TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

Institute of Theoretical Computer Science Chair for Automata Theory

THE BAYESIAN ONTOLOGY REASONER IS BORN!

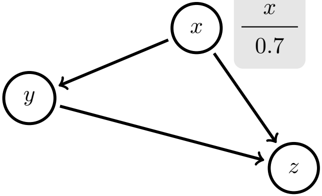
İsmail İlkan Ceylan, Julian Mendez and Rafael Peñaloza

Athens, 6th June 2015

RoSI

Bayesian Networks

Probabilistic graphical models that can compactly represent the joint probability distribution



	y
x	1
$\neg x$	0.5

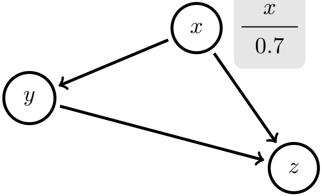
x
0.7

		z
x	y	0.3
x	$\neg y$	0.1
$\neg x$	y	0
$\neg x$	$\neg y$	0.9

Bayesian Networks

Probabilistic graphical models that can compactly represent the joint probability distribution

	y
x	1
$\neg x$	0.5



x
0.7

		z
x	y	0.3
x	$\neg y$	0.1
$\neg x$	y	0
$\neg x$	$\neg y$	0.9

$$\begin{aligned} P(\neg x, \neg y, z) &= P(z|\neg x, \neg y)P(\neg y|\neg x)P(\neg x) \\ &= 0.9 \times 0.5 \times 0.3 \\ &= 0.135 \end{aligned}$$

The Description Logic $\mathcal{BEL}$

Extends $\mathcal{EL}$ by defining a joint probability distribution over the axioms of an $\mathcal{EL}$ ontology

The Description Logic $\mathcal{BEL}$

Extends $\mathcal{EL}$ by defining a joint probability distribution over the axioms of an $\mathcal{EL}$ ontology

$\mathcal{EL}$ concept language	V of propositional variables	contextual TBox
C, D	$\kappa = \{x, \neg y\}$	$\langle C \sqsubseteq D : \kappa \rangle$

The Description Logic $\mathcal{BEL}$

Extends $\mathcal{EL}$ by defining a joint probability distribution over the axioms of an $\mathcal{EL}$ ontology

$\mathcal{EL}$ concept language	V of propositional variables	contextual TBox
C, D	$\kappa = \{x, \neg y\}$	$\langle C \sqsubseteq D : \kappa \rangle$

$\mathcal{BEL}$ Knowledge Base $\mathcal{K} = (\mathcal{T}, \mathcal{B})$

$\left.\begin{array}{l} \mathcal{T} \text{ a (contextual) TBox} \\ \mathcal{B} \text{ a Bayesian network} \end{array}\right\}$ defined over the same variables V

The Description Logic $\mathcal{BEL}$

Extends $\mathcal{EL}$ by defining a joint probability distribution over the axioms of an $\mathcal{EL}$ ontology

$\mathcal{EL}$ concept language	V of propositional variables	contextual TBox
C, D	$\kappa = \{x, \neg y\}$	$\langle C \sqsubseteq D : \kappa \rangle$

$\mathcal{BEL}$ Knowledge Base $\mathcal{K} = (\mathcal{T}, \mathcal{B})$

$\mathcal{T}$ a (contextual) TBox
$\mathcal{B}$ a Bayesian network

} defined over the same variables V

Contextual interpretations

$$\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}, \mathcal{V}^{\mathcal{I}}) \text{ with } \mathcal{I} \models \langle C \sqsubseteq D : \kappa \rangle \text{ iff } \mathcal{V}^{\mathcal{I}} \not\models \kappa \text{ or } C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$$

The Description Logic $\mathcal{BEL}$

Extends $\mathcal{EL}$ by defining a joint probability distribution over the axioms of an $\mathcal{EL}$ ontology

$\mathcal{EL}$ concept language	V of propositional variables	contextual TBox
C, D	$\kappa = \{x, \neg y\}$	$\langle C \sqsubseteq D : \kappa \rangle$

$\mathcal{BEL}$ Knowledge Base $\mathcal{K} = (\mathcal{T}, \mathcal{B})$

$\mathcal{T}$ a (contextual) TBox
$\mathcal{B}$ a Bayesian network
} defined over the same variables V

Contextual interpretations

$$\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}, \mathcal{V}^{\mathcal{I}}) \text{ with } \mathcal{I} \models \langle C \sqsubseteq D : \kappa \rangle \text{ iff } \mathcal{V}^{\mathcal{I}} \not\models \kappa \text{ or } C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$$

Probabilistic interpretation $\mathcal{P} = (\mathfrak{I}, P_{\mathfrak{I}})$

- $\mathfrak{I}$ a set of contextual interpretations
- $P_{\mathfrak{I}}$ a probability distribution over $\mathfrak{I}$

Probabilistic model $\mathcal{P} = (\mathfrak{I}, P_{\mathfrak{I}})$

- $\mathcal{I} \models \mathcal{T}$ for all $\mathcal{I} \in \mathfrak{I}$
- $\sum_{\mathcal{I} \in \mathfrak{I}, \mathcal{V}^{\mathcal{I}} = \mathcal{W}} P_{\mathfrak{I}}(\mathcal{I}) = P_{\mathcal{B}}(\mathcal{W})$

Probabilistic Subsumption

$\mathcal{K} = (\mathcal{T}, \mathcal{B})$ a knowledge base, $\mathcal{P}$ a probabilistic interpretation

Probabilistic Subsumption

$\mathcal{K} = (\mathcal{T}, \mathcal{B})$ a knowledge base, $\mathcal{P}$ a probabilistic interpretation

Probability of a subsumption w.r.t. $\mathcal{P}$

$$P_{\mathcal{P}}(C \sqsubseteq D) = \sum_{\substack{\mathcal{I} \in \mathfrak{I} \\ \mathcal{I} \models C \sqsubseteq D}} P_{\mathfrak{I}}(\mathcal{I})$$

Probabilistic Subsumption

$\mathcal{K} = (\mathcal{T}, \mathcal{B})$ a knowledge base, $\mathcal{P}$ a probabilistic interpretation

Probability of a subsumption w.r.t. $\mathcal{P}$

$$P_{\mathcal{P}}(C \sqsubseteq D) = \sum_{\substack{\mathcal{I} \in \mathfrak{I} \\ \mathcal{I} \models C \sqsubseteq D}} P_{\mathfrak{I}}(\mathcal{I})$$

Probability of a subsumption w.r.t. $\mathcal{K}$

$$P_{\mathcal{K}}(C \sqsubseteq D) = \inf_{\mathcal{P} \models \mathcal{K}} P_{\mathcal{P}}(C \sqsubseteq D)$$

Probabilistic Subsumption

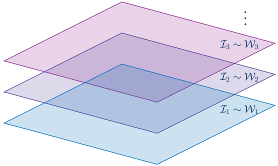
$\mathcal{K} = (\mathcal{T}, \mathcal{B})$ a knowledge base, $\mathcal{P}$ a probabilistic interpretation

Probability of a subsumption w.r.t. $\mathcal{P}$

$$P_{\mathcal{P}}(C \sqsubseteq D) = \sum_{\substack{\mathcal{I} \in \mathfrak{I} \\ \mathcal{I} \models C \sqsubseteq D}} P_{\mathfrak{I}}(\mathcal{I})$$

Probability of a subsumption w.r.t. $\mathcal{K}$

$$P_{\mathcal{K}}(C \sqsubseteq D) = \inf_{\mathcal{P} \models \mathcal{K}} P_{\mathcal{P}}(C \sqsubseteq D)$$



Probabilistic Subsumption

$\mathcal{K} = (\mathcal{T}, \mathcal{B})$ a knowledge base, $\mathcal{P}$ a probabilistic interpretation

Probability of a subsumption w.r.t. $\mathcal{P}$

$$P_{\mathcal{P}}(C \sqsubseteq D) = \sum_{\substack{\mathcal{I} \in \mathfrak{I} \\ \mathcal{I} \models C \sqsubseteq D}} P_{\mathfrak{I}}(\mathcal{I})$$

Probability of a subsumption w.r.t. $\mathcal{K}$

$$P_{\mathcal{K}}(C \sqsubseteq D) = \inf_{\mathcal{P} \models \mathcal{K}} P_{\mathcal{P}}(C \sqsubseteq D)$$

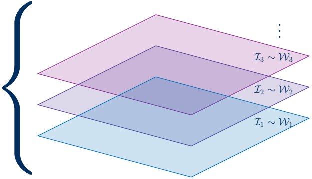
$P_{\mathcal{K}}(C \sqsubseteq D)$ can be computed by

$$\sum_{\substack{\mathcal{T}_{\mathcal{W}} \models C \sqsubseteq D \\ \mathcal{W}(\kappa)=1}} P_{\mathcal{B}}(\mathcal{W})$$

(IJCAR 2014)

and is a PP-complete problem

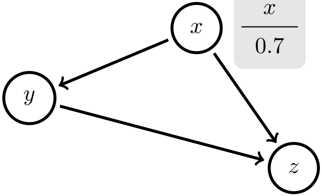
(JELIA 2014)



Example

Given a $\mathcal{BEL}$ KB is $\mathcal{K}_0 = (\mathcal{T}_0, \mathcal{B}_0)$ where $\mathcal{B}_0$ is as depicted:

	y
x	1
$\neg x$	0.5



x
0.7

		z
x	y	0.3
x	$\neg y$	0.1
$\neg x$	y	0
$\neg x$	$\neg y$	0.9

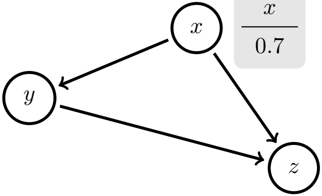
and the contextual TBox $\mathcal{T}_0$ given as:

$$\mathcal{T}_0 := \{\, \langle A \sqsubseteq C : \{x, y\}\rangle \,,\ \langle A \sqsubseteq B : \{\neg x\}\rangle \,,\ \langle B \sqsubseteq C : \{\neg x\}\rangle\}.$$

Example

Given a $\mathcal{BEL}$ KB is $\mathcal{K}_0 = (\mathcal{T}_0, \mathcal{B}_0)$ where $\mathcal{B}_0$ is as depicted:

	y
x	1
$\neg x$	0.5



x
0.7

		z
x	y	0.3
x	$\neg y$	0.1
$\neg x$	y	0
$\neg x$	$\neg y$	0.9

and the contextual TBox $\mathcal{T}_0$ given as:

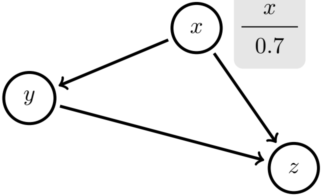
$$\mathcal{T}_0 := \{ \langle A \sqsubseteq C : \{x, y\}\rangle , \langle A \sqsubseteq B : \{\neg x\}\rangle , \langle B \sqsubseteq C : \{\neg x\}\rangle\}.$$

Let $A \sqsubseteq C$ be a subsumption query, then $P_{\mathcal{K}}(A \sqsubseteq C) = 1$

Example

Given a $\mathcal{BEL}$ KB is $\mathcal{K}_0 = (\mathcal{T}_0, \mathcal{B}_0)$ where $\mathcal{B}_0$ is as depicted:

	y
x	1
$\neg x$	0.5



x
0.7

		z
x	y	0.3
x	$\neg y$	0.1
$\neg x$	y	0
$\neg x$	$\neg y$	0.9

and the contextual TBox $\mathcal{T}_0$ given as:

$$\mathcal{T}_0 := \{ \langle A \sqsubseteq C : \{x, y\} \rangle , \langle A \sqsubseteq B : \{\neg x\} \rangle , \langle B \sqsubseteq C : \{\neg x\} \rangle \}.$$

Let $A \sqsubseteq C$ be a subsumption query, then $P_{\mathcal{K}}(A \sqsubseteq C) = 1$

This is a well connected TBox, but are real ontologies really well-connected?

The Bayesian Ontology Reasoner BORN

BORN first computes a module

Consider $\mathcal{BEL}$ KB is $\mathcal{K}_0 = (\mathcal{T}_1, \mathcal{B}_0)$ where:

$$\mathcal{T}_1 := \{ \langle A \sqsubseteq C : \{x, y\}\rangle, \langle A \sqsubseteq B : \{\neg x\}\rangle, \langle B \sqsubseteq C : \{\neg x\}\rangle \\ \langle D \sqsubseteq E : \{\neg y\}\rangle, \langle E \sqsubseteq \exists r.F : \{y\}\rangle, \langle F \sqsubseteq G : \{x, y\}\rangle, ...\}$$

The Bayesian Ontology Reasoner BORN

BORN first computes a module

Consider $\mathcal{BEL}$ KB is $\mathcal{K}_0 = (\mathcal{T}_1, \mathcal{B}_0)$ where:

$$\begin{aligned}\mathcal{T}_1 := \{ & \langle A \sqsubseteq C : \{x, y\}\rangle, \langle A \sqsubseteq B : \{\neg x\}\rangle, \langle B \sqsubseteq C : \{\neg x\}\rangle \\ & \langle D \sqsubseteq E : \{\neg y\}\rangle, \langle E \sqsubseteq \exists r.F : \{y\}\rangle, \langle F \sqsubseteq G : \{x, y\}\rangle, ...\}\end{aligned}$$

New axioms do not carry information about the query $A \sqsubseteq C$.

The idea is to compute a module w.r.t. the query and the TBox.

The Bayesian Ontology Reasoner BORN

BORN first computes a module

Consider $\mathcal{BEL}$ KB is $\mathcal{K}_0 = (\mathcal{T}_1, \mathcal{B}_0)$ where:

$$\begin{aligned}\mathcal{T}_1 := \{ &\langle A \sqsubseteq C : \{x, y\}\rangle, \langle A \sqsubseteq B : \{\neg x\}\rangle, \langle B \sqsubseteq C : \{\neg x\}\rangle \\ &\langle D \sqsubseteq E : \{\neg y\}\rangle, \langle E \sqsubseteq \exists r.F : \{y\}\rangle, \langle F \sqsubseteq G : \{x, y\}\rangle, ...\}\end{aligned}$$

New axioms do not carry information about the query $A \sqsubseteq C$.

The idea is to compute a module w.r.t. the query and the TBox.

Module of $\mathcal{T}_1$ w.r.t. the query $\{A, C\}$ yields:

$$\mathcal{T}_0 := \{ \langle A \sqsubseteq C : \{x, y\}\rangle, \langle A \sqsubseteq B : \{\neg x\}\rangle, \langle B \sqsubseteq C : \{\neg x\}\rangle$$

The Bayesian Ontology Reasoner BORN

BORN converts the knowledge base into a probabilistic logic program:

The Bayesian Ontology Reasoner BORN

BORN converts the knowledge base into a probabilistic logic program:

The subsumption $\langle \mathtt{A} \sqsubseteq \exists \mathtt{r}.\mathtt{B} : x \rangle$ is represented as:

```
con('a'). con('b'). role('r').
subs('a', exists('r', 'b')) :- x0.
```

The Bayesian Ontology Reasoner BORN

BORN converts the knowledge base into a probabilistic logic program:

The subsumption $\langle \mathtt{A} \sqsubseteq \exists \mathtt{r}.\mathtt{B} : x \rangle$ is represented as:

```
con('a'). con('b'). role('r').
subs('a', exists('r', 'b')) :- x0.
```

the Bayesian network $\mathcal{B}_0$ as:

```
.7::x.
 1::y:-x.
.5::y:-\+x.
.3::z:-x,y.
.1::z:-x,\+y.
.0::z:-\+x,y.
.9::z:-\+x,\+y.
```

The Bayesian Ontology Reasoner BORN

BORN converts the knowledge base into a probabilistic logic program:

The subsumption $\langle \mathtt{A} \sqsubseteq \exists \mathtt{r}.\mathtt{B} : x \rangle$ is represented as:

```
con('a'). con('b'). role('r').
subs('a', exists('r', 'b')) :- x0.
```

the Bayesian network $\mathcal{B}_0$ as:

```
.7::x.
 1::y:-x.
.5::y:-\+x.
.3::z:-x,y.
.1::z:-x,\+y.
.0::z:-\+x,y.
.9::z:-\+x,\+y.
```

and the query $\langle \mathtt{A} \sqsubseteq \mathtt{C} \rangle$ as:

```
query(subs('a', 'c')).
```

The Bayesian Ontology Reasoner BORN

BORN converts the knowledge base into a probabilistic logic program:

The subsumption $\langle \texttt{A} \sqsubseteq \exists \texttt{r}.\texttt{B} : x \rangle$ is represented as:

```
con('a'). con('b'). role('r').
subs('a', exists('r', 'b')) :- x0.
```

the Bayesian network $\mathcal{B}_0$ as:

```
.7::x.
 1::y:-x.
.5::y:-\+x.
.3::z:-x,y.
.1::z:-x,\+y.
.0::z:-\+x,y.
.9::z:-\+x,\+y.
```

and the query $\langle \texttt{A} \sqsubseteq \texttt{C} \rangle$ as:

```
query(subs('a', 'c')).
```

This is the syntax of ProbLog, a probabilistic logic programming tool based on efficient techniques such as weighted model counting, etc...

The Bayesian Ontology Reasoner BORN

$\mathcal{EL}$ completion rules added to the program:

The Bayesian Ontology Reasoner BORN

$\mathcal{EL}$ completion rules added to the program:

```
sub(X, B) :- subs(A, B), sub(X, A),
             con(X), con(A), con(B).

sub(X, B) :- subs(and(A1, A2), B), sub(X, A1), sub(X, A2),
             con(X), con(A1), con(A2), con(B).

sub(X, exists(R, B)) :- subs(A, exists(R, B)), sub(X, A),
                        con(X), con(A), con(B), role(R).

sub(X, B) :- subs(exists(R, A), B), sub(X, exists(R, Y)), sub(Y, A),
             con(X), con(Y), con(A), con(B), role(R).
```

The Bayesian Ontology Reasoner BORN

$\mathcal{EL}$ completion rules added to the program:

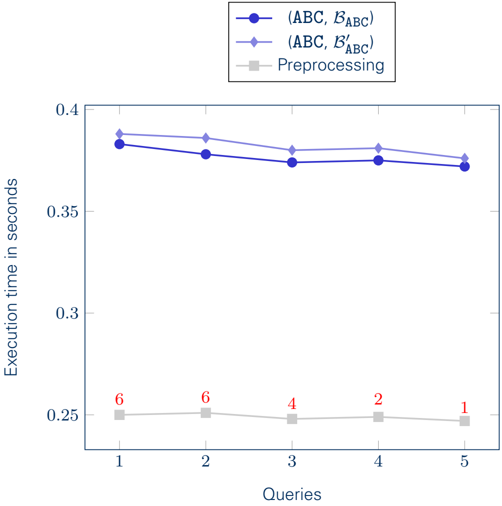
```
sub(X, B) :- subs(A, B), sub(X, A),
             con(X), con(A), con(B).

sub(X, B) :- subs(and(A1, A2), B), sub(X, A1), sub(X, A2),
             con(X), con(A1), con(A2), con(B).

sub(X, exists(R, B)) :- subs(A, exists(R, B)), sub(X, A),
                        con(X), con(A), con(B), role(R).

sub(X, B) :- subs(exists(R, A), B), sub(X, exists(R, Y)), sub(Y, A),
             con(X), con(Y), con(A), con(B), role(R).
```

Inference is performed by ProbLog.

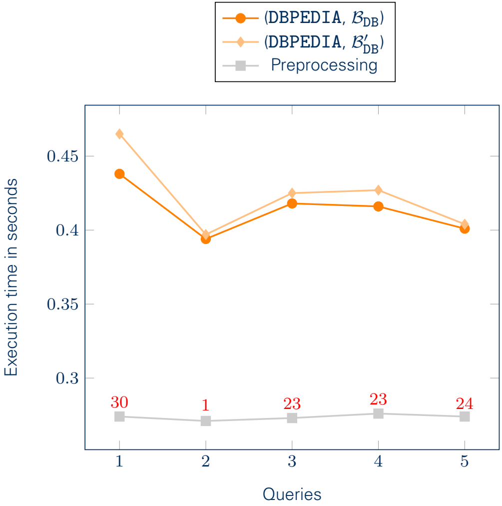
Initial Experiments on Ontologies

Ontology	Size of the terminology	Size of the BN
$(\mathtt{ABC}, \mathcal{B}_{\mathtt{ABC}})$ $(\mathtt{ABC}, \mathcal{B}'_{\mathtt{ABC}})$	6	8
$(\mathtt{DBPEDIA}, \mathcal{B}_{\mathtt{DBPEDIA}})$ $(\mathtt{DBPEDIA}, \mathcal{B}'_{\mathtt{DBPEDIA}})$	266	17
$(\mathtt{GO}, \mathcal{B}_{\mathtt{GO}})$ $(\mathtt{GO}, \mathcal{B}'_{\mathtt{GO}})$	23507	200

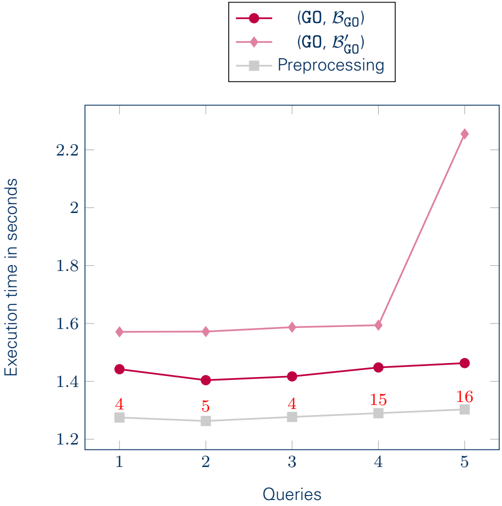
Primitive Experimental Results



Primitive Experimental Results



Primitive Experimental Results



Thanks