HANDBOOK FOR THE M.Sc. MATHEMATICAL MODELLING AND SCIENTIFIC COMPUTING

Issued 2008

Contents

1 The M.Sc. Course

The Course Organiser is Dr Kathryn Gillow.

1.1 Aims

Oxford's M.Sc. in Mathematical Modelling and Scientific Computing aims to train graduates with a strong mathematical background to develop and apply their skills to the solution of real problems. By the end of the course students should be able to formulate a well posed problem in mathematical terms from a possibly sketchy verbal description, carry out appropriate mathematical analysis, select or develop an appropriate numerical method, write a computer program which gives sensible answers to the problem, and present and interpret these results for a possible client. Particular emphasis is placed on the need for all these parts in the problem solving process, and on the fact that they frequently interact and cannot be carried out sequentially.

1.2 The Academic Year

The course lasts almost twelve months, from the beginning of October to the end of the following September. Although the lecture courses are given during the three University terms, the examinations will take place on the Thursdays of the weeks preceding both Hilary and Trinity terms. Additionally, much other work is carried out in the vacations, and students should expect to spend most of the year in Oxford. There will be no time for long holidays.

For the academic year 2008–2009, the course will begin with a week of introductory material based at the Computing Laboratory, beginning at 9.30am on the morning of Monday 6 October 2008.

The dates of the University Full Terms for the Academic Year 2008–2009 are:

MT = Michaelmas Term 2008: Sunday, 12 October – Saturday, 6 December
HT = Hilary Term 2009: Sunday, 18 January – Saturday, 14 March
TT = Trinity Term 2009: Sunday, 26 April – Saturday, 20 June

1.3 Course Structure

1.3.1 Core Courses

There are four core course with a weighting of 1 unit each:

- A1: Mathematical Methods
- A2: Applied Partial Differential Equations
- B1: Numerical Linear Algebra and Numerical Solution of Differential Equations

- B2: Further Numerical Linear Algebra and Finite Element Methods for Partial Differential Equations

A1 and B1 are taken during Michaelmas Term and are examined during Week 0 of Hilary Term. A2 and B2 are taken during Hilary Term and are examined during Week 0 of Trinity Term. [The only exception is that a student who has already completed one of these courses as an Oxford undergraduate may arrange (with the Course Organiser in Week 0 of MT or HT) to take an extra special topic or case study instead.]

1.3.2 Special Topics

All students complete at least 3 special topics with each special topic having a weighting of 1 unit. There is a great variety of special topic lecture courses which are classified under the broad headings of Modelling, Computation or Other. Students should complete at least one Modelling course and one Computation course. A special topic is usually assessed by a mini-project on a topic agreed with the lecturer. Students wishing to do a special topic on one of these courses must inform the lecturer before half the lectures have been given, and the special topic must be handed in with a completed cover sheet within 6 weeks of the end of the term for MT and HT lecture courses, or within 2 weeks of the end of term for TT lecture courses. The lecturer will assess the work and make a recommendation to the examiners.

It is also possible to do other topics if approved by the M.Sc. Supervisory Committee. Students who wish to follow a lecture course not on the list or to do a special topic based on a reading course should submit a short description of the project to the course organiser.

The special topic guidelines are given in Appendix A.

1.3.3 Case Studies

All students will learn MATLAB at the start of the course if they do not already know it. In MT students will take Practical Numerical Analysis classes and Mathematical Modelling classes which will include group work and presentation of results. In HT, students will participate in the Case Studies in Mathematical Modelling and Scientific Computing, and will write up at least one project for assessment for each course. These assessments are worth 1 unit each.

1.3.4 Dissertation

Students normally prepare their dissertations during Trinity Term and the long vacation, but it is often valuable to commence background work earlier. A student's dissertation topic should be selected in consultation with their supervisor and the details of the form and scope of the dissertation are described in the Regulations. Some possible dissertation projects can be viewed at: http://web.comlab.ox.ac.uk/oucl/courses/grad07-08/mmsc/projects.html

Additionally, a number of industrial topics suitable for dissertations will be presented at a meeting in December by companies who sponsor the M.Sc. course. Also students are encouraged to talk to any potential supervisors, which may include most academics or researchers in OCIAM or the NA Group. Note that the supervisor allocated in the first term will not usually turn out to be the supervisor for the dissertation.

Each student will be required to give a short talk and answer questions on the background to their dissertation topic at an open meeting, attended by supervisors, examiners and sponsors, to be held at the end of June.

In normal circumstances the body of the dissertation (excluding appendices etc.) should not exceed 50 pages in length.

Students should submit three bound copies of their dissertation to the Examination Schools by mid-day on Friday 4 September 2009. An electronic copy may also be requested.

The oral examination (viva) will be held in the second half of September and students will be expected to answer questions on their dissertation. Students whose performance was considered to be weak in either the written examination or their special topics, may also expect to be asked questions on these parts of the course at the oral.

2 Core Course Synopses

2.1 A1 Mathematical Methods I

A1 Introduction to Applied Mathematics

(Dr Porter — 6 lectures, MT)

In week one of Michaelmas Term, an introductory course of six lectures will be provided. This course will cover basic material.

Learning Outcomes

Acquired the background knowledge to prepare them for applied mathematics.

Synopsis

Modelling and conservation laws.

Scaling and non-dimensionalisation.

Asymptotic sequences. Regular and singular perturbation methods for algebraic equations. Simple boundary layer theory.

Reading

1. S. D. Howison, *Practical Applied Mathematics: Modelling, Analysis, Approximation* (CUP, Cambridge, 2005). Chs. 1,2,3,13,16.

A1 Mathematical Methods

(Prof Howison — 14 lectures, MT)

Overview

This course develops mathematical techniques which are useful in solving 'real-world' problems involving differential equations, and is a development of ideas which arise in the second year differential equations course. The course aims to show in a practical way how equations 'work', what kinds of solution behaviours can occur, and some techniques which are useful in their solution.

Learning Outcomes

Students will know how differential equations can be used to model real-world phenomena and be able to describe the behaviour of the types of solutions that can occur. They will be familiar with the hysteresis and stability of ODEs and be able to solve Sturm-Liouville systems. They will develop the theory of PDEs, for example to model shocks and know similarity solutions.

Synopsis

Nonlinear oscillations. Multiple scale methods.

Ordinary differential equations: hysteresis and stability.

Sturm-Liouville systems, comparison methods. Integral equations and eigenfunctions.

Partial differential equations: shocks, similarity solutions.

Reading

1. A. C. Fowler, *Techniques of Applied Mathematics*, Mathematical Institute Notes (2005).
2. J. P. Keener, *Principles of Applied Mathematics: Transformation and Approximation* (revised edition, Perseus Books, Cambridge, Mass., 2000).
3. E. J. Hinch, *Perturbation Methods* (CUP, Cambridge, 1991).
4. J. R. Ockendon, S. D. Howison, A. A. Lacey and A. B. Movchan, *Applied Partial Differential Equations* (revised edition, OUP, Oxford, 2003).
5. R. Haberman, *Mathematical Models* (SIAM, Philadelphia, 1998).
6. S. D. Howison, *Practical Applied Mathematics: Modelling, Analysis, Approximation* (CUP, Cambridge, 2005).

A1 Mathematical Methods Supplementary Lectures

(Dr H Ockendon — 8 lectures, MT)

Synopsis

Introduction to distributions.

The delta function.

Green's functions.

Calculus of variations.

Optimal control theory.

2.2 A2 Applied Partial Differential Equations

A2 Applied Partial Differential Equations

(Dr Norbury — 16 lectures, HT)

Learning Outcomes

Students will know a range of techniques to solve PDEs including nonlinear first order and second order and their classification. They will be able to demonstrate various principles for solving PDEs including Green's function, maximum principle and eigenfunctions.

Synopsis

Charpit's equations; eikonal equation.

Systems of partial differential equations, characteristics. Weak solutions. Riemann's function.

Maximum principles, comparison methods, well-posed problems, and Green's functions for the heat equation and for Laplace's equation.

Delta functions. Eigenfunction expansions.

Reading

1. Dr Norbury's web notes.
2. Institute lecture notes are now available (JN).
3. M. Renardy and R. C. Rogers *An Introduction to Partial Differential Equations*, (Springer-Verlag, New York, 2004).
4. J. P. Keener 2000 *Principles of Applied Mathematics: Transformation and Approximation*, (revised edition. Perseus Books, Cambridge, Mass.)
5. J. R. Ockendon, S. D. Howison, A .A. Lacey and A. B. Movchan 2003 *Applied Partial Differential Equations*, (revised edition. OUP, Oxford).

A2 Further Applied Partial Differential Equations — Dr Porter — 8HT

Synopsis

Fredholm alternative and Green's functions for non-self-adjoint problems; application of delta functions. Review of classification of second-order linear equations [3 lectures, 1 class]

Further development of hyperbolic equations: Cauchy-Kovalevskaya theorem, Riemann invariants, shocks and weak solutions, causality. [3 lectures, 1 class]

2.3 B1 Numerical Linear Algebra & Numerical Solution of Differential Equations

B1 Numerical Linear Algebra

(Dr Wathen — 8 lectures, MT)

Overview

Linear Algebra is a central and widely applicable part of Mathematics. It is estimated that many (if not most) computers in the world are computing with matrix algorithms at any moment in time whether these be embedded in visualization software in a computer game or calculating prices for some financial option. This course builds on elementary linear algebra and in it we derive, describe and analyse a number of widely used constructive methods (algorithms) for various problems involving matrices.

Synopsis

Numerical Methods for solving linear systems of equations, computing eigenvalues and singular values and various related problems involving matrices are the main focus of this course.

Syllabus

Common problems in linear algebra. Matrix structure, singular value decomposition. QR factorization, the QR algorithm for eigenvalues. Direct solution methods for linear systems, Gaussian elimination and its variants. Iterative solution methods for linear systems.

Chebyshev polynomials and Chebyshev semi-iterative methods, conjugate gradients, convergence analysis, preconditioning.

Reading

1. L. N. Trefethen and D. Bau III, *Numerical Linear Algebra* (SIAM, 1997).

2. J. W. Demmel, *Applied Numerical Linear Algebra* (SIAM, 1997).

3. A. Greenbaum, *Iterative Methods for Solving Linear Systems* (SIAM, 1997).

4. G. H. Golub and C. F. van Loan, *Matrix Computations* (John Hopkins University Press, 3rd edition, 1996).

5. H. C. Elman, D. J. Silvester and A. J. Wathen, *Finite Elements and Fast Iterative Solvers* (OUP, 1995), only chapter 2.

B1 Numerical Solution of Differential Equations

(Prof Süli — 16 lectures, MT)

Overview

To introduce and give an understanding of numerical methods for the solution of ordinary and partial differential equations, their derivation, analysis and applicability.

The MT lectures are devoted to numerical methods for initial value problems, while the HT lectures concentrate on the numerical solution of boundary value problems.

Learning Outcomes

At the end of the course the student will be able to:

1. Construct one-step and linear multistep methods for the numerical solution of initial-value problems for ordinary differential equations and systems of such equations, and to analyse their stability and accuracy properties;

2. Construct finite difference methods for the numerical solution of initial-boundary-value problems for second-order parabolic partial differential equations, and first-order hyperbolic equations, and to analyse their stability and accuracy properties.

Syllabus

Initial value problems for ordinary differential equations: Euler, multistep and Runge-Kutta; stiffness; error control and adaptive algorithms.

Initial value problems for partial differential equations: parabolic equations, hyperbolic equations; explicit and implicit methods; accuracy, stability and convergence, Fourier analysis, CFL condition.

Synopsis

The MT part of the course is devoted to the development and analysis of numerical methods for initial value problems. We begin by considering classical techniques for the numerical solution of ordinary differential equations. The problem of stiffness is then discussed in tandem with the associated questions of step-size control and adaptivity.

Initial value problems for ordinary differential equations: Euler, multistep and Runge-Kutta; stiffness; error control and adaptive algorithms. [Introduction (1 lecture) + 5 lectures]

The remaining lectures focus on the numerical solution of initial value problems for partial differential equations, including parabolic and hyperbolic problems.

Initial value problems for partial differential equations: parabolic equations, hyperbolic equations; explicit and implicit methods; accuracy, stability and convergence, Fourier analysis, CFL condition. [10 lectures]

Reading

The course will be based on the following textbooks:

1. K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations* (Cambridge University Press, 1994). ISBN 0-521-42922-6 (Paperback edition) [Chapters 2, 3 (Secs. 3.1, 3.2), Chapter 4 (Secs. 4.1–4.6), Chapter 5].

2. E. Süli and D. Mayers, *An Introduction to Numerical Analysis* (Cambridge University Press, 2003). ISBN 0-521-00794-1 (Paperback edition) [Chapter 12].

3. A. Iserles, *A First Course in the Numerical Analysis of Differential Equations* (Cambridge University Press, 1996). ISBN 0-521-55655-4 (Paperback edition) [Chapters 1–5, 13, 14].

2.4 B2 Further Numerical Linear Algebra & Finite Element Methods for PDEs

B2 Further Numerical Linear Algebra

(Dr Wathen — 8 lectures, MT)

Overview

Linear Algebra is a central and widely applicable part of Mathematics. It is estimated that many (if not most) computers in the world are computing with matrix algorithms at any moment in time whether these be embedded in visualization software in a computer game or calculating prices for some financial option. This course builds on elementary linear algebra and in it we derive, describe and analyse a number of widely used constructive methods (algorithms) for various problems involving matrices.

Synopsis

Numerical Methods for solving linear systems of equations, computing eigenvalues and singular values and various related problems involving matrices are the main focus of this course.

Syllabus

Common problems in linear algebra. Matrix structure, singular value decomposition. QR factorization, the QR algorithm for eigenvalues. Direct solution methods for linear systems, Gaussian elimination and its variants. Iterative solution methods for linear systems.

Chebyshev polynomials and Chebyshev semi-iterative methods, conjugate gradients, convergence analysis, preconditioning.

Reading

1. L. N. Trefethen and D. Bau III, *Numerical Linear Algebra* (SIAM, 1997).
2. J. W. Demmel, *Applied Numerical Linear Algebra* (SIAM, 1997).
3. A. Greenbaum, *Iterative Methods for Solving Linear Systems* (SIAM, 1997).
4. G. H. Golub and C. F. van Loan, *Matrix Computations* (John Hopkins University Press, 3rd edition, 1996).
5. H. C. Elman, D. J. Silvester and A. J. Wathen, *Finite Elements and Fast Iterative Solvers* (OUP, 1995), only chapter 2.

B2 Finite Element Methods for Partial Differential Equations

(Dr Wathen — 16 lectures, HT)

Overview

Computational algorithms are now widely used to predict and describe physical and other systems. Underlying such applications as Weather Forecasting, Civil Engineering (design of structures) and Medical Scanning are numerical methods which approximately solve partial differential equation problems. This course gives a mathematical introduction to one of the more widely used methods: the finite element method.

Synopsis

Finite element methods represent a powerful and general class of techniques for the approximate solution of partial differential equations. The aim of this course is to introduce these methods for boundary value problems for the Poisson and related elliptic partial differential equations. Attention will be paid to formulation, analysis, implementation and applicability of these methods.

Syllabus

Elliptic boundary value problems in 1-dimension and 2-dimensions; variational and weak forms. Galerkin finite element methods; piecewise polynomials; implementation issues; a priori and a posteriori error analysis. Mixed finite element methods and the Stokes problem.

Reading

The main text will be

1. Howard Elman, David Silvester and Andy Wathen, *Finite Elements and Fast Iterative Solvers* (Oxford University Press, 2005) [mainly Chapters 1 and 5]

and some of the introductory material is usefully covered in

2. Endre Süli and David Mayers, *An Introduction to Numerical Analysis* (Cambridge University Press, 2003) [Chapter 11 and in particular Chapter 14]

3. David Silvester, *A Finite Element Primer*, notes which can be found at http://www.maths.manchester.ac.uk/∼djs/primer.pdf

Another book on finite elements which may be useful for different parts of the course is

4. Claes Johnson, *Numerical Solution of Partial Differential Equations by the Finite Element Method* (Cambridge University Press, 1990). (Which is unfortunately out of print) [Chapter 1–4]

and the Computing Lab lecture notes

5. Endre Süli, *Finite Element Methods for Partial Differential Equations*, Oxford University Computing Laboratory, 2001.

are also useful.

3 Special Topics

These are the special topic courses available for the Academic year 2008–2009. Each falls under the broad heading of Modelling [M], Computation [C] or Other [O].

Michaelmas Term 2008

- Approximation of Functions [C], Dr I J Sobey
- Martingales Through Measure Theory [O], Dr P Tarres
- Mathematical Ecology and Biology [M] Dr A Fletcher
- Mathematics and the Environment [M], Dr A C Fowler and Dr G Sander
- Methods of Functional Analysis for Partial Differential Equations [O], Dr J Dyson
- Perturbation Methods [O], Dr M Porter
- Solid Mechanics [M], Prof J M Ball
- Stochastic Differential Equations [O], Prof T Lyons
- Topics in Fluid Mechanics [M], Prof S J Chapman and Dr Whittaker
- Viscous Flow [M], Dr J Oliver

Hilary Term 2009

- Applied Complex Variables [O], Prof S J Chapman
- Elasticity and Plasticity [M],
- Mathematical Methods for Signal Processing [O], Dr C Orphanidou and Dr I Drobnjak
- Mathematical Models of Financial Derivatives [M], Dr H Jin
- Mathematical Physiology [M], Prof P Maini
- Nonlinear Systems [O], Dr I M Moroz
- Numerical Solution of Differential Equations II [C], Dr I J Sobey
- Topics in Numerical Optimization [C], Dr C Ortner
- Waves and Compressible Flow [M],

Trinity Term 2009

- C++ for Scientific Computing [C],

- Mathematics for Geoscience [M], Prof C L Farmer (Schlumberger)
- Numerical Multiphysics Modelling in Biology and Physiology [C], Dr J P Whiteley and Dr N Smith
- Spectral Methods for ODE and PDE [C], Prof L N Trefethen
- Stochastic Modelling and Simulation of Biological Processes [C], Prof K Burrage

3.1 Special Topic Synopses

3.1.1 Approximation of Functions

(Dr Sobey — 16 lectures, MT)

Overview

The central idea in approximation of functions can be illustrated by the question: Given a set of functions A and an element $u \in A$, if we select a subset $B \subset A$, can we choose an element $U \in B$ so that U approximates u in some way? The course focuses on this question in the context of functions when the way we measure 'goodness' of approximation is either with an integral least square norm or with an infinity norm of the difference $u - U$. The choice of measure leads to further questions: is there a best approximation; if a best approximation exists, is it unique, how accurate is a best approximation and can we develop algorithms to generate good approximations? This course aims to give a grounding in the advanced theory of such ideas, the analytic methods used and important theorems for real functions. As well as being a beautiful subject in its own right, approximation theory is the foundation for many of the algorithms of computational mathematics and numerical analysis.

Synopsis

Introduction to approximation. Approximation in L^2. Approximation in L^∞: Oscillation Theorem, Exchange Algorithm. Approximation with splines. Rational approximation. Approximation of periodic functions.

Reading

1. M. J. D. Powell, *Approximation Theory and Methods* (CUP).
2. P. J. Davis, *Interpolation and Approximation* (Dover).

3.1.2 Martingales Through Measure Theory

(Dr Tarres — 16 lectures, MT)

Overview

Probability theory arises in the modelling of a variety of systems where the understanding of the"unknown" plays a key role, such as population genetics in biology, market evolution in financial mathematics, and learning features in game theory. It is also very useful in various areas of mathematics, including number theory and partial differential equations. The course introduces the basic mathematical framework underlying its rigorous analysis, and is therefore meant to provide some of the tools which will be used in more advanced courses in probability.

The first part of the course provides a review of measure theory from Integration Part A, and develops a deeper framework for its study. Then we proceed to develop notions of conditional expectation, martingales, and to show limit results for the behaviour of these martingales which apply in a variety of contexts.

Learning Outcomes

The students will learn about measure theory, random variables, independence, expectation and conditional expectation, product measures and discrete-parameter martingales.

Synopsis

A branching-process example. Review of σ-algebras, measure spaces. Uniqueness of extension of π-systems and Carathéodory's Extension Theorem [both without proof], monotone-convergence properties of measures, $\limsup$ and $\liminf$ of a sequence of events, Fatou's Lemma, reverse Fatou Lemma, first Borel-Cantelli Lemma.

Random variables and their distribution functions, representation of a random variables with prescribed distribution function, σ-algebras generated by a collection of random variables. Independence of events, random variables and σ-algebras, π-systems criterion for independence, second Borel-Cantelli Lemma. The tail σ-algebra, Kolomogorov's 0–1 Law.

Integration and expectation, review of elementary properties of the integral and L^p spaces [done in Part A Integration for the Lebesgue measure on $\mathbb{R}$]. Scheffé's Lemma, Jensen's inequality, orthogonal projection in L^2. Product measures, [Fubini's Theorem, infinite products of probability triples]. The Kolmogorov Theorem and definition of conditional expectation, proof as least-squares-best predictor, elementary properties. [The Radon-Nikodym Theorem.]

Filtrations, martingales, stopping times, discrete stochastic integrals, Doob's Optional-Stopping Theorem, Doob's Upcrossing Lemma and "Forward" Convergence Theorem, martingales bounded in L^2, Doob decomposition.

Uniform integrability and L^1 convergence, Levy's "Upward" and "Downward" Theorem, corollary to the Kolmogorov's Strong Law of Large Numbers, Doob's submartingale and L^p inequalities. Examples and applications, including branching processes, harmonic functions with boundary conditions on connected finite subsets of $\mathbb{Z}^d$ [not examinable].

[ ]=covered informally, without proofs.

Reading

1. D. Williams, *Probability with Martingales* (Cambridge University Press, 1995).

Further Reading

1. Z. Brzeźniak and T. Zastawniak, *Basic Stochastic Processes. A course through exercises*, Springer Undergraduate Mathematics Series (Springer-Verlag London, Ltd., 1999) [more elementary than D. Williams' book, but can provide with a complementary first reading].
2. M. Capinski and E. Kopp, *Measure, Integral and Probability*, Springer Undergraduate Mathematics Series (Springer-Verlag London, Ltd., second edition, 2004).
3. R. Durrett, *Probability: Theory and Examples,* (Second Edition, Duxbury Press, Wadsworth Publishing Company, 1996).
4. A. Etheridge, *A Course in Financial Calculus* (Cambridge University Press, 2002).
5. J. Neveu, *Discrete-parameter Martingales* (North-Holland, Amsterdam, 1975).
6. S. I. Resnick, *A Probability Path*, (Birkhäuser, 1999).

3.1.3 Mathematical Ecology and Biology

(Dr Fletcher — 14 lectures, MT)

Overview

Mathematical Ecology and Biology introduces the applied mathematician to practical applications in an area that is growing very rapidly. The course mainly focusses on situations where continuous models are appropriate and where these may be modelled by deterministic ordinary and partial differential equations. By using particular modelling examples in ecology, chemistry, biology, physiology and epidemiology, the course demonstrates how various applied mathematical techniques, such as those describing linear stability, phase planes, singular perturbation and travelling waves, can yield important information about the behaviour of complex models.

Learning Outcomes

Students will have developed a sound knowledge and appreciation of the ideas and concepts related to modelling biological and ecological systems using ordinary and partial differential equations.

Synopsis

Continuous and discrete population models for a single species, including Ludwig's 1978 insect outbreak model for spruce budworm and hysteresis. Harvesting and strategies for sustainable fishing. Modelling interacting populations, including the Lotka-Volterra model for predator-prey (with application to hare-lynx interactions), and Okubo's 1989 model for red-grey squirrel competition. Principle of competitive exclusion.

Epidemic models.

Michaelis-Menten model for enzyme-substrate kinetics.

Excitable systems. Threshold phenomena (nerve pulses).

Travelling wave propagation with biological examples.

Biological pattern formation. Turing's model for animal coat markings.

Nerve signal propagation.

Reading

J.D. Murray, *Mathematical Biology, Volume I: An Introduction (2002); Volume II: Spatial Models and Biomedical Applications (2003)* (3rd edition, Springer-Verlag).

1. Volume I: 1.1, 1.2, 1.6, 2.1–2.4, 3.1, 3.3–3.6, 3.8, 6.1–6.3, 6.5, 6.6, 8.1, 8.2, 8.4, 8.5, 10.1, 10.2, 11.1–11.5, 13.1–13.5, Appendix A.

2. Volume II: 1.6, 2, 3.1, 3.2, 5.1, 5.2, 13.1–13.4.

Further Reading

1. J. Keener and J. Sneyd, *Mathematical Physiology* (Springer, Berlin, 1998) 1.1, 1.2, 9.1, 9.2.

2. H. Meinhardt, *The Algorithmic Beauty of Sea Shells* (2nd enlarged edition, Springer, Berlin, 2000).

3.1.4 Mathematics and the Environment

(Dr Fowler & Dr Sander — 16 lectures, MT)

Overview

The aim of the course is to illustrate the techniques of mathematical modelling in their particular application to environmental problems. The mathematical techniques used are drawn from the theory of ordinary differential equations and partial differential equations. However, the course does require the willingness to become familiar with a range of different scientific disciplines. In particular, familiarity with the concepts of fluid mechanics will be useful.

Synopsis

Applications of mathematics to environmental or geophysical problems involving the use of models with ordinary and partial differential equations. Examples to be considered are: Climate dynamics. River flow and sediment transport. Glacier dynamics.

Reading

1. A. C. Fowler, *Mathematics and the Environment*, Mathematical Institute Notes (Revised edition, September 2004.)
2. K. Richards, *Rivers* (Methuen, 1982).
3. G. B. Whitham, *Linear and Nonlinear Waves* (Wiley, New York, 1974).
4. W. S. B. Paterson, *The Physics of Glaciers* (3rd edition, Pergamon Press, 1994).
5. J. T. Houghton, *The Physics of Atmospheres* (3rd ed., CUP, Cambridge, 2002).

3.1.5 Methods of Functional Analysis for Partial Differential Equations

(Dr Dyson — 16 lectures, MT)

Overview

The course will introduce some of the modern techniques in partial differential equations that are central to the theoretical and numerical treatments of linear and nonlinear partial differential equations arising in science, geometry and other fields.

Learning Outcomes

Students will learn techniques and results, such as Sobolev spaces, weak convergence, weak solutions, the direct method of calculus of variations, embedding theorems, the Lax-Milgram theorem, the Fredholm Alternative and the Hilbert-Schmidt theorem and how to apply these to obtain existence and uniqueness results for linear and nonlinear elliptic partial differential equations.

Synopsis

Part I Function Spaces:

Why are function spaces important for partial differential equations?

Definition of Banach spaces, separability and dual spaces. Definition of Hilbert space. The spaces $L^p(\Omega)$, $1 \leq p \leq \infty$, where $\Omega \subset \mathbb{R}^n$ is open. Minkowski and Hölder inequalities. Statement that $L^p(\Omega)$ is a Banach space, and that the dual of L^p is $L^{p'}$, for $1 \leq p < \infty$. Statement that L^2 is a Hilbert space.

Weak and weak* convergence in L^p spaces. Examples. A bounded sequence in the dual of a separable Banach space has a weak* convergent subsequence.

Mollifiers and the density of smooth functions in L^p for $1 \leq p < \infty$.

Definition of weak derivatives and their uniqueness. Definition of Sobolev space $W^{m,p}(\Omega)$, $1 \leq p \leq \infty$. $H^m(\Omega) = W^{m,2}(\Omega)$. Definition of $W_0^{1,p}(\Omega)$, $1 \leq p < \infty$. Brief introduction to distributions.

Part II Elliptic Problems:

The direct method of calculus of variations: The Poincaré inequality. Proof of the existence and uniqueness of a weak solution to Poisson's equation $-\Delta u = f$, with zero Dirichlet boundary conditions and $f \in L^2(\Omega)$, with Ω bounded. Discussion of regularity of solutions.

The Lax-Milgram theorem and Gårding's inequality. Existence and uniqueness of weak solutions to general linear uniformly elliptic equations.

Embedding theorems (proofs omitted except $W^{1,1}(a,b) \hookrightarrow C[a,b]$).

Compact operators and self adjoint operators. Fredholm Alternative and Hilbert-Schmidt Theorem. Examples including $-\Delta$ with zero Dirichlet boundary conditions.

A nonlinear elliptic problem treated by the direct method.

Reading

1. Lawrence C. Evans, *Partial Differential Equations,* Graduate Studies in Mathematics (American Mathematical Society, 2004).

2. M. Renardy and R. C. Rogers, *An Introduction to Partial Differential Equations* (Springer-Verlag, New York, 2004).

Additional Reading

1. E. Kreyszig, *Introductory Functional Analysis with Applications* (Wiley, revised edition, 1989).
2. J. Rauch, *Partial Differential Equations* (Springer-Verlag, New York, 1992).

3.1.6 Perturbation Methods

(Dr Porter — 16 lectures, MT)

Overview

Perturbation methods underlie almost all applications of physical applied mathematics: for example, boundary layers in viscous flow, celestial mechanics, optics, shock waves, reaction-diffusion equations and nonlinear oscillations. The aims of the course are to give a clear and systematic account of modern perturbation theory and to show how it can be applied to differential equations.

Synopsis

Asymptotic expansions. Asymptotic evaluation of integrals (including Laplace's method, method of stationary phase, method of steepest descent). Stokes phenomenon. Regular and singular perturbation theory. Methods of multiple scales. WKB theory and semiclassics. Boundary layers and related topics. Applications to nonlinear oscillators. Applications to partial differential equations.

Reading

1. E. J. Hinch, *Perturbation Methods* (CUP, 1991), Chs. 1–3, 5–7.
2. C. M. Bender and S. A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers* (Springer, 1999), Chs. 6, 7, 9–11.
3. J. Kevorkian and J. D. Cole, *Perturbation Methods in Applied Mathematics* (Springer–Verlag, 1981), Chs. 1, 2.1–2.5, 3.1, 3.2, 3.6, 4.1, 5.2.

3.1.7 Solid Mechanics

(Prof Ball — 16 lectures, MT)

Overview

Solid mechanics is a vital ingredient of materials science and engineering, and is playing an increasing role in biology. It has a rich mathematical structure. The aim of the course is to derive the basic equations of elasticity theory, the central model of solid mechanics, and give some interesting applications to the behaviour of materials.

Learning Outcomes

Students will learn basic techniques of modern continuum mechanics, such as kinematics of deformation, stress, constitutive equations and the relation between nonlinear and linearized models. They will also gain an insight into some recent developments in applications of mathematics to a variety of different materials.

Synopsis

(1) Nonlinear and linear elasticity (6 lectures)

Lagrangian and Eulerian descriptions of motion, analysis of strain. Balance laws of continuum mechanics. Frame-indifference. Cauchy and Piola-Kirchhoff stress. Constitutive equations for a nonlinear elastic material. Material symmetry, isotropy. Linear elasticity as a linearization of nonlinear elasticity.

(2) Exact solutions in elastostatics. (6 lectures)

Universal deformations for compressible materials. Incompressibility and models of rubber. Exact solutions for incompressible materials, including the Rivlin cube, simple shear, torsion of a cylinder, inflation of a balloon. Cavitation in polymers.

(3) Phase transformations in solids (4 lectures)

Martensitic phase transformations, twins and microstructure.

Austenite-martensite interfaces. The shape-memory effect.

Reading

1. R. J. Atkin and N. Fox, *An Introduction to the Theory of Elasticity* (Longman, 1980).
2. M. E. Gurtin, *An Introduction to Continuum Mechanics* (Academic Press, 1981).

Further Reading

1. Stuart S. Antman, *Nonlinear Problems of Elasticity,* Applied Mathematical Sciences v. 107 (Springer-Verlag, 1995).
2. Jerrold E. Marsden, Thomas J.R. Hughes, *Mathematical Foundations of Elasticity* (Prentice-Hall, 1983).
3. Philippe G. Ciarlet, *Mathematical Elasticity,* Studies in Mathematics and its Applications; v. 20, 27, 29 (North-Holland, 1988).
4. Kaushik Bhattacharya, *Microstructure of Martensite — Why it forms and how it gives rise to the shape-memory effect* (Oxford University Press, 2003).

3.1.8 Stochastic Differential Equations

(Prof Lyons — 16 lectures, MT)

Overview

Stochastic differential equations have been used extensively in many areas of application, including finance and social science as well as Chemistry. This course develops the basic theory of Itô's calculus and stochastic differential equations, and gives a few applications.

Learning Outcomes

The student will have developed an appreciation of stochastic calculus as a tool that can be used for defining and understanding diffusive systems.

Synopsis

Itô's calculus: stochastic integrals with respect to martingales, Itô's lemma, Levy's theorem on characteristic of Brownian motion, exponential martingales, exponential inequality, Girsanov's theorem, The Martingale Representation Theorem. Stochastic differential equations: strong solutions, questions of existence and uniqueness, diffusion processes, Cameron-Martin formula, weak solution and martingale problem. Some selected applications chosen from option pricing, stochastic filtering etc.

Reading

1. Dr Qian's online notes:

   http://people.maths.ox.ac.uk/∼qianz/_private/SDE-2007M.pdf

2. B. Oksendal, *Stochastic Differential Equations: An introduction with applications* (Universitext, Springer, 6th edition). Chapters II, III, IV, V, part of VI, Chapter VIII (F).
3. F. C. Klebaner, *Introduction to Stochastic Calculus with Applications* (Imperial College Press, 1998, second edition 2005). Sections 3.1 – 3.5, 3.9, 3.12. Chapters 4, 5, 11.

Alternative Reading

1. H. P. McKean, *Stochastic Integrals* (Academic Press, New York and London, 1969).

Further Reading

1. N. Ikeda and S. Watanabe, *Stochastic Differential Equations and Diffusion Processes* (North-Holland Publishing Company, 1989).
2. I. Karatzas and S. E. Shreve, *Brownian Motion and Stochastic Calculus,* Graduate Texts in Mathematics 113 (Springer-Verlag, 1988).
3. L. C. G. Rogers and D. Williams, *Diffusions, Markov Processes and Martingales Vol1 (Foundations) and Vol 2 (Itô Calculus)* (Cambridge University Press, 1987 and 1994).

3.1.9 Topics in Fluid Mechanics

(Prof Chapman & Dr Whittaker — 16 lectures, MT)

Overview

The course will expand and illuminate the 'classical' fluid mechanics taught in the third year course B6, and illustrate its modern application in a number of different areas in industry and geoscience.

Synopsis

Convection: Earth's mantle and core; magma chambers. Stability, boundary layers, parameterised convection.

Rotating flows: atmosphere and oceans. Waves, geostrophy, quasi-geostrophy, baroclinic instability.

Two-phase flows: boilers, condensers, fluidised beds. Flow régimes. Homogeneous, drift-flux, two-fluid models. Ill-posedness, waves, density wave oscillations.

Thin film flows: coatings and foams. Lubrication theory: gravity flows, Marangoni effects. Droplet dynamics, contact lines, menisci. Drying and wetting. Foam drainage.

Reading

1. J. S. Turner, *Buoyancy Effects in Fluids* (CUP, Cambridge, 1973).
2. A. E. Gill, *Atmosphere-Ocean Dynamics* (Academic Press, San Diego, 1982).
3. J. Pedlosky, *Geophysical Fluid Dynamics* (Springer-Verlag, Berlin, 1979).
4. D. A. Drew and S. L. Passman,*Theory of Multicomponent Fluids* (Springer-Verlag, Berlin, 1999).
5. P. B. Whalley, *Boiling, Condensation and Gas-Liquid Flow* (OUP, Oxford, 1987).
6. D. Weaire and S. Hutzler, *The Physics of Foams* (OUP, Oxford, 1999).

Further Reading

1. G. K. Batchelor, H. K. Moffatt and M. G. Worster (eds.), *Perspectives in Fluid Dynamics* (CUP, Cambridge, 2000).

3.1.10 Viscous Flow

(Dr Oliver — 14 lectures, MT)

Overview

Viscous fluids are important in so many facets of everyday life that everyone has some intuition about the diverse flow phenomena that occur in practice. This course is distinctive in that it shows how quite advanced mathematical ideas such as asymptotics and partial differential equation theory can be used to analyse the underlying differential equations and hence give scientific understanding about flows of practical importance, such as air flow round wings and flow in oil reservoirs.

Learning Outcomes

Students will have developed an appreciation of diverse flow phenomena in various mediums including Poiseuille flow, Rayleigh flow, airflow around wings and flow in oil reservoirs. They will have a demonstrable knowledge of the mathematical theory necessary to analyse such phenomena.

Synopsis

Derivation of Navier-Stokes equations for an incompressible Newtonian fluid. Vorticity. Energy equation and dissipation. Exact solutions for unidirectional flows; shear flow, Poiseuille flow, Rayleigh flow. Dimensional analysis, Reynolds number. Derivation of equations for high and low Reynolds number flows.

Derivation of Prandtl's boundary-layer equations. Similarity solutions for flow past a semi-infinite flat plate and for jets. Discussion of separation and application to the theory of flight. Jeffery-Hamel flow.

Slow flow past a circular cylinder and a sphere. Non-uniformity of the two dimensional approximation; Oseen's equation. Lubrication theory: bearings, thin films and Hele-Shaw cell. Flow in a porous medium. Stability and the transition to turbulence.

Reading

1. D. J. Acheson, *Elementary Fluid Dynamics* (OUP, 1990), Chs 2, 6, 7, 8.
2. H. Ockendon and J. R. Ockendon, *Viscous Flow* (Cambridge Texts in Applied Mathematics, 1995).
3. M. E. O'Neill and F. Chorlton, *Viscous and Compressible Fluid Dynamics* (Ellis Horwood, 1989), Chs 2, 3, 4.1–4.3, 4.19–4.20, 4.22–4.24, 5.1–5.2, 5.6.

3.1.11 Applied Complex Variables

(Prof Chapman — 16 lectures, HT)

Overview

The course begins where core second-year complex analysis leaves off, and is devoted to extensions and applications of that material. It is assumed that students will be familiar with inviscid two-dimensional hydrodynamics to the extent of the existence of a harmonic stream function and velocity potential in irrotational incompressible flow, and Bernoulli's equation.

Synopsis

1–2 Review of core real and complex analysis, especially contour integration, Fourier transforms.

2–4 Conformal mapping. Riemann mapping theorem (statement only). Schwarz-Christoffel formula. Solution of Laplace's equation by conformal mapping onto a canonical domain.

5–6 Applications to inviscid hydrodynamics: flow past an aerofoil and other obstacles by conformed mapping; free streamline flows, hodograph plane.

7–8 Flow with free boundaries in porous media. Construction of solutions using conformal mapping. The Schwarz function.

9–15 Transform methods, complex Fourier transform. Solution of mixed boundary value problems motivated by thin aerofoil theory and the theory of cracks in elastic solids. Index. Riemann Hilbert problems, Wiener-Hopf method.

16 Stokes phenomenon.

Reading

1. M. J. Ablowitz and A. S. Fokas, *Complex Variables: Introduction and Applications* (2nd edition, CUP, Cambridge, 2003). ISBN 0521534291.

2. J. Ockendon, S. Howison, A. Lacey and A. Movchan, *Applied Partial Differential Equations* (Oxford, 1999) Pages 195–212.

3. G. F. Carrier, M. Krook and C. E. Pearson, *Functions of a Complex Variable* (McGraw-Hill, New York, 1966). (Reprinted by Hod Books, 1983.) ISBN 0962197300 (Out of print).

3.1.12 Elasticity and Plasticity

(Lecturer t.b.c. — 16 lectures, HT)

Please note that it has not been confirmed that this course will be running in 2008-09.

Overview

Propagating disturbances, or waves, occur frequently in applied mathematics, especially in applied mechanics. This course will be centred on some prototypical examples from fluid dynamics, the two most familiar being surface gravity waves and waves in gases. The models for compressible flow will be derived and then analysed for small amplitude motion. This will shed light on the important phenomena of dispersion, group velocity and resonance, and the differences between supersonic and subsonic flow, as well as revealing the crucial dependence of the waves on the number of space dimensions.

Larger amplitude motion of liquids and gases will be described by incorporating nonlinear effects, and the theory of characteristics for partial differential equations will be applied to understand the shock waves associated with supersonic flight.

Learning Outcomes

Students will have developed a sound knowledge of a range of mathematical models used to study waves (both linear and nonlinear); will be able to describe examples of waves from fluid dynamics and will have analysed a model for compressible flow. They will have an awareness of shock waves and how the theory of characteristics for PDEs can be applied to study those associated with supersonic flight.

Synopsis

1–2 Equations of inviscid compressible flow including flow relative to rotating axes.

3–6 Models for linear wave propagation including Stokes waves, Inertial waves, Rossby waves and simple solutions.

7–10 Theories for Linear waves: Fourier Series, Fourier integrals, method of stationary phase, dispersion and group velocity. Flow past thin wings, Huyghens principle.

11–12 Nonlinear Waves: method of characteristics, simple wave flows applied to one-dimensional unsteady gas flow and shallow water theory.

13–16 Shock Waves: weak solutions, Rankine-Hugoniot relations, oblique shocks, bores and hydraulic jumps.

Reading

1. H. Ockendon and J. R. Ockendon, *Waves and Compressible Flow* (Springer, 2004).
2. J. R. Ockendon, S. D. Howison, A. A. Lacey and A. B. Movchan, *Applied Partial Differential Equations* (revised edition, OUP, Oxford, 2003). Chs 2.5, 4.5–7.
3. D. J. Acheson, *Elementary Fluid Dynamics* (OUP, 1990). Ch 3
4. J. Billingham and A. C. King, *Wave Motion* (CUP, 2000). Ch 1–4, 7,8.

Background Reading

1. M. J. Lighthill, *Waves in Fluids* (CUP, 1978).
2. G. B. Whitham, *Linear and Nonlinear Waves* (Wiley, 1973).

3.1.13 Mathematical Methods for Signal Processing

(Dr Orphanidou & Dr Drobnjak — 12 lectures, HT)

Aims and Objectives

The course introduces the mathematical techniques used and applied in modern signal processing systems; in particular focusing on linear transforms and linear algebra. Signal processing is the analysis and manipulation of signals including sound, image and video, as well as biomedical signals such as the ECG, EEG and MRI. The course aims to introduce the main algorithms employed for performing operations on these signals, in either discrete or continuous time, such as data compression, denoising, filtering, event detection and identification, classification and a variety of other interesting and useful operations.

Synopsis

Introduction to signals and systems.

Fundamentals of digital signal processing.

Linear signals and systems.

Transform Theory.

Filter design.

Applications in biomedical and speech processing.

Introduction to image analysis, image registration, image segmentation using Markov Random Field Theory.

Model-based data analysis: General Linear Models (GLM), Model-free data analysis, Independent Component Analysis (ICA).

Applications in image processing.

3.1.14 Mathematical Models of Financial Derivatives

(Dr Jin — 16 lectures, HT)

Overview

The course aims to introduce students to mathematical modelling in financial markets. At the end of the course the student should be able to formulate a model for an asset price and then determine the prices of a range of derivatives based on the underlying asset using arbitrage free pricing ideas.

Learning Outcomes

Students will have a familiarity with the mathematics behind the models and analytical tools used in Mathematical Finance. This includes being able to formulate a model for an asset price and then determining the prices of a range of derivatives based on the underlying asset using arbitrage free pricing ideas.

Synopsis

Introduction to markets, assets, interest rates and present value; arbitrage and the law of one price: European call and put options, payoff diagrams. Introduction to Brownian motion, continuous time martingales, informal treatment of Itô's formula and stochastic differential equations. Discussion of the connection with PDEs through the Feynman-Kac formula.

The Black-Scholes analysis via delta hedging and replication, leading to the Black-Scholes partial differential equation for a derivative price. General solution via Feynman-Kac and risk neutral pricing, explicit solution for call and put options.

Extensions to assets paying dividends, time-varying parameters. Forward and future contracts, options on them. American options, formulation as a free-boundary problem and a linear complementarity problem. Simple exotic options. Weakly path-dependent options including barriers, lookbacks and Asians.

Reading

1. T. Bjork, *Arbitrage Theory in Continuous Time* (OUP, 1998).
2. P. Wilmott, S. D. Howison and J. Dewynne, *Mathematics of Financial Derivatives* (CUP, 1995).
3. A. Etheridge, *A Course in Financial Calculus* (CUP, 2002).

Background Reading

1. J. Hull, *Options Futures and Other Financial Derivative Products* (4th edition, Prentice Hall, 2001).
2. N. Taleb, *Dynamic Hedging* (Wiley, 1997).
3. P. Wilmott, *Derivatives* (Wiley, 1998).

3.1.15 Mathematical Physiology

(Prof Maini — 16 lectures, HT)

Overview

The course aims to provide an introduction which can bring students within reach of current research topics in physiology, by synthesising a coherent description of the physiological background with realistic mathematical models and their analysis. The concepts and treatment of oscillations, waves and stability are central to the course, which develops ideas introduced in the more elementary B8a course. In addition, the lecture sequence aims to build understanding of the workings of the human body by treating in sequence problems at the intracellular, intercellular, whole organ and systemic levels.

Synopsis

Review of enzyme reactions and Michaelis-Menten theory. Trans-membrane ion transport: Hodgkin-Huxley and Fitzhugh-Nagumo models.

Excitable media; wave propagation in neurons.

Calcium dynamics: calcium-induced calcium release. Intracellular oscillations and wave propagation.

The electrochemical action of the heart. Spiral waves, tachycardia and fibrillation. The heart as a pump. Regulation of blood flow.

Respiration and CO_2 control. Mackey and Grodins models.

Regulation of stem cell and blood cell production. Dynamical diseases.

Reading

The principal text is:

1. J. Keener and J. Sneyd, *Mathematical Physiology* (Springer-Verlag, 1998). Chs. 1, 4, 5, 9–12, 14–17.

Subsidiary mathematical texts are:

1. J. D. Murray, *Mathematical Biology* (Springer-Verlag, 2nd ed., 1993).
2. L. A. Segel, *Modeling Dynamic Phenomena in Molecular and Cellular Biology* (CUP, 1984).
3. L. Glass and M. C. Mackey, *From Clocks to Chaos* (Princeton University Press, 1988).
4. P. Grindrod, *Patterns and Waves* (Oxford University Press, 1991).

General physiology texts are:

1. R. M. Berne and M. N. Levy, *Principles of Physiology* (2nd ed., Mosby, St. Louis, 1996).
2. J. R. Levick, *An Introduction to Cardiovascular Physiology* (3rd ed. Butterworth-Heinemann, Oxford, 2000).
3. A. C. Guyton and J. E. Hall, *Textbook of Medical Physiology* (10th ed., W. B. Saunders Co., Philadelphia, 2000).

3.1.16 Nonlinear Systems

(Dr Moroz — 16 lectures, HT)

Overview

This course aims to provide an introduction to the tools of dynamical systems theory, which are essential in the realistic modelling and study of many disciplines, including Mathematical Ecology and Biology, Fluid mechanics, Economics, Mechanics and Celestial Mechanics. The course will include the study of both deterministic ordinary differential equations, as well as nonlinear difference equations, drawing examples from the various areas of application, whenever possible and appropriate. The course will include the use of numerical software including Matlab.

Learning outcomes

Students will have developed a sound knowledge and appreciation of some of the tools and concepts used in the study of dynamical systems.

Synopsis

Bifurcations for Ordinary Differential Equations

Bifurcations for simple ordinary differential equations: saddle-node, transcritical, pitchfork, Hopf. Centre stable and unstable manifolds. Normal forms. The Hopf Bifurcation Theorem. Lyapunov functions. [6 lectures]

Bifurcations For Maps

Poincaré section and first-return maps. Brief review of multipliers, stability and periodic cycles.Elementary bifurcations of one-dimensional maps: saddle-node, transcritical, pitchfork, period-doubling. Two-dimensional maps. Hénon and Standard map. [6 lectures]

Chaos

Logistic map. Bernoulli shift map and symbolic dynamics. Smale Horseshoes. Lorenz equations. [4 lectures]

Reading

1. G. L. Baker and J. P. Gollub, *Chaotic Dynamics: An Introduction* (2nd ed., CUP, Cambridge, 1996).

2. P. G. Drazin, *Nonlinear Systems* (CUP, Cambridge, 1992).

3.1.17 Numerical Solution of Differential Equations II

(Dr Sobey — 16 lectures, HT)

Overview

To introduce and give an understanding of numerical methods for the solution of ordinary and partial differential equations, their derivation, analysis and applicability.

These lectures concentrate on the numerical solution of boundary value problems.

Learning Outcomes

Students will understand and have experience of the theory for:

(i) Construction of shooting methods for boundary value problems in one independent variable

(ii) Elementary numerical analysis of elliptic partial differential equations

(iii) Analysis of iterative methods for solution of large linear systems of equations

Syllabus

Boundary value problems for ordinary differential equations: shooting and finite difference methods.

Boundary value problems for PDEs: finite difference discretisation; Poisson equation. Associated methods of sparse numerical algebra: brief consideration of sparse Gaussian elimination, classical iterations, multigrid iterations.

Synopsis

The course is concerned with numerical methods for boundary value problems. We begin by developing numerical techniques for the approximation of boundary value problems for second-order ordinary differential equations.

Boundary value problems for ordinary differential equations: shooting and finite difference methods. [Introduction (1 lecture) + 2 lectures]

Then we consider finite difference schemes for elliptic boundary value problems. This is followed by an introduction into the theory of direct and iterative algorithms for the solution of large systems of linear algebraic equations which arise from the discretisation of elliptic boundary value problems.

Boundary value problems for PDEs: finite difference discretisation; Poisson equation. Associated methods of sparse numerical algebra: sparse Gaussian elimination, classical iterations, multigrid iterations. [13 lectures]

Reading

This course does not follow any particular textbook, but the following essentially cover the material:

1. A. Iserles, *A First Course in the Numerical Analysis of Differential Equations* (Cambridge University Press, 1996), Chapters 7,10,11.
2. K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations* (Cambridge University Press, 1994).
   Or the more recent 2nd edition (2005), Chapters 6 and 7.
3. G. D. Smith, *Numerical Solution of Partial Differential Equations: Finite Difference Methods* (Clarendon Press, Oxford, 1985 (and any later editions)), has some of the material in Chapter 5.

3.1.18 Waves & Compressible Flow

(Lecturer t.b.c. — 16 lectures, HT)

Overview

Propagating disturbances, or waves, occur frequently in applied mathematics. This course will be centred on some prototypical examples from fluid dynamics, the two most familiar being surface gravity waves and waves in gases. The models for compressible flow will be derived and then analysed for small amplitude motion. This will shed light on the important phenomena of dispersion, group velocity and resonance, and the differences between supersonic and subsonic flow, as well as revealing the crucial dependence of the waves on the number of space dimensions.

Larger amplitude motion of liquids and gases will be described by incorporating nonlinear effects, and the theory of characteristics for partial differential equations will be applied to understand the shock waves associated with supersonic flight.

Learning Outcomes

Students will have developed a sound knowledge of a range of mathematical models used to study waves (both linear and nonlinear); will be able to describe examples of waves from fluid dynamics and will have analysed a model for compressible flow. They will have an awareness of shock waves and how the theory of characteristics for PDEs can be applied to study those associated with supersonic flight.

Synopsis

1–2 Equations of inviscid compressible flow including flow relative to rotating axes.

3–6 Models for linear wave propagation including Stokes waves, Inertial waves, Rossby waves and simple solutions.

7–10 Theories for Linear waves: Fourier Series, Fourier integrals, method of stationary phase, dispersion and group velocity. Flow past thin wings, Huyghens principle.

11–12 Nonlinear Waves: method of characteristics, simple wave flows applied to one-dimensional unsteady gas flow and shallow water theory.

13–16 Shock Waves: weak solutions, Rankine-Hugoniot relations, oblique shocks, bores and hydraulic jumps.

Reading

1. H. Ockendon and J. R. Ockendon, *Waves and Compressible Flow* (Springer, 2004).
2. J. R. Ockendon, S. D. Howison, A. A. Lacey and A. B. Movchan, *Applied Partial Differential Equations* (revised edition, OUP, Oxford, 2003). Chs 2.5, 4.5–7.
3. D. J. Acheson, *Elementary Fluid Dynamics* (OUP, 1990). Ch 3
4. J. Billingham and A. C. King, *Wave Motion* (CUP, 2000). Ch 1–4, 7,8.

Background Reading

1. M. J. Lighthill, *Waves in Fluids* (CUP, 1978).
2. G. B. Whitham, *Linear and Nonlinear Waves* (Wiley, 1973).

3.1.19 C++ for Scientific Computing

(Lecturer t.b.c. — week 1, TT)

Synopsis

Producing almost any numerical software requires writing codes that manipulate matrices and vectors, making Matlab a natural choice as an introductory programming language for scientific computing. However, the ease of programming in Matlab comes at a cost: the codes take a relatively long time to execute; and the software is commercial. While the use of procedural languages such as Fortran and C will overcome these limitations, they do not allow the straightforward coding of operations between matrices and vectors permitted by Matlab. An alternative approach is to use an object-oriented language such as C++ , where vectors and matrices can be represented as classes. Writing subroutines for these classes that have the same syntax as employed by Matlab allows code developed in Matlab to be translated with minimal effort into C++ code. Such an approach combines the ease of development in the Matlab environment without the associated drawbacks.

Pre-requisites

Basic coding of numerical methods in Matlab.

Syllabus

C++ programming fundamentals:

1. Variables and expressions
2. Input and output
3. Flow of control: if, switch, while, for
4. Pointers and references
5. Arrays
6. Functions

Object orientation in C++:

1. Concept of a class
2. Private, public and protected class members
3. Constructors
4. Operator overloading
5. Function overloading — templates
6. Defining functions that may take default values
7. Exceptions
8. Derived classes using the example of sparse matrices from Matlab, where a matrix is represented by three vectors

3.1.20 Mathematics for Geoscience

(Prof Farmer — 12 lectures, TT)

Synopsis

The course starts with some background to the geology, geophysics and engineering involved in the recovery of oil from deep inside the Earth. The lectures then fall into three areas:

Flow through porous media — balance laws, Darcy's law, analytical and numerical methods, upscaling.

Grid generation and geometric modelling — surface modelling, structured and unstructured grids.

Spatial Statistics — Kriging, stochastic sampling.

All topics involve much application of partial differential equations and the calculus of variations. The mathematical theory applies equally to groundwater modelling, the study of subsurface pollution and remediation, and even to problems outside the geosciences.

Reading

1. Murray R. Spiegel, *Vector Analysis and an Introduction to Tensor Analysis*, (Schaum's Outline Series, 1959).
2. G. F. Carrier and C. E. Pearson, *Partial Differential Equations: Theory and Technique*, (Academic Press, 1988).
3. P. M. Knupp and S.Steinberg, *Fundamentals of Grid Generation*, (CRC Press, Boca Raton, 1993).
4. J. J. Binney, N. J. Dowrick, A. J. Fisher and M. E. J. Newman, *The Theory of Critical Phenomena*, (Oxford University Press, Oxford, 1992), (Appendix L in particular.)
5. D. W. Peaceman, *Fundamentals of Numerical Reservoir Simulation*, (Elsevier, 1977).

Background Reading

1. R. C. Selley, *Elements of Petroleum Geology*, (Academic Press, San Diego, 1998).

3.1.21 Numerical Multiphysics Modelling in Biology and Physiology

(Dr Whiteley & Dr Smith — 12 lectures, TT)

Please note it has not been confirmed that this course will be running in 2008-09.

Overview

Many problems across the physical and life sciences require the coupling of mathematical models of various physical processes. For example, when modelling the heart we must model biochemical reactions, electrical conduction, mechanical deformation and fluid mechanics. Calculating a numerical solution of this large system is often prohibitively computationally expensive, and so we resort to decomposing the large system into smaller, computationally tractable subsystems. The accuracy, efficiency and stability of the combined numerical scheme is heavily dependent on the way in which the system is decomposed. In this course we use simple example problems to develop techniques that may be used to assess the stability, efficiency and accuracy of a given decomposition.

Synopsis

1. Derivation of an example simplified model of electromechanical coupling, i.e. a beating heart, that consists of two components: a model of the electrical activity; and a model of the mechanical activity. Demonstration of the dangers of using a naive numerical scheme when coupling these models.

2. Problems where the coupling is in one direction only — i.e. assuming the electrical model is independent of the mechanical model. Proof of stability of the resulting numerical scheme. A posteriori error analysis of the effect of transferring error from one component of the system to the other.

3. Problems where the system is coupled in both directions. Solution by iteration, proof of stability, a posteriori error analysis.

4. The approximation of weak coupling in one direction by a one-way coupled system.

Prerequisites

Prerequisites for this course are the B2 course on Finite Element Methods for Partial Differential Equations.

3.1.22 Spectral Methods for ODE & PDE

(Prof Trefethen — 12 lectures, TT)

Please note it has not been confirmed that this course will be running in 2008-09.

Synopsis

This lecture course is a hands-on introduction to spectral methods for ordinary and partial differential equations. The emphasis is on spectral collocation ("pseudospectral") methods, including both fundamental principles and applications realised in Matlab. Applications include eigenvalue problems, Schrödinger's equation, hydrodynamic stability, boundary-value problems, and wave equations. Notes for the course will be handed out by the lecturer.

Reading

1. L. N. Trefethen, *Spectral methods in MATLAB*, (SIAM, 2000).

3.1.23 Stochastic Modelling & Simulation of Biological Processes

(Prof Burrage — 12 lectures, TT)

Synopsis

This course will address some of the important issues associated with the modelling and simulation of processes taking place on or inside a biological cell. The following topics will be covered:

- A brief introduction to important aspects of Molecular Cellular Biology.
- How noise arises in cellular processes.
- Intrinsic and Extrinsic noise.
- Discrete stochastic modelling and simulation through Markov processes.
- Continuous stochastic modelling and simulation through stochastic differential equations.
- Trajectorial simulations versus probability density calculations.
- The role of delays in genetic regulation.
- Spatial modelling: granular transport versus reaction diffusion equations.
- Towards multiscale simulation.

Many of these issues will be discussed in the light of a number of case studies

- The Hes1 gene as a molecular clock.
- Cascading reactions: Michaelis-Menten and MAPK.
- Stochastic switches.
- Insulin/glucose dynamics.

All simulations to be done in MATLAB.

Reading

1. Christopher Fall, Eric Marland, John Wagner and John Tyson (eds.) *Computational Cell Biology* (Springer, 2002).
2. Linda J. S. Allen, *An Introduction to Stochastic Processes with Applications to Biology*, (Pearson/Prentice Hall, 2003).
3. D. J. Wilkinson, *Stochastic Modelling for Systems Biology* (Taylor and Francis, 2006).

A Special Topic Guidelines

Special Topics usually take the form of a short essay based on a topic relevant to one of the listed lecture courses. Students are expected to read beyond the lectures and to write about 10–15 pages.

The subject of the essay **must** be agreed between the lecturer and the student and the student should then write a short plan (2 or 3 sentences including 1 or 2 references) which **must** be approved by the lecturer before the end of the term in which the lectures take place. The lecturer will keep a copy of this plan.

The student will then write the special topic, usually without further assistance from the lecturer, but he/she should consult his/her supervisor on general issues. In particular a draft of the special topic should be shown to the supervisor before final submission.

The lecturer will mark the special topic themselves or arrange for an alternative marker to do so.

Students are advised to read the University's policy on plagiarism which may be found in Appendix C and is also available online at: http://www.admin.ox.ac.uk/epsc/plagiarism/

B Exam Conventions

The board of examiners will consist of 4 internal members (2 from the Numerical Analysis Group and 2 from OCIAM) and 1 external examiner. The examiners will appoint assessors to help with the assessment of special topics and dissertations.

All students must complete 13 units. Each unit will carry the same weight. Marks will be given in terms of USMs out of 100 with the usual conventions: 0–49 fail, 50–69 pass, 70–100 distinction.

A student generally takes and is assessed on: 4 core courses; 3 special topics; 2 case studies (one in modelling and one in scientific computing); a dissertation.

1. Core Courses (1 unit each). There are four courses, two in Michaelmas Term and two in Hilary Term. Each course will be assessed by a written examination paper on Thursday of Week 0 of the following term. Each paper will be two hours long and contain 6 questions. The best 4 answers will count and students will be given a USM for each paper, with a weighting of 1 unit.

2. Special topics (1 unit each). Each student must do at least one special topic in the area of Modelling (M) and one in the area of Computation (C). For each special topic taken the student must submit a mini-project. Mini-projects will be marked by the appropriate lecturer. For each mini-project they will be given a USM (0–49 fail, 50–69 pass, 70–100 distinction):

    - 70–100: High quality work at distinction level. The project should show excellent understanding of the relevant concepts, skill and competence in the mathematical description and clarity in the presentation.
    - 50–69: Satisfactory work corresponding to a pass. The project should show some understanding with reasonable arguments and layout but may contain minor flaws.
    - 49 or less: Poor work corresponding to a fail.

3. Case Studies in Modelling and in Scientific Computing (1 unit each). Each student must do at least one modelling case study and at least one scientific computing case study. Each case study involves 4 weeks of group work, an oral presentation, and a report. Each student will write an individual report which will be marked by the appropriate lecturer. 20% of the mark will be for the oral presentation. For each Case Study they will be given a USM (0–49 fail, 50–69 pass, 70–100 distinction), with a weighting of 1 unit.

4. Dissertation (4 units). Dissertations will be read and marked by at least two examiners/assessors, neither of whom is the student's supervisor and at least one of whom will be an examiner. Dissertations should be approximately 50 pages long, and not necessarily original, and will be given a USM with a weighting of 4 units. The USM marks will include performance in the viva if one is held.

The USMs, weighted as above, are averaged to give an Average USM.

Students will only be able to obtain a Distinction if they fulfil all the following criteria:

- Average USM ≥ 70
- All partial USM > 50
- Dissertation/Viva of sufficient merit (examiners' discretion).

Students will fail if the average USM< 50. Candidates who fail 4 or more units of assessment may also fail even if their average USM ≥ 50.

Otherwise, students will be awarded a Pass.

C University's Policy on Plagiarism

C.1 What is plagiarism?

Plagiarism is the copying or paraphrasing of other people's work or ideas into your own work without full acknowledgement. All published and unpublished material, whether in manuscript, printed or electronic form, is covered under this definition. Collusion is another form of plagiarism involving the unauthorised collaboration of students (or others) in a piece of work. Cases of suspected plagiarism in assessed work are investigated under the disciplinary regulations concerning conduct in examinations. Intentional or reckless plagiarism may incur severe penalties, including failure of your degree or expulsion from the university.

C.2 Why does plagiarism matter?

It would be wrong to describe plagiarism as only a minor form of cheating, or as merely a matter of academic etiquette. On the contrary, it is important to understand that plagiarism is a breach of academic integrity. It is a principle of intellectual honesty that all members of the academic community should acknowledge their debt to the originators of the ideas, words, and data which form the basis for their own work. Passing off another's work as your own is not only poor scholarship, but also means that you have failed to complete the learning process. Deliberate plagiarism is unethical and can have serious consequences for your future career; it also undermines the standards of your institution and of the degrees it issues.

C.3 What forms can plagiarism take?

- Verbatim quotation of other people's intellectual work without clear acknowledgement. Quotations must always be identified as such by the use of either quotation marks or indentation, with adequate citation. It must always be apparent to the reader which parts are your own independent work and where you have drawn on someone else's ideas and language.
- Paraphrasing the work of others by altering a few words and changing their order, or by closely following the structure of their argument, is plagiarism because you are deriving your words and ideas from their work without giving due acknowledgement. Even if you include a reference to the original author in your own text you are still creating a misleading impression that the paraphrased wording is entirely your own. It is better to write a brief summary of the author's overall argument in your own words than to paraphrase particular sections of his or her writing. This will ensure you have a genuine grasp of the argument and will avoid the difficulty of paraphrasing without plagiarising. You must also properly attribute all material you derive from lectures.

- Cutting and pasting from the Internet. Information derived from the Internet must be adequately referenced and included in the bibliography. It is important to evaluate carefully all material found on the Internet, as it is less likely to have been through the same process of scholarly peer review as published sources.

- Collusion. This can involve unauthorised collaboration between students, failure to attribute assistance received, or failure to follow precisely regulations on group work projects. It is your responsibility to ensure that you are entirely clear about the extent of collaboration permitted, and which parts of the work must be your own.

- Inaccurate citation. It is important to cite correctly, according to the conventions of your discipline. Additionally, you should not include anything in a footnote or bibliography that you have not actually consulted. If you cannot gain access to a primary source you must make it clear in your citation that your knowledge of the work has been derived from a secondary text (e.g. Bradshaw, D. Title of Book, discussed in Wilson, E., Title of Book (London, 2004), p. 189).

- Failure to acknowledge. You must clearly acknowledge all assistance which has contributed to the production of your work, such as advice from fellow students, laboratory technicians, and other external sources. This need not apply to the assistance provided by your tutor or supervisor, nor to ordinary proofreading, but it is necessary to acknowledge other guidance which leads to substantive changes of content or approach.

- Professional agencies. You should neither make use of professional agencies in the production of your work nor submit material which has been written for you. It is vital to your intellectual training and development that you should undertake the research process unaided.

- Autoplagiarism. You must not submit work for assessment which you have already submitted (partially or in full) to fulfil the requirements of another degree course or examination.

C.4 Not just printed text!

The necessity to reference applies not only to text, but also to other media, such as computer code, illustrations, graphs etc. It applies equally to published text drawn from books and journals, and to unpublished text, whether from lecture handouts, theses or other student's essays. You must also attribute text or other resources downloaded from web sites.

All matters relating to plagiarism are taken very seriously and would lead to a Disciplinary matter.

For further information see The Proctors' and Assessors' Memorandum *Essential Information for Students* Section 9, also available online at
http://www.admin.ox.ac.uk/proctors/info/pam/section9.shtml.

D Electronic Resources for Mathematics

Oxlip — Oxford Libraries Information Portal http://www.bodley.ox.ac.uk/oxlip/

Physical & Mathematical Sciences

Databases

Resource name	Subject coverage	Access
MathSciNet	Mathematical Reviews produced by the AMS	Oxlip
Zentralblatt MATH European Mathematical Society	Pure and Applied Mathematics and History of Mathematics http://www.zentralblatt-math.org/zmath/en/advanced/	Internet
INSPEC	Physics, Engineering, Computing and Applied Mathematics	Oxlip
Web of Knowledge — Web of Science — Science Citation Index	Science and Technology	Oxlip

Electronic Journals

Oxford University e-journals portal http://sfx7.exlibrisgroup.com/oxford/az provides an extensive collection of e-journals published by the main societies and publishers: Association for Computing — ACM Digital Archive, American Mathematical Society, London Mathematical Society, IEEE Electronic Library — Computer Society (Digital Library), SIAM journals including LOCUS archive, Cambridge University Press — Computer Science and Mathematics, Science Direct, Springer journals, Wiley InterScience, etc.

Resource name	Subject coverage	Access
DOAJ Directory of Open Access Journals	Full-text journals	Oxlip
E-print and preprint servers ArXiv.org	Full-text articles in physics, mathematics, computing uk.ArXiv.org at http://xxx.soton.ac.uk/	Open Access
The Computing Research Repository (CoRR)	Online repository. Full-text articles. http://arxiv.org/corr/home	Open Access
The Mathematical Institute E-prints Archive (Oxford University)	Open access resource, full-text articles http://eprints.maths.ox.ac.uk/	Open Access
Oxford University Research Archive — ORA	Full-text articles, conference proceedings, theses, reports	Oxlip
ZETOC journals — British Library	Electronic Tables of Contents Bibliographic details (references only)	Oxlip

Electronic Books

Resource name	Subject coverage	Access
Encyclopaedia of Mathematical Physics	e-book	Oxlip
Handbook of Mathematical Functions	e-book	Oxlip
Handbook in Economics series (Science Direct) including: Handbook of Mathematical Economics; Handbook of Computational Economics; Handbook of the Economics of Finance; more ...	E-book series in Economics, Finance, Computer Science	Oxlip
Handbook of Statistics (Science Direct)	e-book	Oxlip
Lecture Notes in Mathematics (Springer)	e-book series	Oxford e-journals
Lecture Notes in Computer Science (Springer)	e-book series	Oxford e-journals
CREDO Reference Science dictionaries online	e-books	Oxlip
Oxford Reference Online Oxford University Press	e-books, English Dictionaries and Science Dictionaries	Oxlip
Oxford Scholarship Online — OUP	e-books, Economics and Finance Collection	Oxlip

Dissertations

Resource name	Subject coverage	Access
Dissertation Abstracts Online	Doctoral dissertations and Masters Theses	Oxlip
Electronic Index to Theses	Dissertations from UK universities	Oxlip
Oxford Doctoral Dissertations	Science, Technology and Medicine	Print/RSL

Conference Proceedings

Resource name	Subject coverage	Access
British Standards On-Line — BSOL	Full-text standards	Oxlip
IIEE Conference Proceedings, Standards	Engineering, Physics, Applied Sciences http://www.ieee.org/web/publications/home/index.html	Internet
Index to Scientific and Technical Proceedings	ISI Web of Knowledge — Science, Technology and Engineering	Oxlip
Lecture Notes in Mathematics	Conference Proceedings, e-books	Oxford e-journals

General and reference resources and software

Resource name	Subject coverage	Access
EndNoteWeb/EndNote	Reference management software (web version)	Oxlip
RefWorks	Reference management software (web version)	Oxlip

RefWorks and EndNoteWeb are online research management, writing and collaboration tool, designed to help researchers easily gather, manage, store and share all types of information, as well as generate citations and bibliographies. Any bona fide member of the University may freely create an account, though you must be within the Oxford Internet domain to do so.

Internet Gateways (quality assessed Internet Resources)

Resource name	Subject coverage	Access
American Mathematical Society (AMS)	Directory of Mathematics Pre-print and e-Print Servers http://www.ams.org/global-preprints/	Internet
EMIS — The European Mathematical Information Service	Pure and Applied Mathematics, Statistics. http://www.maths.soton.ac.uk/EMIS/ European Mathematical Society — EMS	Internet
Intute — Internet resources for Science, Engineering and Technology	Quality assessed Internet Resources Mathematics http://www.intute.ac.uk/sciences/mathematics/	Oxlip/ Internet
The London Mathematical Society	LMS http://www.lms.ac.uk/	Internet
PhysMathCentral	Peer reviewed e-journals (physics and maths) http://www.physmathcentral.com/	Open Access
The Royal Society	Independent scientific academy of the UK http://royalsociety.org/	Internet
Society for Industrial and Applied Mathematics SIAM	Books, Journals, Conference Proceedings http://www.siam.org/	Internet

Accessing electronic resources:
Electronic resources can be accessed directly on the University of Oxford network. For remote access to databases, electronic reference works and e-book or e-journals packages: Use OxLIP+ beta version (http://oxlip-plus.ouls.ox.ac.uk) and login with your Oxford Single Sign-On (SSO) (http://www.oucs.ox.ac.uk/registration/oxford/). Alternatively, click directly on a resource name and it will prompt you for your SSO authentication (which will override the padlock).

For remote access to individual e-journals:
Use OU E-journals (http://ejournals.ouls.ox.ac.uk/). Users will be prompted to login with their Oxford Single Sign-On (SSO) when clicking through from the list of e-journal titles to the individual journal homepage.

For print resources check OLIS Library Catalogue http://www.lib.ox.ac.uk/olis/
Please note that this List of resources is not exhaustive.

Your subject librarian can advice on other relevant resources for your research topic. Contact: Ljilja Ristic, Physical Sciences Librarian Subject Consultant, Radcliffe Science Library Ljilja.Ristic@bodley.ox.ac.uk, Tel. (01865) 272816.