \underset{a}{\to} U\cap V} \; \sum_{U\cap V \underset{b}{\to} V} \tilde{E}_{ab} \\
\tilde{E}_{ab} &= \left\{ \begin{array}{ccc} i & \xrightarrow{f_0} & j \\ a \big\downarrow & & \big\downarrow b \\ \ell & \xrightarrow{f_1} & k \end{array} \right\}
\end{aligned}$$

Outline

Summary

- extended the ranking methods to paths
  - path rank is a measure of nonlocal information
  - allows estimating the flow bias to extract modules

Summary

- extended the ranking methods to paths
  - path rank is a measure of nonlocal information
  - allows estimating the flow bias to extract modules
- extracted modules (= communities = concepts)
  - parametric, richer structure for simpler algorithmics
  - concept networks for latent semantics

Current work

Is this method "real"?

- experimental validation
  - PL networks
  - IMDB
- relate with spectral methods
- algorithmics, convergence. . .