*Article*

# A Compositional model of Consciousness based on subjectivity as a fundamental feature of nature

**Camilo Miguel Signorelli [1,2,*] 0000-0002-2110-7646, Quanlong Wang [1,*]**

[1] Department of Computer Science, University of Oxford; cam.signorelli@cs.ox.ac.uk
[2] Cognitive Neuroimaging Unit, INSERM U992, NeuroSpin
* Correspondence: cam.signorelli@cs.ox.ac.uk; quaang@cs.ox.ac.uk

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**Abstract:** The science of consciousness have gained considerable understanding on objective neural mechanisms of consciousness, however this strategy has also failed in recovering subjective features from the objective and measurable mechanisms. One alternative to current models of consciousness is starting by the assumption of subjectivity as fundamental, instead of the physical realm. Then, from that subjectivity try to recover the "objective world". These ideas have motivated the conscious agent model, and here, alternative forms of this hypothesis are explored in terms of diagrammatic reasoning, string diagrams and category theory. Firstly, philosophical considerations are introduced, arguing the difference between our philosophical stand point and what would be dualism, idealism, or other types of monism. Secondly, the definition of conscious experience is constructed via entangled features of that experience, together with empirical distinctions. Then, the biding problem is defined and after the introduction of our reasoning model, the main issues about that problem admit a simple solution when the questions are stated differently and according the model discussed. Finally, we explore and discuss the viability of using string diagrams and process theories, adding only a new interpretation and linking the results with conscious experience.

**Keywords:** Consciousness; Conscious Agents; Compositionality; Binding problem; Mathematics of Conciousness; Monoidal Categories

## 1. Introduction

The science of consciousness has proved elusive. On the one hand, biology and neuroscience have acquired considerable comprehension of objective neural mechanisms of consciousness [1]. On the other hand, the subjective aspects of conscious experience are mainly neglected by these approaches [2,3] or at least postponed for further developments [4]. The basic assumption is that subjective aspects of experience would emerge from the objective physical properties of the brain. In other words, the world, considered as both objective and subjective, might be entirely constructed by measurable physical generators, and subjective features of reality are merely consequences of the objective and measurable properties of the world. In this line, one would expect that taking a physical and mathematical theory, the subjective aspects of the experience may naturally emerge from the interaction and combination of these physical and mathematical generators. Nevertheless, scientific approaches to consciousness have failed in recovering subjectivity from the objective and measurable reality [2,3,5].

It is well recognised that objectivity is a basic assumption of science. Objectivity relates to a perceived or unperceived object while subjectivity to a perceiving subject. The object is meant to exist independently of any subject to perceive it, and as such, objectivity is commonly associated with concepts like truth and reliability [6]. This assumption is deeply grounded in classical neuroscience, as well as other scientific fields [7–9]. Contemporary theories of consciousness tend to focus on the physical parts from which, for example, the unity of experience would emerge as a whole. The parts are considered cells, neurons, brain regions, and the whole being the unified conscious experience. This is called building blocks models [10] or reductionist approaches [8].

Nevertheless, there is an epistemological issue: "our knowledge is limited to the realm of our own subjective impressions, allowing us no knowledge of objective reality as it is in itself" [6]. One alternative to deal with that issue is to remove the assumption of objectivity, and take consciousness as fundamental property of the world. One theoretical example is the conscious agent model [11,12], where the world consists of conscious agents and their experiences. Once the original question is solved, now the inverse problem comes into play: how does objective phenomenon such as quantum physics or relativity arise from? Thus, the aim of such models is recovering fundamental physical theories from the agent's interactions, for example, quantum mechanics [11]. Ontologically, this type of idealism is different than current scientific approaches to consciousness and cognition. Moreover, there is still much work to satisfactorily reach that goal, and it is not so evident that the current versions of conscious agent models are capable to recover the entire objective realm (see objections and replies section in [11]).

Through these pages, we propose some new ideas toward answering these questions. Subjective aspects of reality are here postulated as primitive, and used as mathematical generators of the objective aspects, without falling into idealism nor dualism. Differently than reductionist approaches, our model take advantage of compositional principles which define the unity of elements as primarily and basic to any potential division of components [13]. These compositional models have been successful at the moment to understand basic computational capabilities of physical theories such as quantum theory [14,15], causal models [16,17], relativity [18] and interestingly also natural language [19] and cognition [20,21]. Therefore, in this article, a new language to reason about consciousness features is defined, taking inspiration from the hypothesis of conscious agents [12], phenomenology [22], Yogacara school [23,24], as well as other elements from the unified field hypothesis [10] and compositionality [13,25]. The mathematical language is based on diagrammatic calculus, following similar principles and mathematical structures that have proved useful in the understanding of fundamental physical theories [26]. Using this new compositional framework and primitive mathematical generators as essential features of subjective experience and consciousness, different aspects of experience, such as binding properties or external and internal subjective distinction, emerge naturally. Eventually, the future goal is recovering objective physical theories from primitive notions of subjectivity that indeed would correspond to each other, avoiding ontological claims and without the need of invoking any physical realization but pure mathematical entities.

Following these comments, our study starts with an introduction of the Yogacara philosophical understanding of consciousness, in terms of fundamental existence and epistemic restrictions (Section 2). Then, a brief discussion of the irreducible features of consciousness and subjective experience (Section 3) serves to re-define them in terms of primitive diagrams (Section 4). From these definitions, other aspects will naturally emerge (Section 5). As an example of application, the binding problem is examined to then propose a simple and elegant solution (Section 6), followed by the main conclusions (Section 7).

## 2. Yogacara Philosophy and Phenomenology

Starting from subjective aspects of reality may sound new to modern science, but the discussion of epistemic restrictions have been part of millenary traditions such as Buddhism and its Yogacara school, long before phenomenology appears as the science of phenomena and experience. Yogacara (Sanskrit for *Yoga Practice*), also called Vijnanavada (*Doctrine of Consciousness*) or Vijnaptimatra (*Consciousness Only*), is one of the two main branches of Mahayana (*Great Vehicle*) Buddhism (the other being Madhyamaka, *Middle way*). All the alternative names of Yogacara philosophy involve the key concept of consciousness, and specifically, consciousness-only. This concept is sometimes wrongly interpreted. Nevertheless, the meaning behind is closer to epistemic limitations mentioned in modern phenomenology than variants of philosophical idealism [23,27].

To understand consciousness-only, another concept from the Yogacara philosophy is needed: Trisvabhāva or the three natures. Trisvabhāva is the premise that all the possible forms of existence are

divided into three types: i) Parikalpita-svabhāva, the *fully conceptualized* nature, ii) Paratantra-svabhāva, the *other dependent* nature, and iii) Parinipanna-svabhāva, the *perfect-accomplished-real* nature. As explained by [24]: "The first nature is the nature of existence produced from attachment to imaginatively constructed discrimination. The second nature is the nature of existence arising from causes and conditions. The third nature is the nature of existence being perfectly accomplished (real)". The third nature of existence is "the ultimate reality, something that never changes". An important remark is that this nature does not correspond to mind or the "ultimate mind" from which everything would originate. The ultimate reality is invariant and can not be directly depicted, it is neither objective nor subjective.

The confusing move is the statement that these three natures are inseparable from consciousness, as mentioned in Cheng Weishi Lun [28] and translated to English by [29]. There, it is argued that the three natures of existence are equivalently characterised in terms of the mind, its attributes (*Citta-Caittas*) and the manifestations emerged by it (*darsana* and *nimittabhaga*). In this form, the natures of existence involve many conditioning factors which all together generate the illusion of existence. This illusion is called the "nature of dependence on others" (*Paratantra*), and it is manifested as *Ataman* and *Dharmas*. They may exist or not, being identical or different, inclusive or exclusive, among other qualities. In other words, distinctions appear. According the Yogacara philosophy, all these distinctions are the "nature of mere-imagination" (*Parikalpita*). Every aspect that is dependent on others is indeed void (*Sunya*) and the only genuine nature is revealed by this *voidness*, which is finally called "nature of ultimate reality" (*Parinispanna*). Therefore, all these three natures are inseparable from the perceiving mind, and what Yogacara called consciousness-only is this inseparability between consciousness and the three natures of existence. However, this inseparability does not imply that existence is the mind, or reduced to mind. Consciousness-only does not claim that mind nor consciousness correspond to the ultimate reality and unique category from which everything emerges (like different types of idealism and monism would claim), but the epistemic restriction that mind and its limitations imposes to the access and description of that ultimate reality [23]. In this sense, Yogacara philosophy is the first phenomenological approach to the physical and mental world that indeed relate each other to the same *voidness*.

This remark might become clear when mind is defined as possessed by sentient beings. The second nature or the other dependent nature is what Yogacara refers to the mind and its attributes. On that framework, the mind, as part of sentient beings, is divided into eight types of consciousnesses, what in modern science one would call senses or ways of perceiving: the five **sense-consciousnesses** (eye or visual, ear or auditory, nose or olfactory, tongue or gustatory, body or tactile consciousnesses), **mental consciousness**, **manas consciousness** (the seventh or thought-centre consciousness), and **alaya consciousness** (the eighth or storehouse consciousness). Each type of consciousness manifests itself in two forms: the perceived division (nimittabhaga in Sanskrit) and the perceiving division (darsanabhaga in Sanskrit). Here, mental consciousness becomes relevant because it is closer to modern notions of awareness in the form of phenomenal consciousness and access consciousness (Section 3). Therefore, the mind is not related to an original and invariant nature, but one more illusion, and indeed the major mechanism why illusions appear to us, sentient beings [23].

Contrary to dualism, the notions above deny any conceptual duality (e.g. physical-non-physical, external-internal) regarding the *perfect-accomplished-real* nature. Different than idealism [27], the mind is not seen as cause effective of the rest of the world, by only of the illusion of distinctions on that world. Consciousness is essential because everything considered, affirmed or denied, even the idea of objectivity, occur to us only in consciousness. However, consciousness is not the ultimate reality. Therefore, the ontological query is suspended while an epistemic caution is reinforced: "all our efforts to get beyond ourselves are nothing but projections of our consciousness"[23]. In modern words, consciousness-only would be better understood as a claim of awareness-only, or perception-only, much closer to phenomenology.

## 3. Features of Consciousness

The discussion above sets the background for following sections: our model aims to describe the perceived and perceiving division of mental consciousness. Consciousness as part of mental attributes of sentient agents is quite tricky to define, especially because the word is used in different contexts with dissimilar connotations [30] and even distinct verbal times (transitive and intransitive) [31]. Nevertheless, what is quite clear is that phenomenal consciousness involves qualitative and subjective aspects of experience [10], associated with inner processes of sentient agents. Therefore, the meaning of conscious experience will be constructed from the discussion of some, but not all, different features involved, assuming the reader is familiar with part of these concepts, thus leaving aside some details. The main focus is on the attributes that are quite accepted, up to certain subtleties [32], inside the scientific community on consciousness research. So, the model avoids any further assumptions such as the extra axioms stated in [33]. Interestingly, the few ingredients discussed below are enough to recover important properties of consciousness within our formal semantics (Section 5).

### *3.1. Essential features of Consciousness*

Common accounts of consciousness would start describing the intuitions behind the understanding of consciousness. These intuitions are formalized in terms of **features of consciousness**, such as qualitativeness, subjectivity, unity, intentionality [10], and **aspects of consciousness**, phenomenal and access consciousness [34]. Among features, qualitativeness is one of the most important notions, closely connected with the concept of qualia. According to Searle, the first three features qualitativeness, subjectivity and unity are intrinsically related to each other and should be considered the same. To those accounts, an extra feature is added, the property of distinction (Section 2), which is commonly missed on these descriptions of conscious experience. Therefore, our starting point will be a bit unconventional, empathising what we called the **3+1 essential properties of consciousness** that we consider any theory should aim to explain. These concepts are strongly entangled between each other, as the reader will notice.

The first attribute discussed here is **unity**. Any experience is given as a unified single moment. Some may argue this experience is continuous, others that it is discrete [35,36], it may contain one or many different contents, etc. Independently, the subsumed experience is one unified coexistence, a unified conscious field [10] that may be just conceptually subdivided into different notions of unity (Objectual, Spatial, Subjective, Subsumptive) [32]. In the followed model, unity is not only a property of experience, it is inherited from an essential property of what was called the ultimate reality or *voidness*.

One aspect of this unity is what some authors consider the distinct feature of **qualitativeness**. Every experience is mostly qualitative, instead of quantitative [37]. In words of Nagel, there is a kind of "it feels like" or "what is like to be" something or someone having certain experience [2]. The qualitativeness character of experience may come from external perceptions or internal thoughts. In this work, the former and the latter are not differentiated [10]. This property is what distinguishes between the experience of red and green: the irreducible phenomenology of consciousness, phenomenal consciousness or qualia [34]. Moreover, qualitativeness implies distinctions among experiences. One experience that has certain quality may be different than other experience that generate the feeling of having other quality.

Qualitativeness leads us to the second side of the unity, or the third feature of consciousness: **subjectivity**. It seems that experiences only exist if there are subjects or agents (sentient beings) to experience something. Neither a rock appears to have any kind of experience, nor particles or atoms. Qualitativeness would imply subjectivity since, to exist a qualitative feeling regarding some event, there must exist a subject to experience that event [10]. This experience is part of the called first-person data, corresponding to elements of reality that do not exist without a subject, such as perceptual experiences (e.g. the experience of color), bodily experiences (e.g., pain and hunger), emotional experiences, mental imagery, among others [38]. First-person data contrasts with the third-person data, related to objective

and quantitative measurements such as brain signatures of perceptual discrimination or differences between sleep and wakefulness [3]. More generally, brain signatures of conscious experience would correspond to objective elements that seem to exist without the need of subjects to perceive them. That division is also called the hard and easy problem of consciousness [3]. Nevertheless, conscious experiences only exist when there are agents to experience: some "I" owner of that experience. This feature admits a connection with conscious access aspect of experience, since the existence of the subject or agent owner of that experience imposes another distinction, a physical or virtual boundary that perceived elements must "cross" to become part of that experience.

Furthermore, qualitativeness and subjectivity as features of conscious experience imply two different kinds of unities that in turns generate distinctions. The former corresponds to the **phenomenal unity** and the latter to the **access unity** [32]. The distinctions correspond to experiences and content, respectively. The "how" we are conscious and "what" we are conscious of [39]. This fits quite well with our previous discussions, mind and consciousness generate distinctions that break the invariability of the *voidness* and creates different ways to discriminate between agent-subject and object, qualitative and quantitative features, inside or outside, identical or different, among others. This last attribute is called the **distinction** property. In other words, phenomenal subjective experiences are unified in terms of phenomenology as well as distinctive regarding accessibility. Distinction applies between experiences but also distinguishing among elements on that experience. While the phenomenal unity is associated with high-levels of the binding problem reviewed below, the access unity is connected with the segregation aspect of that problem, and as we will see, the binding problem becomes a problem of distinction instead of unity.

The linage generated by qualitativeness and subjectivity is in our framework what in science of consciousness is commonly called phenomenal consciousness and access consciousness (Figure 1). Phenomenal experience, the what is like to be, is differentiated from access consciousness, the accessibility of content in certain moment of time [34,40] and self-reference, linked with confidence, control and meta-cognition [41,42]. The difference is not only conceptual, it also involves empirical evidence of different brain signatures [34,40]. Therefore, it is an empirical and conceptual distinction. This mention is important because axioms on the model discussed bellow do correspond to these conceptual but also empirical division. The model assumes nothing extra than starting from these features of consciousness as fundamental axioms of the model.

Summarizing, the full story looks like this (Figure 1): Unity is a primary notion of mind, while mind generates experiences which are qualitative and subjective aspects of that unity, namely phenomenal unity and access unity, the perceiving and the perceived. Both types of unity-experience generates distinctions among experiences and contents of experience, as well as between the agent-subject and the object, qualitative an quantitative features, inside or outside, which are summarized by the feature of distinction imposed by the conscious experience (Figure 1). Therefore, a conscious experience would be, by definition, unified in terms of its phenomenal aspect or phenomenal unity, but not necessarily in terms of its access unity. In other words, different evolving subjective experiences correspond to modifications or modulations of a unified and already existing qualitative subjectivity, the intrinsic mental consciousness (independent of the five senses [43]). Moreover, Unity is more than just a mental attribute, it is inherited from a fundamental property of the world, that also appear to us in what is called the physical realm. In terms of Yogacara school, this fundamental property may correspond to the *voidness* or emptiness of existence, the lack of distinctions that reveals the ultimate existence as being part of an invariant world. All these concepts are dependent of each other, they are indeed entangled and interdependent. The conceptual distinction here is just that, a conceptual distinction to make easy the understanding of our reasoning.

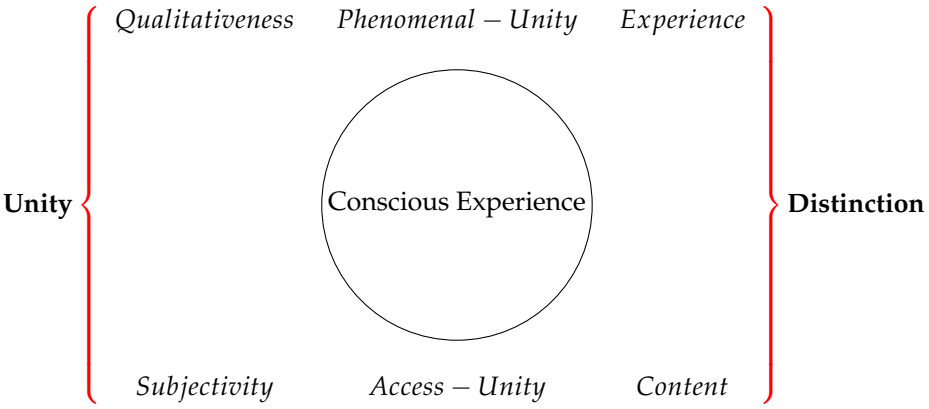



**Figure 1.** Essential features of Consciousness. A conceptual division map to organize the discussion about features and aspects of consciousness inside our framework. The upper part corresponds to Phenomenal Consciousness and the lower to Access Consciousness. In the centre, Conscious experience is the entangled combination of all these concepts. Unity is here an essential feature of experience, from which qualitativeness and subjectivity are part, as the perceiving and perceived division that all together define conscious experience. From experience itself, distinctions emerge among experiences and contents of these experiences.

### *3.2. Unbinding the Binding Problem*

Unity and distinction are crucial features of experience, and commonly associated with, but not reduced to, the binding properties of perception, neurons and brain regions. The so-called binding problem [44,45]. In a materialistic and reductionist formulation, the problem is stated as the need of a neural mechanism from which unified experiences emerge by combination of separated elements, e.g. how to bind different features of a perceived object, such as colour and shape (Figure 2A). This corresponds to a low level of the **combination aspect** of the binding problem [45]. A high level would be another instance where these combined objects are thought to be bound with other background features, as well as emotional feelings to create one single unified phenomenal subjective experience [32]. This is the phenomenal unity of the combination problem [32,44], the intuition that regardless the distinct neural paths, our experiences are integrate-wholes. This is the subjective or **phenomenal binding problem**. According to this construction, the subjective unified experience seems inconsistent with the many separate brain activities from which the whole experience is thought to emerge, there is not a single module or region where that integration may take place [45,46]. Furthermore, there is a **segregation aspect** of binding, i.e. having a blue square and a red triangle how one can recognize that the blue belongs to the square and the red to the triangle and not vice versa [45–47]. In other words, how sensory inputs are allocated to recognize "discrete objects" and not just a collection of separated colours and shapes (Figure 2B). Mechanistically, the question is how cells and neurons recognize that they are being activated either by different objects or by only one complex object. This is a discrimination issue, the **feature binding problem**, associated with distinctive properties of experience and access unity. Both, the combination and segregation, are considered part of one and the same binding problem. Hence, the question becomes to understand how properties of objects are first combined, then segregated to later being recombined or unified in one whole experience together with all the extra features of the experienced context.

Unfortunately, these two aspects and versions of the problem are not always differentiated, making the discussion sometimes ambiguous [48]. In this line, Revonsuo and others clearly stated different related binding problems, some associated with consciousness and others not. At least three levels are distinguished: phenomenal, neural and cognitive [44]. In turns that Feldman describes four binding problems [45]: Coordination, Subjective, Feature and Variable binding, all of them regarded to different tasks, time scales and brain circuits. According to these definitions, there are different models trying to solve the questions about combination and segregation, mainly regarding feature binding.

Some of them are combination coding, population coding, binding by synchrony [49], and feature integration model [50]. However, none of them is theoretically and empirically satisfactory, they seem acting at different hierarchical levels and stages of perception [46,47], leaving important open questions that will be reconsidered in light of the new compositional and diagrammatic model presented in next sections (Section 5 and 6). There, the difference between phenomenal unity and access unity as separate features of experience plus the distinction feature emerged from them will become important for a tentative solution or reinterpretation of the binding problems. To avoid any confusion, our focus will be the subjective/phenomenal and feature binding regarding the combination and segregation aspects.

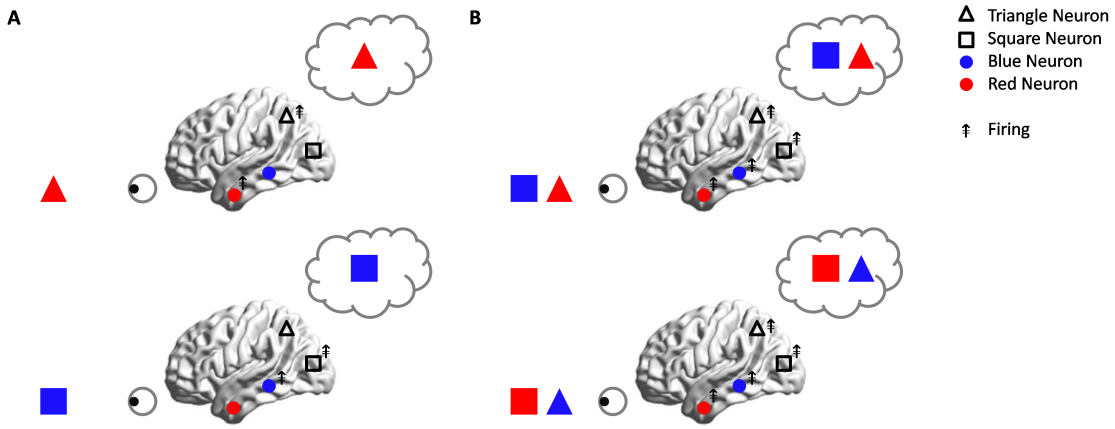



**Figure 2.** The Feature Binding Problem. A) The combining aspect of binding is about how cells and neurons integrate different features, for instance, shape and colour. Based on the assumption of independent neurons or modular brain regions processing different features, the integration may take place if neurons corresponding to each feature are simultaneously activated. In the upper figure, a red triangle activates the triangle shape neuron and the colour red neuron, in lower figure an example for blue square. B) The problem arises when the triangle and the square are presented simultaneously, first with one combination of colours (top) and then inverting them (bottom). In both cases all the neurons or regions are activated at the same time. Therefore, the question becomes how the brain can segregate each colour to the corresponding shape. One alternative is a combination coding, such as new layers of neurons would bind the previous ones. Another is binding by synchrony, i.e. neurons with correlated firing would bind features together. Unfortunately, for these and others possible solutions, there are theoretical and empirical concerns. The main objection is indeed the original assumption of independent processing features or modular paths.

## 4. Mathematical Consciousness Generators

The mathematical formalism in next sections is based on basic notions of Category theory. The core definitions of that framework are presented to then introduce a graphical diagrammatic language allowing a mathematical reasoning about previously mentioned features of consciousness and phenomenal subjective experience. This diagrammatic language is mathematically rigorous [15] and has proven useful to reconstruct different aspects of physical theories [26,51,52].

### *4.1. Preliminaries*

#### 4.1.1. Category

A category $\mathfrak{C}$ consists of:

- a class of objects $ob(\mathfrak{C})$;
- for each pair of objects $A, B$, a set $\mathfrak{C}(A, B)$ of morphisms from $A$ to $B$;

- for each triple of objects $A, B, C$, a composition map

$$\begin{array}{ccc} \mathfrak{C}(B,C) \times \mathfrak{C}(A,B) & \longrightarrow & \mathfrak{C}(A,C) \\ (g,f) & \mapsto & g \circ f; \end{array}$$

- for each object $A$, an identity morphism $1_A \in \mathfrak{C}(A,A)$,

satisfying the following axioms:

- associativity: for any $f \in \mathfrak{C}(A,B), g \in \mathfrak{C}(B,C), h \in \mathfrak{C}(C,D)$, there holds $(h \circ g) \circ f = h \circ (g \circ f)$;
- identity law: for any $f \in \mathfrak{C}(A,B), 1_B \circ f = f = f \circ 1_A$.

A morphism $f \in \mathfrak{C}(A,B)$ is an *isomorphism* if there exists a morphism $g \in \mathfrak{C}(B,A)$ such that $g \circ f = 1_A$ and $f \circ g = 1_B$. A *product category* $\mathfrak{A} \times \mathfrak{B}$ can be defined componentwise by two categories $\mathfrak{A}$ and $\mathfrak{B}$.

4.1.2. Functor

Given categories $\mathfrak{C}$ and $\mathfrak{D}$, a functor $F : \mathfrak{C} \longrightarrow \mathfrak{D}$ consists of:

- a mapping

$$\begin{array}{ccc} \mathfrak{C} & \longrightarrow & \mathfrak{D} \\ A & \mapsto & F(A); \end{array}$$

- for each pair of objects $A, B$ of $\mathfrak{C}$, a map

$$\begin{array}{ccc} \mathfrak{C}(A,B) & \longrightarrow & \mathfrak{D}(F(A),F(B)) \\ f & \mapsto & F(f), \end{array}$$

satisfying the following axioms:

- preserving composition: for any morphisms $f \in \mathfrak{C}(A,B), g \in \mathfrak{C}(B,C)$, there holds $F(g \circ f) = F(g) \circ F(f))$;
- preserving identity: for any object $A$ of $\mathfrak{C}$, $F(1_A) = 1_{F(A)}$.

A functor $F : \mathfrak{C} \longrightarrow \mathfrak{D}$ is *faithful (full)* if for each pair of objects $A, B$ of $\mathfrak{C}$, the map

$$\begin{array}{ccc} \mathfrak{C}(A,B) & \longrightarrow & \mathfrak{D}(F(A),F(B)) \\ f & \mapsto & F(f) \end{array}$$

is injective (surjective).

4.1.3. Natural transformation

Let $F, G : \mathfrak{C} \longrightarrow \mathfrak{D}$ be two functors. A natural transformation $\tau : F \to G$ is a family $(\tau_A : F(A) \longrightarrow G(A))_{A \in \mathfrak{C}}$ of morphisms in $\mathfrak{D}$ such that the following square commutes:

$$\begin{array}{ccc} F(A) & \xrightarrow{\tau_A} & G(A) \\ {\scriptstyle F(f)}\downarrow & & \downarrow{\scriptstyle G(f)} \\ F(B) & \xrightarrow{\tau_B} & G(B) \end{array}$$

for all morphisms $f \in \mathfrak{C}(A,B)$. A natural isomorphism is a natural transformation where each of the $\tau_A$ is an isomorphism.

#### 4.1.4. Strict monoidal category

A strict monoidal category consists of:

- a category $\mathfrak{C}$;
- a unit object $I \in ob(\mathfrak{C})$;
- a bifunctor $- \otimes - : \mathfrak{C} \times \mathfrak{C} \longrightarrow \mathfrak{C}$,

satisfying

- associativity: for each triple of objects $A, B, C$ of $\mathfrak{C}$, $A \otimes (B \otimes C) = (A \otimes B) \otimes C$; for each triple of morphisms $f, g, h$ of $\mathfrak{C}$, $f \otimes (g \otimes h) = (f \otimes g) \otimes h$;
- unit law: for each object $A$ of $\mathfrak{C}$, $A \otimes I = A = I \otimes A$; for each morphism $f$ of $\mathfrak{C}$, $f \otimes 1_I = f = 1_I \otimes f$.

#### 4.1.5. Strict symmetric monoidal category

A strict monoidal category $\mathfrak{C}$ is symmetric if it is equipped with a natural isomorphism

$$\sigma_{A,B} : A \otimes B \to B \otimes A$$

for all objects $A, B, C$ of $\mathfrak{C}$ satisfying:

$$\sigma_{B,A} \circ \sigma_{A,B} = 1_{A \otimes B}, \;\; \sigma_{A,I} = 1_A, \;\; (1_B \otimes \sigma_{A,C}) \circ (\sigma_{A,B} \otimes 1_C) = \sigma_{A,B \otimes C}.$$

#### 4.1.6. Self-dual strict compact closed category

A self-dual strict compact closed category is a strict symmetric monoidal category $\mathfrak{C}$ such that for each object $A$ of $\mathfrak{C}$, there exists two morphisms

$$\epsilon_A : A \otimes A \to I, \;\; \eta_A : I \to A \otimes A$$

satisfying:

$$(\epsilon_A \otimes 1_A) \circ (1_A \otimes \eta_A) = 1_A, \;\; (1_A \otimes \epsilon_A) \circ (\eta_A \otimes 1_A) = 1_A.$$

#### 4.1.7. PROP

A PROP is a strict symmetric monoidal category having the natural numbers as objects, with the tensor product of objects given by addition [53].

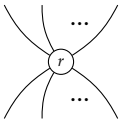
As shown in [54], any PROP can be represented by generators and rewriting rules. This enables PROPs to be understood intuitively by string diagrams without the burden of heavy math. We will pursue this diagrammatic way through out the next sections.

### *4.2. Interpretations*

Given a theory, a generator is the basic mathematical structure, together with an specific interpretation, that generates the calculus on that theory. Following our previous discussion, the four essential features of conscious experience are re-defined here in terms of the four main mathematical generators or processes. These generators and its interpretations are the main components in our theory, the essential or fundamental processes that lead the rest of interactions among elements inside the model.

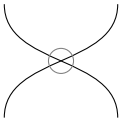
**Unity** $\mapsto$

**Qualitativeness** $\mapsto$




**Subjectivity** $\mapsto$

**Distinction** $\mapsto$



Taking these interpretations and mathematical processes in form of a theory of consciousness, extra operations and basic rules correspond to the axioms of the theory. These rules allow us to reason about the interpretations of the theory, prove and infer new results, following them as a frame of theoretical work. These rules will not be defined here, but as an example, the next section shows the way to define and implement them.

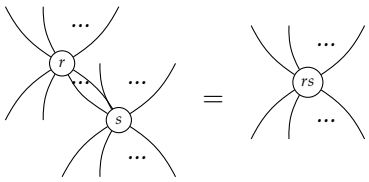
## 5. Properties of Consciousness

The rationale behind axiomatic mathematical models is deducing new properties from the primitive definitions within the theory. Therefore, our framework focuses on what can be done with the generators and rules more than if our description is ontologically true. As an example, we start defining the unity and what means composition for the generators of the theory. Following the compositional framework, our model recovers interesting and desirable properties for a model of consciousness.

Unity is interpreted as connectivity, the unity generators ensure that connectivity, where different paths on a circuit, cross or connect at least in one moment and place. Consequences of the Unity generators are the possibilities to connect, combine and compound in many different ways the other generators of the model, as well as manipulating and transforming the circuits, accommodating and visualizing their interactions.

For instance, the composition for Qualitativeness becomes simply:

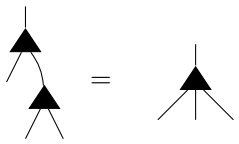
**Definition 1.** *Qualitativeness compounds as follows:*




(1)

It means that any qualitative aspect of the experience (given by the process of quality) is fused and glued by default, just by means of being connected. Similar rule is stated for subjectivity.

**Definition 2.** *Subjectivity compounds as follows:*



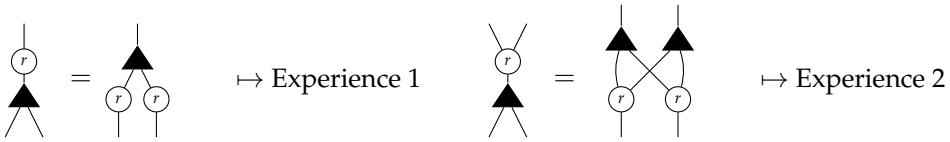
(2)

These compositions or rules ensure the continuity of unity of experience and its compositional nature across different instances of experience. In both cases, composition take the form of a fusion rule.

Moreover, a definition for conscious experience in terms of composition of our generators is of course desirable. As discussed above, conscious experience was conceptually defined as the interaction or composition between all the aspects summarized in Figure 1. Therefore a diagrammatic proposition regarding these ideas takes the next form:

**Proposition 1.** ***Conscious experience****. Conscious experiences correspond to compositions of Qualitativeness and Subjectivity processes, such as the interaction generates a new shape, informing of one or another kind of experience.*

**Example 1.**

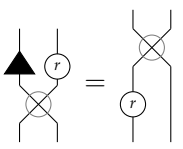



The way how to read these diagrams is from top to bottom, i.e. imagine the  "crossing"  . The effect of the interaction is given by the rules or axioms of the theory. In our example, experience 1 is the composition of one input quality process $r$ and one subjective process. This composition generates a copy of that quality. Experience 2 is the composition between two inputs quality and one subjective process again, generating a different experience. The subtle difference is the number of inputs in the qualitative process. Therefore, conscious experiences are unified composition of qualitative and subjective generators related to the shape-effect, or circuit reorganization, generated by rules of composition.

According to that an previous discussion, the main problem of feature binding is not to recognize the mechanisms of network integration or feature combination, but the mechanisms of distinction, segregation or division of features. The model tries to solve the binding problem by inverting the question from “how to bind two elements together?” to “how to distinguish two elements already binded?” (Section 6).

To target this question, the distinction component defined above seems to present a compelling property:

**Proposition 2.** ***Distinction.*** *Distinction differentiates between qualitativeness and subjectivity as follows:*

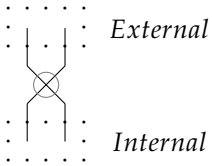



(3)

In other words, only qualitative aspects can go through a distinction. If subjectivity is associated with subject-agents and qualitativeness as basic process triggered by objects, then qualitative aspects of experience are the features that cross distinctions while subjective features do not. It generates a

distinction between subjects-objects and between internal-external experiences, since another subject is always external to the observing subject separated by a distinction process.

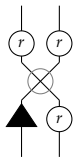
**Proposition 3.** ***Boundary.*** *Distinction generates a boundary between external/internal.*




(4)

Now all the basic compositions and definitions of the model are in place to define Perception as the simple composition all these generators:

**Proposition 4.** ***Perception.*** *Perception corresponds to the composition of Qualitativeness, Subjectivity and Distinction.*



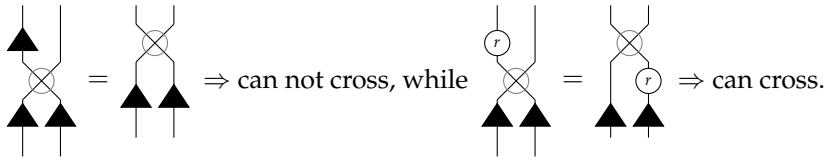

$\mapsto$ *Perception of r* (5)

Perception is not only a conscious experience, it is also a kind of conscious experience that allows distinctions between external and internal contents of that experience, as well as objective-subjective instanciations, since an externally trigger experience is associated with objective perception, while internally trigger experiences (what happens after crossing the distinction boundary) are commonly related to subjective inner experiences.

As a way of example, this last proposition sediments the path to prove the next theorem.

**Theorem 1.** ***Unreadability of external subjectivity.*** *It is impossible to access, read or compound others/external conscious experiences.*

**Proof.** From proposition 1 and 4, a conscious perception involves qualitativeness, subjectivity and distinction, such as the distinction imposes a boundary between external and internal experiences (equation 4). Moreover, the equation 3 forces a restriction to subjectivity process, preventing it from crossing the boundary. It means




It completes the proof. □

This very simple example shows the power of diagrammatic reasoning, recovering one of the main and recognized underpin of personal subjective conscious experience: the inaccessibility of others "what is like to be" becomes a consequence of a simple and “geometrical” property of the calculus introduced here.

## 6. The Unbinding Problem

In previous section two variants of the binding problem were mentioned, the subjective/phenomenal and the feature binding problem. According previous discussions, the phenomenal binding is giving by primitive notions of unity, while feature binding is the disruption or distortion of that unity. A toy model is given for feature binding case.

### 6.1. Phenomenal Subjective Binding

The brain seems not a unified and holistic system in the same way that conscious experience seems to be [10]. Since the brain is thought as discrete group of neurons and regions, how that system produce a unified conscious experience? This is the classical phenomenal binding problem. In our compositional model, the question is stated differently. The combination and segregation aspects are reduced to only a discrimination issue, since the unity of experience is given by default, just by means of being connected, using the fusion and other rules. Therefore, the phenomenal level is assumed as primitive, reducing the neural and cognitive dimension to a merely "distinction mechanism" without sacrificing the phenomenology. According to our model, the unity of consciousness or phenomenal unity is given by the primitive diagrammatic generators, it is a fundamental property of the model. So, the question is reformulated: how systems, such as the visual system, generates or introduces visual experiences into an already unified conscious field?

This switch of paradigm implies that different conscious perceptions are indeed modifications of an already existing field of consciousness, instead of constructed by various disparate bits of reality [10]. The phenomenal unity of experiences corresponds to the effect of the qualitativeness and subjective processes interacting and reorganizing the circuit formed by them. It translates in a primary basic total field or state of consciousness from which then, its modifications inform about particular perceptual states. A total phenomenal unity subsumes any other distinctive perception [32]. In other words, the most basic conscious experience is the total phenomenal experience, the phenomenal field. This a basic but not less complex structure, given by different types of sub-circuits from which distinctions arise, and eventually conveying a clean explanation of the phenomenal subjective unity of experience.

While, totally neutral to the possible "neural mechanisms", our model solves the combination problem stating that everything is already phenomenologically combined by means of topological connection, in turns that the segregation problem is targeted by the distinction property of consciousness. This aspect may find connection with access unity defined above. In other words, the hard phenomenal binding problem is relaxed, becoming a problem of modification or distinction [10]. The main conclusion guide us to search for mechanisms of separation and distinction that make elements of our perception look segregated, instead of looking how to unify elements already unified. In our framework, the mechanism of segregation related to the feature binding problem would be the geometry modification driven by transformation or rule-based re-configurations of the circuit.

### 6.2. A Toy model for Feature Binding

The original version of feature binding problem comes from the apparent modular codification observed in neurons of the primary visual cortex, which seemed to respond selectively to single features, such as colour or shape. It creates the paradox that any original combination or relationships between stimuli features is lost when decomposed into independent modules, and the need of a recombination somewhere later [48]. Nevertheless, this modular independence is misleading and disconfirmed by modern experiments [55–57]. The same neuron is activated by multiple stimuli and features, and indeed it is also concurrently selective to combinations of features [48]. The brain, in fact, works in parallel where different circuits and tasks are performed simultaneously. Therefore, the unbinding, the separations of the causes of an input, seems more relevant than ever before [45].

In this subsection, a toy model for feature binding is given. Assume there are two choices for colour: green and red; and two choices for shape: square and triangle. The scenario of the feature binding is as follows: given a shape and a colour at the same time, say, square and red, one can see a combined object– red square; given two combined objects, say green square and red triangle, one can see the two objects simultaneously. Then the binding problem is simply restated as: what is the mechanism/transformation for realising the above scenario?

One alternative to solve this question borrows an idea from quantum theory. Firstly, the two shapes are encoded into a two-state system $A_2$: $square \mapsto |0\rangle$ , $triangle \mapsto |1\rangle$ . Secondly, the two colours into another two-state system $B_2$: $green \mapsto |0\rangle$ , $red \mapsto |1\rangle$ . Thirdly, the combined objects is described as

a four-state system $C_4$: *green square* $\mapsto |0\rangle$ , *red square* $\mapsto |1\rangle$ , *green triangle* $\mapsto |2\rangle$ , *red triangle* $\mapsto |3\rangle$ . Then the binding mechanism is realised by the following linear map:

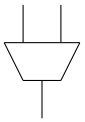
$$\begin{array}{ccc} L : A_2 \otimes B_2 & \longrightarrow & C_4 \\ |00\rangle & \mapsto & |0\rangle \\ |01\rangle & \mapsto & |1\rangle \\ |10\rangle & \mapsto & |2\rangle \\ |11\rangle & \mapsto & |3\rangle \end{array}$$

Here two combined objects presented at the same time are modelled by the superposition of the two states representing the two objects. For example, green square and red triangle shown simultaneously are represented as $|00\rangle + |11\rangle$. Then one can check that the linear map $L$ is the mechanism that realises the binding: given square and green, a green square can be obtained via $L$; given green square and red triangle simultaneously, a green square and a red triangle can be obtained simultaneously via $L$; and so on.

This toy model can be generalised to a generic situation:

$$\begin{array}{ccc} L : A_m \otimes B_n & \longrightarrow & C_{mn} \\ |ij\rangle & \mapsto & |in + j\rangle \end{array}$$

diagrammatically represented by:



where $0 \leq i \leq m-1, 0 \leq j \leq n-1$.

## 7. Conclusions

The main motivation for our theoretical and very simple exercise is bringing back consciousness into formal mathematical and physical discussions. As stated in [37], science has thrown away the subjective features of consciousness, leaving us in strange position, without rigorous tools to describe qualitative aspects of reality that we experience everyday. A common example is the paradox of the falling tree: if a tree falls and nobody is there to hear it, does the tree make any sound? Yes, of course the tree generates vibrations, but the quality of sounds are only assigned by the observer. In other words, there are objective realities (vibrations), but subjective and qualitative features such sounds, colours, smells and tastes are there only if a conscious mind is ready to experience them. The vibration is characterized by common mathematical language and physical mechanisms. We claim here that diagrammatic calculus may bring consciousness back to maths and the physical world, constructing a new form of describing the structure of experience and therefore, a new science of consciousness.

These ideas may inspire great debate and we are willing to motivate that discussion. The study of structures of consciousness also implies to change the paradigm and basic assumptions. Using consciousness as fundamental means starting from its features as axioms, true mathematical axioms and not merely manifestos or empty claims that are not translated to concrete mathematical definitions or propositions. Moreover, generators and processes become abstract mathematical structures, independent of their realizations. Interestingly, it conveys into a clear advantage over other models: the direct connection with physical theories and structures. Our approach is related, almost tautologically, to quantum theory, since the same generators and rewriting rules as the ZW-ZX-calculus are used, just changing their interpretations. The ZW-ZX-calculus are graphical languages for quantum theory, when stated under vector space interpretation. Therefore, the reconstruction goal pursued by conscious agent models is reached here for free, only invoking phenomenal aspects (no need of any monism or dualism). In other words, it doesn't matter whether a process is a consciousness process or

a physical process, they share similar structure. These conclusions and interpretations may inspire debate and their eventual contradictions need to be further discussed.

Finally, our approach is ontologically neutral. Generators do not need to have physical interpretation but a combination of them may have. For example, the two spiders which compose the quantum gate CNOT in ZX-calculus do not represent any physical realization, but their combination do [15]. Similarly, our definitions do not find biological implementation by their own, but the combination of processes and circuits may find concrete realizations into physical and biological mechanisms. Rocks, atoms or electrons do not present any traces of subjective experiences, therefore only sensed/living beings would have both qualitativeness and subjectivity processes distributed and organized in such a way that the interaction leads to conscious and perceptual experience. This is something to explore further, especially in light of other inspirational works about conscious research and mathematical structures. For instance, the distinction process, that indeed is a composition of qualitativeness and subjectivity processes simplified as a generator, may represent biological boundaries and eventually take the form of a second order metabolic closure membrane, as hypothesized in ([58], regarding a more complex categorical structure). Similarly, it is valid to ask how the brain creates the unified field of experience mentioned above. We consider, however, that phenomenal science of consciousness have much more to gain and understand, using high-abstract mathematical formalism, axiomatizing the phenomenology of conscious experience, to later search for the physical structures that may realize that mathematical properties.

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**Sample Availability:** Samples of the compounds ..... are available from the authors.