Game Semantics for Access Control

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Paper in MFPS XXV, Oxford April 3–7 2009

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Then $\ell < \ell'$ means that ℓ is (relatively) Lo and ℓ' is (relatively) Hi.

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In this reading, the underlying principle we want to enforce is:

> No proof of a formula of the form "P says ϕ" can make any essential use of formulas of the form "Q says ψ" unless Q is at the same or higher security level as P. In other words, we cannot rely on a lower standard of "evidence" or authorization in passing to a higher level.

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In this context, it is natural to read the security lattice in the opposite direction!

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The main results are *non-interference theorems*, stating that the desired restrictions are enforced by the type system.

A Small Example (Garg and Abadi)

Let there be two principals, Bob (a user) and admin (standing for administration). Let dfile stand for the proposition that a certain file should be deleted. Consider the collection of assertions:

1. (admin says dfile) $\Rightarrow$ dfile
2. admin says ((Bob says dfile) $\Rightarrow$ dfile)
3. Bob says dfile

Using the unit of the monad with (3) yields (admin says (Bob says dfile)). Using modal consequence with (2) yields:

- (admin says (Bob says dfile)) $\Rightarrow$ (admin says dfile)

dfile now follows using modus ponens.

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- Advantages of the semantic approach: more robust and general. Still applicable to syntactic systems.
- Some novelties in the Game Semantics: justified AJM games (with no justification pointers), eliminating the need for an 'intensional equivalence' on strategies.

Game Semantics

Justified AJM Games

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Intuition for justifiers in terms of procedural control-flow:

- A call of procedure P will have as its justifier the currently active call of the procedure in which P was (statically) declared. ‘Link in the “static chain”’.
- A procedure return will have the corresponding call as its justifier.

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So our games will have a ‘static’ justification function

$$\mathrm{j}_A : M_A \rightharpoonup M_A$$

but no justification pointers — plays are just sequences of moves.

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Global conditions on plays $s \in M_A^{\circledast}$:

(p1) Opponent starts If s is non-empty, it starts with an O-move.

(p2) Alternation Moves in s alternate between O and P.

(p3) Linearity Any move occurs at most once in s.

(p4) Well-bracketing Write each answer a as $)_a$ and the corresponding question $q = \mathrm{j}_A(a)$ as $(_a$. Then we require that s is well-bracketed in the obvious sense.

(p5) Justification If m occurs in s, $s = s_1 m s_2$, then the justifier $\mathrm{j}_A(m)$ must occur in s_1.

Constructions: Bang

The game $!A$ is defined as the "infinite symmetric tensor power" of A. The symmetry is built in via the equivalence relation on positions.

- $M_{!A} = \omega \times M_A = \sum_{i \in \omega} M_A$.
- Labelling is by source tupling: $\lambda_{!A}(i, a) = \lambda_A(a)$.
- Justification is componentwise: $\mathsf{j}_{!A}(i, m) = (i, \mathsf{j}_A(m))$.
- We write $s \upharpoonright i$ to indicate the restriction to moves with index i.

$$P_{!A} = \{s \in M_{!A}^{\circledast} \mid (\forall i \in \omega)\ s \upharpoonright i \in P_A\}\,.$$

- Let $S(\omega)$ be the set of permutations on ω. Then $s \approx_{!A} t$ iff:

$$(\exists \pi \in S(\omega))[(\forall i \in \omega.\, s \upharpoonright i \approx_A t \upharpoonright \pi(i))\ \wedge\ (\pi \circ \mathtt{fst})^*(s) = \mathtt{fst}^*(t)].$$

Linear Implication

- $M_{A \multimap B} = (\Sigma_{b \in \mathsf{Init}_B} M_A) + M_B$.
- $\lambda_{A \multimap B} = [[\overline{\lambda_A} \mid b \in \mathsf{Init}_B], \lambda_B]$.
- We define justification by cases. We write m_b, for $m \in M_A$ and $b \in \mathsf{Init}_B$, for the b-th copy of m.

$$\begin{array}{rcl} \mathsf{j}_{A \multimap B}(m_b) & = & \begin{cases} b, & m \in \mathsf{Init}_A \\ (\mathsf{j}_A(m))_b, & m \notin \mathsf{Init}_A \end{cases} \\ \mathsf{j}_{A \multimap B}(m) & = & \mathsf{j}_B(m), \qquad m \in M_B \,. \end{array}$$

- We write $s{\upharpoonright}A$ to indicate the restriction to moves in $\Sigma_{b \in \mathsf{Init}_B} M_A$, replacing each m_b by m.

$$P_{A \multimap B} = \{s \in M_{A \multimap B}^{\circledast} \mid s{\upharpoonright}A \in P_A \ \wedge \ s{\upharpoonright}B \in P_B\}$$

Note that Linearity for A implies that *only one copy* m_b of each $m \in M_A$ can occur in any play $s \in P_{A \multimap B}$.

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A *strategy* on a game A is a non-empty set $\sigma \subseteq P_A^{\mathsf{even}}$ of even-length plays satisfying the following conditions:

Causal Consistency $sab \in \sigma \implies s \in \sigma$

Representation Independence $s \in \sigma \ \wedge \ s \approx_A t \implies t \in \sigma$

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Everything works out just fine!

The Model

Games Over A Lattice

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Given a security semilattice $(\mathcal{L}, \sqcup, \perp)$, we define a category $\mathcal{G}_{\mathcal{L}}$ with objects

$$A = (M_A, \lambda_A, \mathsf{j}_A, P_A, \approx_A, \mathsf{lev}_A)$$

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(p6) Levels A non-initial move m can only be played if $\mathsf{lev}_A(m) \leq \mathsf{lev}_A(\mathsf{j}_A(m))$.

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Note that formally, this is a purely *static constraint* (on types rather than strategies)!

The Level Monads

Fixing a level ℓ, we can embed $\mathcal{G}$ fully and faithfully into $\mathcal{G}_\mathcal{L}$ by giving every move of every game the level ℓ. Interesting things start to happen when there are moves at different levels.

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We define, for each $\ell \in \mathcal{L}$, a construction T_ℓ on games, which acts only on the level assignment:

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The following commutation properties of T_ℓ are immediate.

Proposition 1 *The following equations hold:*

$$\begin{array}{lcl}
T_\ell I & = & I \\
T_\ell(A \otimes B) & = & T_\ell A \otimes T_\ell B \\
T_\ell(A \multimap B) & = & T_\ell A \multimap T_\ell B \\
T_\ell(A \,\&\, B) & = & T_\ell A \,\&\, T_\ell B \\
T_\ell\, !A & = & !T_\ell A \\
T_\ell(A \Rightarrow B) & = & T_\ell A \Rightarrow T_\ell B
\end{array}$$

The semilattice structure on $\mathcal{L}$ acts on the $\mathcal{L}$-indexed family of monads in the evident fashion:

Proposition 2 *The following equations hold:*

$$\begin{array}{rcl} T_\ell(T_{\ell'}A) & = & T_{\ell \sqcup \ell'}A \\ T_\perp A & = & A. \end{array}$$

We can extend each T_ℓ with a functorial action: if $\sigma : A \rightarrow B$ then we can define $T_\ell\sigma : T_\ell A \rightarrow T_\ell B$ simply by taking $T_\ell\sigma = \sigma$. To justify this, note that

$$P_{A \multimap B} = P_{T_\ell(A \multimap B)} = P_{T_\ell A \multimap T_\ell B}$$

Copycats and Levels

Proposition 3 *The copy-cat strategy is well defined on* $A \multimap T_\ell A$.

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Proof Consider a play of the copy-cat strategy

$$
\begin{array}{cccc}
 & A & \multimap & T_\ell A \\
 & \vdots & & \vdots \\
O & & & m_1 \\
P & m_1 & & \\
O & m_2 & & \\
P & & & m_2
\end{array}
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One shows that the Level condition holds for each of these moves. □

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Thus we can define a natural transformation $\eta_A : A \rightarrow T_\ell A$, where η_A is the copy-cat strategy. Furthermore, by Proposition 2, $T_\ell T_\ell A = T_\ell A$.

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Proposition 4 *Each* T_ℓ *is an idempotent commutative monad.*

Results

No Natural Contraflow

Firstly, we prove a strong form of converse of Proposition 3.

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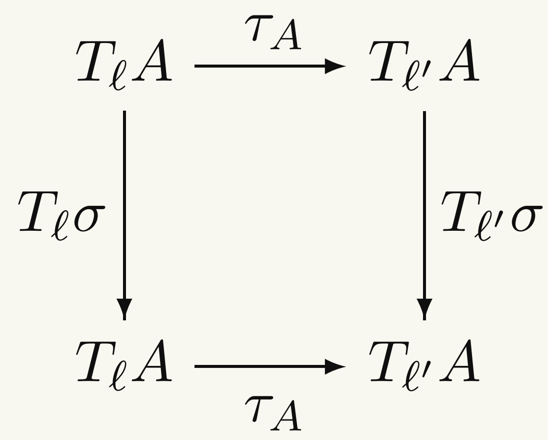
Proof Suppose for a contradiction that there is such a natural transformation τ. Given any flat game $X_\perp^\flat$, with $\mathsf{lev}_{X_\perp^\flat}(m) = \perp$ for all moves $m \in M_{X_\perp^\flat}$, the strategy $\tau_{X_\perp^\flat} : T_\ell X_\perp^\flat \to T_{\ell'} X_\perp^\flat$ can only play in $T_{\ell'} X_\perp^\flat$, since playing the initial move in $T_\ell X_\perp^\flat$ would violate the Level condition.
We now work the naturality square



with $A = \mathbf{Nat}_\perp^\flat$ to yield the required contradiction. $\square$

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Consider the following situation. We have a term in context $\Gamma \vdash t : T$, and we wish to guarantee that t is not able to access some part of the context. For example, we may have $\Gamma = x : U, \Gamma'$, and we may wish to verify that t cannot access x. Rather than analyzing the particular term t, we may wish to guarantee this purely at the level of the types, in which case it is reasonable to assume that this should be determined by the types U and T, and independent of Γ'.

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This can be expressed in terms of the categorical semantics as follows. Note that the denotation of such a term in context will be a morphism of the form $f : A \otimes C \to B$, where $A = [\![U]\!]$, $C = [\![\Gamma']\!]$, $B = [\![T]\!]$.

Formalizing Non-Flow

Consider the following situation. We have a term in context $\Gamma \vdash t : T$, and we wish to guarantee that t is not able to access some part of the context. For example, we may have $\Gamma = x : U, \Gamma'$, and we may wish to verify that t cannot access x. Rather than analyzing the particular term t, we may wish to guarantee this purely at the level of the types, in which case it is reasonable to assume that this should be determined by the types U and T, and independent of Γ'.

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Definition 6 *Let $\mathcal{C}$ be an affine category, i.e. a symmetric monoidal category in which the tensor unit I is the terminal object. We write $\top_A : A \to I$ for the unique arrow. We define $A \not\rightarrow B$ if for all objects C, and $f : A \otimes C \to B$, f factors as*

$$f = A \otimes C \xrightarrow{\top_A \otimes \mathsf{id}_C} I \otimes C \xrightarrow{\cong} C \xrightarrow{g} B.$$

The idea is that no information from A can be used by f — it is "constant in A". Note that $\mathcal{G}_{\mathcal{L}}$ and $\mathcal{G}_{\mathcal{L}}^{\mathsf{hf}}$ are affine, so this definition applies directly to our situation.

The No-Flow Theorem

Firstly, we characterize this notion in $\mathcal{G}_{\mathcal{L}}$ and $\mathcal{G}_{\mathcal{L}}^{\mathsf{hf}}$.

Lemma 7 *In $\mathcal{G}_{\mathcal{L}}$ and $\mathcal{G}_{\mathcal{L}}^{\mathsf{hf}}$, $A \not\rightarrow B$ if and only if, for any strategy $\sigma : A \otimes C \rightarrow B$, σ does not play any move in A.*

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Given a game A, we define:

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\begin{aligned}
\mathsf{Level}(A) \quad &= \quad \{\mathsf{lev}_A(m) \mid m \in \mathsf{Init}_A\} \\
A \rhd B \quad &\equiv \quad \forall \ell \in \mathsf{Level}(A), \ell' \in \mathsf{Level}(B). \neg(\ell \leq \ell')
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Theorem 8 (No-Flow) *For any games A, B in $\mathcal{G}_{\mathcal{L}}$:*

$$A \not\rightarrow B \iff A \rhd B.$$

Computing Levels

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We consider a syntax of types built from basic types (to be interpreted as flat games at a stipulated level) using the connectives of ILL extended with the level monads. For any such type T, we can give a simple inductive definition of $\mathsf{Level}(A)$ where $A = [\![T]\!]$:

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\begin{array}{lcl}
\mathsf{Level}(X^{\flat}_{\ell}) & = & \{\ell\} \\
\mathsf{Level}(I) & = & \varnothing \\
\mathsf{Level}(A \otimes B) & = & \mathsf{Level}(A) \cup \mathsf{Level}(B) \\
\mathsf{Level}(A \multimap B) & = & \mathsf{Level}(B) \\
\mathsf{Level}(A \,\&\, B) & = & \mathsf{Level}(A) \cup \mathsf{Level}(B) \\
\mathsf{Level}(A \Rightarrow B) & = & \mathsf{Level}(B) \\
\mathsf{Level}(!A) & = & \mathsf{Level}(A) \\
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This yields a simple, computable analysis which by Theorem 8 can be used to guarantee access constraints of the kind described above.

Protected Types

We give a semantic account of *protected types*, which play a key rôle in the DCC type system (Abadi, Bannerjee, Heintze, Riecke).

Definition 9 *We say that a game A is* protected at level ℓ *if* $\mathsf{Level}(A) \geq \ell$, *meaning that $\ell' \geq \ell$ for all $\ell' \in \mathsf{Level}(A)$.*

This notion extends immediately to types via their denotations as games.
The following (used as an inductive *definition* of protection in Abadi et al.) is an immediate *consequence* of our definition.

Lemma 10

1. *If $\ell \leq \ell'$, then $T_{\ell'} A$ is protected at level ℓ.*
2. *If B is protected at level ℓ, so are $A \multimap B$ and $A \Rightarrow B$.*
3. *If A and B are protected are level ℓ, so are $A \,\&\, B$ and $A \otimes B$.*
4. *If A is protected at level ℓ, so is $!A$.*
5. *I is protected at level ℓ.*

We also have the following *protected promotion* lemma, which shows the soundness of the key typing rule in DCC.

Lemma 11 *If $\sigma : !A \rightarrow T_\ell B$, $\tau : !B \rightarrow C$, and C is protected at level ℓ, then the coKleisli composition*

$$\sigma^\dagger ; \tau : !A \rightarrow C$$

is well-defined.

Proof Firstly, by Proposition 1, $T_\ell\, !B = !T_\ell B$. So it suffices to show that τ is well-defined as a strategy $\tau : T_\ell\, !B \rightarrow C$. If we consider an initial move m in $T_\ell\, !B$ played by τ, we must have $\mathsf{lev}_{!B}(m) \leq \mathsf{lev}(\mathsf{j}(m))$ since $\tau : !B \rightarrow C$ is well-defined. Moreover, $\ell \leq \mathsf{lev}(\mathsf{j}(m))$ since C is protected at ℓ. Hence $\mathsf{lev}_{T_\ell\, !B}(m) \leq \mathsf{lev}(\mathsf{j}(m))$. □

Stability Under Erasure

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Firstly, given $\ell \in \mathcal{L}$, we define the erasure A^{ℓ} of a type A, which replaces every sub-expression of A of the form $T_{\ell'} B$, with $\ell' \geq \ell$, by $\top$. Semantically, this corresponds to erasing all moves m in the game (denoted by) A such that lev$(m) \geq \ell$, and all plays containing such moves.

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Abadi's result is that, if we can derive a typed term in context $\Gamma \vdash e : A$, then we can derive a term $\Gamma^\ell \vdash e' : A^\ell$. To obtain an appropriate semantic version, we need to introduce the notion of *total* strategies. A strategy σ is total if when $s \in \sigma$, and $sa \in P_A$, then $sab \in \sigma$ for some b.

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Theorem 12 *Suppose that $\sigma : A \to B$ is a total strategy. Then so is $\sigma' : A^\ell \to B^\ell$ for any $\ell \in \mathcal{L}$, where σ' is the restriction of σ to plays in $A^\ell \multimap B^\ell$.*

Proof Suppose for a contradiction that σ' is not total, and consider a witness $sab \in \sigma \setminus \sigma'$, with $sa \in P_{A^\ell \multimap B^\ell}$. Then $\mathsf{lev}(b) \geq \ell$; but by the Level constraint, we must have $\mathsf{lev}(\mathsf{j}(b)) \geq \ell$, which by the Justification condition contradicts $sa \in P_{A^\ell \multimap B^\ell}$. $\square$

Further Directions

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- We have considered a semantic setting which is adequate for both intuitionistic and (intuitionistic-)linear type theories. It would also be interesting to look at access control in the context of *classical type theories* such as $\lambda\mu$, particularly since it is suggested by Abadi and Garg and Pfenning that there are problems with access control logics in classical settings.

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- The development of algorithmic game semantics suggests that it may be promising to look at automated analysis based on our semantic approach.

- We have developed our semantics in the setting of AJM games, equipped with a notion of justification. One could alternatively take HO-games as the starting point, but these would also have to be used in a hybridized form, with "AJM-like" features, in order to provide models for linear type theories. In fact, one would like a form of game semantics which combined the best features (and minimized the disadvantages) of the two approaches. Some of the ideas introduced in the present paper may be useful steps in this direction.