Categories of Matroids

Vaia Patta

March 6, 2015

What is a matroid?

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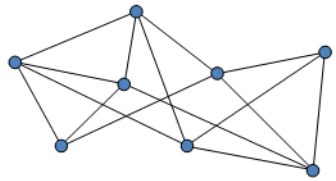

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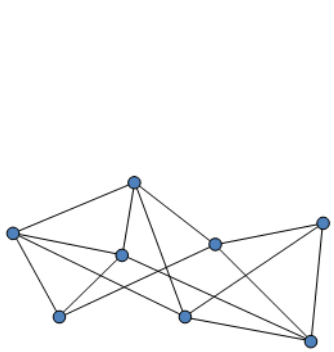

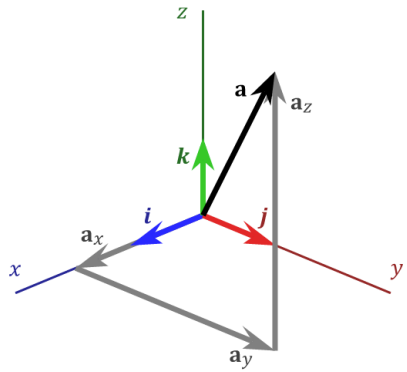

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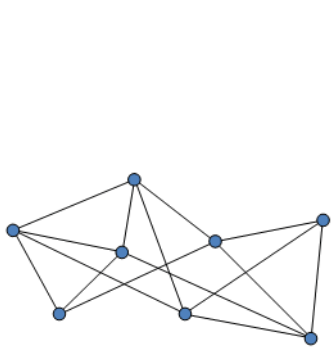

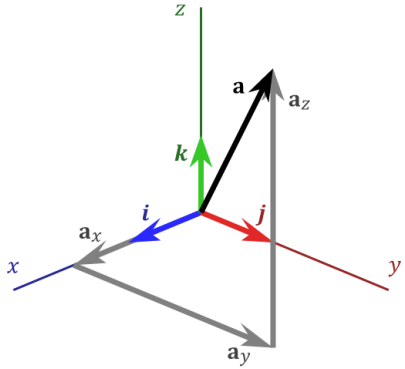

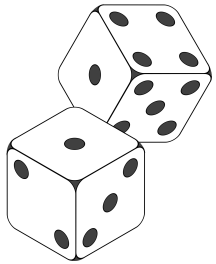

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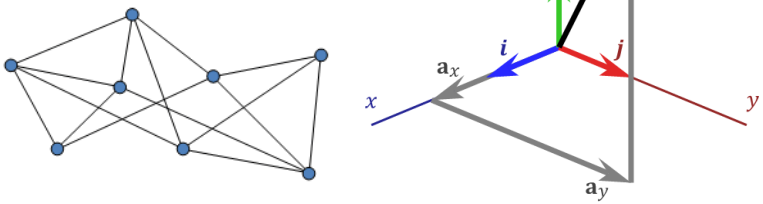

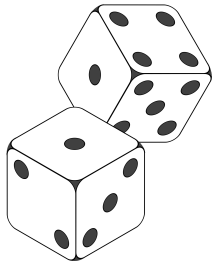

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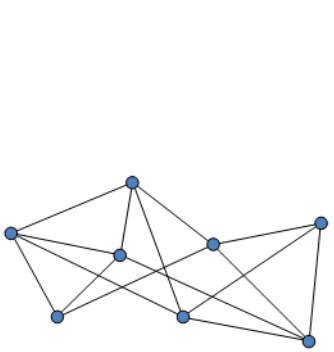

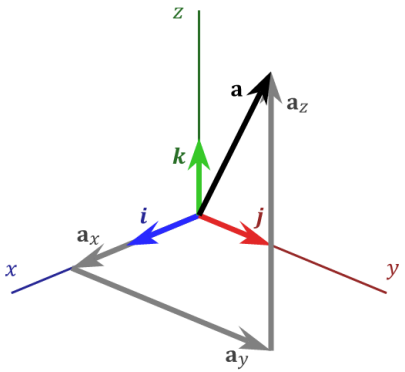

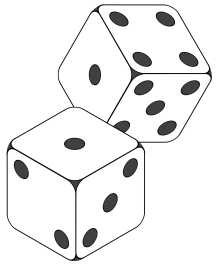

$$x = ax_1 + bx_2 \qquad \text{...etc}$$

Interesting classes of matroids

- *Pointed matroids*: distinguished element $\bullet$, where $\{\bullet\}$ is dependent (i.e. $\bullet$ is a *loop*)

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- *Representable matroid*: those arising from linear dependence of vectors

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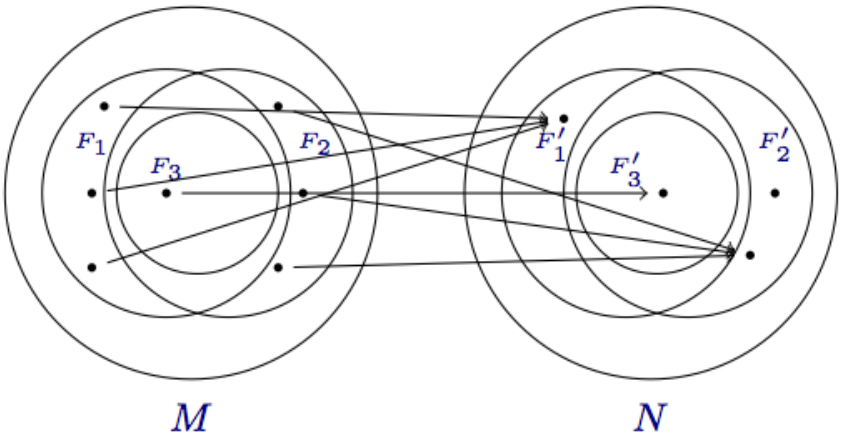
Strong maps: Functions between matroids for which the preimage of a closed set is closed



$f : M \to N$

Limits and Colimits

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Limits:

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Products

Limits and Colimits

	Matroids•	Matroids	Loopless•	Simple	Simple•	Repr. over k	Repr. over k•
Limits:							
Products	✗	✗	✗	✗	✗	✗	✗

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Pullbacks	✗	✗	✗	✗	✗	✗	✗

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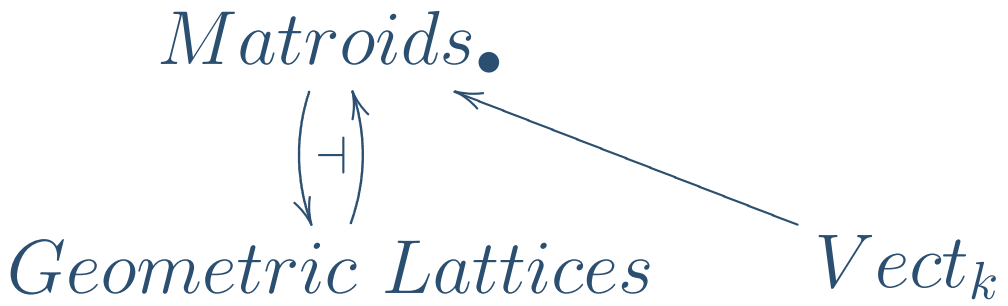
Functors and adjunctions

$$\begin{array}{c} Matroids. \\ \left(\dashv\right) \\ Geometric\ Lattices \end{array}$$

Functors and adjunctions



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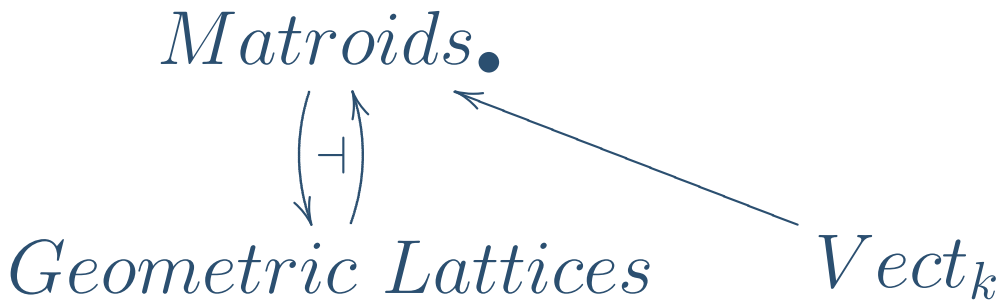

Free matroids: Every set is independent.

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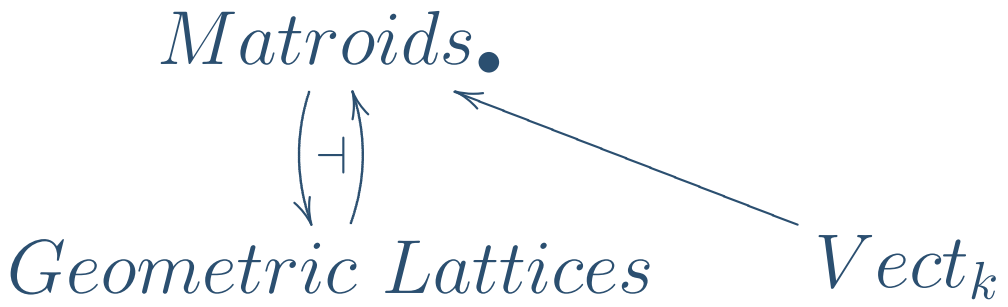

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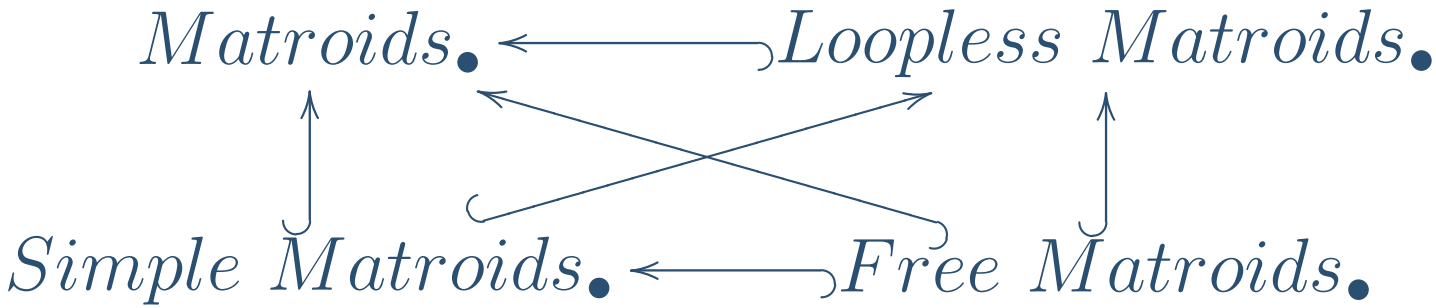

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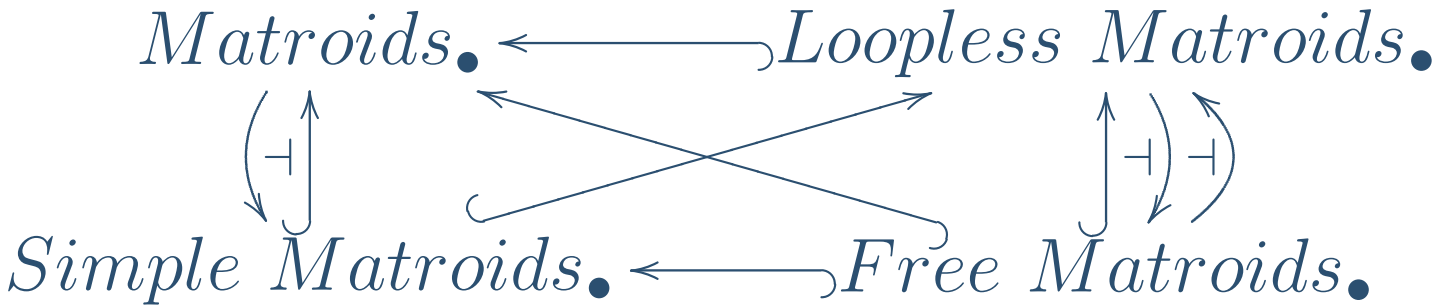

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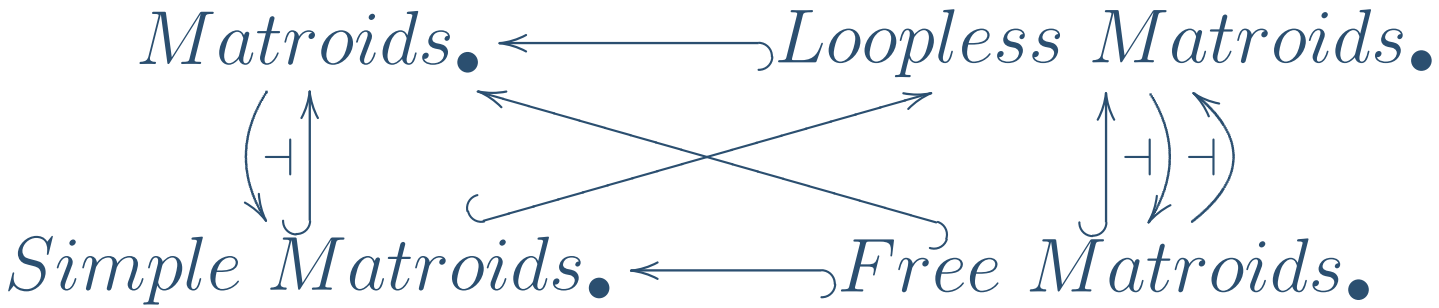

Matroids as functors:

$$Geometric\ Lattices \longrightarrow Sub$$