

Appendix A

Definition of DGLP Syntax

A.1 BNF Rules for DGLP Syntax

For the following BNF derivation rules, terminal symbols appear surrounded by single quotes and non-terminal symbols are shown in bold face; symbols that may appear zero or one time (optional) are enclosed in square brackets ($[...]$) and symbols that may appear zero or more times are enclosed in curly brackets ($\{...\}$); the vertical bar ($|$) indicates an alternative choice and an expression of the form $s_1..s_n$ denotes choice between the elements of the interval from s_1 to s_n inclusive. For Table A.1 we require that for each **DescriptionGraph** the nodes are provided sequentially w.r.t. the node identifier and that the node identifiers **NI** used in the definition of each **Edge** should occur in the definition of a **Node** of the same **DescriptionGraph**; similarly, for each **DescriptionGraphNegLabels** expression. Finally, for Table A.1 we require that $B \rightarrow H$ is an expression of the form (2.1).

UppercaseLetter ::= ‘A’ .. ‘Z’
LowercaseLetter ::= ‘a’ .. ‘z’
Letter ::= **UppercaseLetter** | **LowercaseLetter**
Digit ::= ‘0’ .. ‘9’
Alphanumeric ::= **Letter** | **Digit**
LowercaseString ::= **LowercaseLetter**{**Alphanumeric**}
NaturalNumber ::= **Digit**{**Digit**}
ClassName ::= **LowercaseString**
PropertyName ::= **LowercaseString**
NI ::= **NaturalNumber**
Literal ::= **ClassName** | ‘NOT’ **ClassName**
Ontology ::= **DGLP** {, **DGLP** }
DGLP ::= **Rule** | **Implies** | **ImpliedBy** | **ImpliesAndImpliedBy**
Rule ::= ‘NERule’ ‘(’ **B** ‘→’ **H** ‘)’
Implies ::= ‘begin’ **ClassName** ‘SubClassOf’
{ **ClassName** ‘AND’ } **Consequent**
‘end’
Consequent ::= **Property** ‘SOME’ **DescriptionGraph**
Property ::= **PropertyName** | ‘Inverse’ ‘(’ **PropertyName** ‘)’
DescriptionGraph ::= ‘Graph’ ‘(’ **NodeSet** **EdgeSet** ‘)’
NodeSet ::= ‘Nodes’ ‘(’ **Node** {, **Node** } ‘)’
Node ::= **NI** { **ClassName** }
EdgeSet ::= ‘Edges’ ‘(’ [**Edge** {, **Edge** }] ‘)’
Edge ::= **NI** **NI** **PropertyName** { **PropertyName** }

Table A.1: BNF rules for the definition of structured objects

```

ImpliedBy ::= ClassImpliedBy | PropertyImpliedBy
ClassImpliedBy ::= 'begin' ClassName 'SuperClassOf'
                   Antecedent { 'OR' Antecedent }
                   'end'
Antecedent ::= ClassExpression
                | [ ClassExpression 'AND' ] Predicate
ClassExpression ::= ClassName { 'AND' Literal }
Predicate ::= Existential | CardinalityRestriction | Universal
Existential ::= Property 'SOME' [ Predicative ]
Predicative ::= ClassExpression | DescriptionGraphNegLabels
DescriptionGraphNegLabels ::= 'GraphNL' '(' NodeSetNL EdgeSetNL ')'
NodeSetNL ::= NodesMode '(' NodeNL { , NodeNL } ')'
NodesMode ::= 'Nodes' | 'DisjointNodes'
NodeNL ::= NI { Literal }
EdgeSetNL ::= 'Edges' '(' [ EdgeNL { , EdgeNL } ] ')'
EdgeNL ::= NI NI PropertyName
            { PropertyName } { 'NOT' PropertyName }
PropertyImpliedBy ::= 'begin' PropertyName 'SuperPropertyOf'
                       PropertyExpression { 'OR' PropertyExpression }
                       'end'
PropertyExpression ::= PropertyName { 'AND' PropertyName }
                       { 'AND' 'NOT' PropertyName }
                       | PropertyName 'o' PropertyName
                       { 'o' PropertyName }
                       | 'Inverse' '(' PropertyName ')'

```

Table A.2: BNF rules for recognition of classes and properties

NonZeroDigit	::=	'1' .. '9'
NonZeroNaturalNumber	::=	NonZeroDigit { Digit }
N	::=	NonZeroNaturalNumber
ZN	::=	NaturalNumber
CardinalityRestriction	::=	Property 'AT LEAST' N [Predicative] ClassName 'AND' Property 'AT MOST' ZN [Predicative] Property 'EXACTLY' N [Predicative]
Universal	::=	ClassExpression 'AND' Property 'ONLY' Disjunction
Disjunction	::=	'(' Literal { 'OR' Literal } ')'
ImpliesAndImpliedBy	::=	'begin' ClassName 'EquivalentTo' Consequent { 'AND' ClassName } 'end'

Table A.3: BNF rules for cardinality constraints and equivalence axioms

A.2 Mappings of DGLP Axioms to Nonmonotonic Existential Rules

The mapping of the following translation rules is defined recursively, that is in certain cases the mapping of a construct relies on the mapping of its subconstruct(s), where the logical expression generated by the recursive invocation of the mapping is placed in the position of the recursive invocation. Additionally, the auxiliary operator HA is used to specify that the elements upon which HA operates shall be translated into head atoms; the operator BA is analogous for body atoms.

Element E of the BNF grammar	Logical expression $\text{NER}(E)$
$\mathbf{DGLP}_1, \dots, \mathbf{DGLP}_n$	$\{\text{NER}(\mathbf{DGLP}_i)\}_{i=1}^n$
begin ClassName SubClassOf ClassName₁ AND ... AND ClassName_n AND Consequent end	ClassName (x) $\rightarrow \bigwedge_{i=1}^n \mathbf{ClassName}_i(x) \wedge$ HA (Consequent)
Element E of the BNF grammar	Logical expression $\text{HA}(E)$
PropertyName SOME Graph (Nodes (Node₁ , ... , Node_n) EdgeSet)	$\exists_{i=1}^n y_i \cdot \bigwedge_{i=1}^n \mathbf{PropertyName}(x, y_i)$ $\wedge \bigwedge_{i=1}^n \text{HA}(\mathbf{Node}_i)$ $\wedge \text{HA}(\mathbf{EdgeSet})$
Inverse (PropertyName) SOME Graph (Nodes (Node₁ , ... , Node_n) EdgeSet)	$\exists_{i=1}^n y_i \cdot \bigwedge_{i=1}^n \mathbf{PropertyName}(y_i, x)$ $\wedge \bigwedge_{i=1}^n \text{HA}(\mathbf{Node}_i)$ $\wedge \text{HA}(\mathbf{EdgeSet})$
(NI ClassName₁ ... ClassName_n)	$\bigwedge_{i=1}^n \mathbf{ClassName}_i(y_{\text{NI}})$
Edges (Edge₁ , ... , Edge_n)	$\bigwedge_{i=1}^n \text{HA}(\mathbf{Edge}_i)$
(NI1 NI2 PropertyName₁ ... PropertyName_n)	$\bigwedge_{i=1}^n \mathbf{PropertyName}_i(y_{\text{NI1}}, y_{\text{NI2}})$

Table A.4: Mappings to NER for **Implies** expressions; we have $n \geq 1$ where applicable

Element E of the BNF grammar	Logical expression $\text{NER}(E)$
begin ClassName SuperClassOf Antecedent₁ OR ... OR Antecedent_n end	$\text{BA}(\text{Antecedent}_i)$ $\rightarrow \text{ClassName}(x)$ $i = 1, \dots, n$
NERule ($B \rightarrow H$)	$B \rightarrow H$
Element E of the BNF grammar	Logical expression $\text{BA}(E)$
ClassName₁ AND ... AND ClassName_m AND NOT ClassName_{m+1} ... AND NOT ClassName_n	$\bigwedge_{i=1}^m \text{ClassName}_i(x) \wedge$ $\bigwedge_{i=m+1}^n \text{not } \text{ClassName}_i(x)$
ClassExpression AND Predicate	$\text{BA}(\text{ClassExpression}) \wedge$ $\text{BA}(\text{Predicate})$
PropertyName SOME ClassName₁ AND ... AND ClassName_k AND NOT ClassName_{k+1} ... AND NOT ClassName_l	PropertyName $(x, z) \wedge$ $\bigwedge_{i=1}^k \text{ClassName}_i(z) \wedge$ $\bigwedge_{i=k+1}^l \text{not } \text{ClassName}_i(z)$
Inverse(PropertyName) SOME ClassName₁ AND ... AND ClassName_k AND NOT ClassName_{k+1} ... AND NOT ClassName_l	PropertyName $(z, x) \wedge$ $\bigwedge_{i=1}^k \text{ClassName}_i(z) \wedge$ $\bigwedge_{i=k+1}^l \text{not } \text{ClassName}_i(z)$

Table A.5: Mappings to NER for **ImpliedBy** expressions; we have $n \geq 1$, $n \geq m \geq 1$ and $l \geq k \geq 0$ where applicable

Element E of the BNF grammar	Logical expression $BA(E)$
PropertyName SOME GraphNL (Nodes (NodeNL ₁ , . . . , NodeNL _{n}) EdgeSetNL)	$\bigwedge_{i=1}^n \mathbf{PropertyName}(x, z_i) \wedge \bigwedge_{i=1}^n BA(\mathbf{NodeNL}_i) \wedge BA(\mathbf{EdgeSet})$
Inverse(PropertyName) SOME GraphNL (Nodes (NodeNL ₁ , . . . , NodeNL _{n}) EdgeSetNL)	$\bigwedge_{i=1}^n \mathbf{PropertyName}(z_i, x) \wedge \bigwedge_{i=1}^n BA(\mathbf{NodeNL}_i) \wedge BA(\mathbf{EdgeSetNL})$
PropertyName SOME GraphNL (DisjointNodes (NodeNL ₁ , . . . , NodeNL _{n}) EdgeSetNL)	$\bigwedge_{1 \leq i < j \leq n} z_i \neq z_j \wedge \bigwedge_{i=1}^n \mathbf{PropertyName}(x, z_i) \wedge \bigwedge_{i=1}^n BA(\mathbf{NodeNL}_i) \wedge BA(\mathbf{EdgeSetNL})$
Inverse(PropertyName) SOME GraphNL (DisjointNodes (NodeNL ₁ , . . . , NodeNL _{n}) EdgeSetNL)	$\bigwedge_{1 \leq i < j \leq n} z_i \neq z_j \wedge \bigwedge_{i=1}^n \mathbf{PropertyName}(z_i, x) \wedge \bigwedge_{i=1}^n BA(\mathbf{NodeNL}_i) \wedge BA(\mathbf{EdgeSetNL})$
(NI ClassName ₁ . . . ClassName _{m} NOT ClassName _{$m+1$} . . . NOT ClassName _{n})	$\bigwedge_{i=1}^m \mathbf{ClassName}_i(z_{NI}) \wedge \bigwedge_{i=m+1}^n \text{not } \mathbf{ClassName}_i(z_{NI})$
Edges (EdgeNL ₁ , . . . , EdgeNL _{n})	$\bigwedge_{i=1}^n BA(\mathbf{EdgeNL}_i)$
(NI1 NI2 PropertyName ₁ . . . PropertyName _{m} NOT PropertyName _{$m+1$} . . . NOT PropertyName _{n})	$\bigwedge_{i=1}^m \mathbf{PropertyName}_i(z_{NI1}, z_{NI2}) \wedge \bigwedge_{i=m+1}^n \text{not } \mathbf{PropertyName}_i(z_{NI1}, z_{NI2})$

Table A.6: Mappings to NER for **DescriptionGraphNegLabels** expressions; we have $n \geq 1$ and $n \geq m \geq 1$ where applicable

Element E of the BNF grammar	Logical expression $\text{NER}(E)$
begin PropertyName SuperPropertyOf PropertyExpression ₁ OR ... OR PropertyExpression _{n} end	$\text{BA}(\text{PropertyExpression}_i)$ $\rightarrow \text{PropertyName}(x_0, x_n)$ $i = 1, \dots, n$
Element E of the BNF grammar	Logical expression $\text{BA}(E)$
PropertyName ₁ AND ... AND PropertyName _{m} AND NOT PropertyName _{$m+1$} ... AND NOT PropertyName _{n}	$\bigwedge_{i=1}^m \text{PropertyName}_i(x_0, x_n) \wedge$ $\bigwedge_{i=m+1}^n \text{not } \text{PropertyName}_i(x_0, x_n)$
PropertyName ₁ \circ ... \circ PropertyName _{n}	$\bigwedge_{i=1}^n \text{PropertyName}_i(x_{i-1}, x_i)$
‘Inverse’ ‘(’ PropertyName ‘)’	PropertyName (x_n, x_0)

Table A.7: Mappings to NER for **PropertyImpliedBy** expressions; we have $n \geq 1$ and $n \geq m \geq 1$ where applicable

Element E of the BNF grammar	Logical expression $\text{NER}(E)$
ClassExpression AND PropertyName ONLY (ClassName ₁ OR ... ClassName _{m} OR NOT ClassName _{$m+1$} OR ... NOT ClassName _{n})	$\text{BA}(\text{ClassExpression}) \wedge$ $\text{not } \text{AuxClassName}(x)$ and also add the following rule PropertyName (x, z) \wedge $\text{not } \text{ClassName}_1(z) \wedge \dots \wedge$ $\text{not } \text{ClassName}_m(z) \wedge$ $\text{ClassName}_{m+1}(z) \wedge \dots \wedge$ $\text{ClassName}_n(z)$ $\rightarrow \text{AuxClassName}(x)$

Table A.8: Mappings to NER for **Universal** expressions; we have $m \geq 0$ and $n \geq 0$ where applicable

Element E of the BNF grammar	Logical expression $BA(E)$
Property AT LEAST N CN ₁ AND ... CN _{k} AND NOT CN _{$k+1$} AND ... NOT CN _{ℓ}	$BA (\text{Property } \text{SOME } \text{GraphNL} ($ $\text{DisjointNodes} (1 \text{CN}_1 \dots \text{CN}_k$ $\text{NOT } \text{CN}_{k+1} \dots \text{NOT } \text{CN}_\ell ,$... , $N \text{CN}_1 \dots \text{CN}_k$ $\text{NOT } \text{CN}_{k+1} \dots \text{NOT } \text{CN}_\ell)$ $\text{Edges} ()))$
Property AT LEAST N GraphNL (NodesMode (NI_1 NodeLabel ₁ , ... , NI_m NodeLabel _{m}) Edges (S_1 D_1 EdgeLabel ₁ , ... , S_ℓ D_ℓ EdgeLabel _{ℓ}))	$BA (\text{Property } \text{SOME } \text{GraphNL} ($ $\text{DisjointNodes} (NI_1^1 \text{NodeLabel}_1 , \dots ,$ $NI_m^1 \text{NodeLabel}_m , \dots ,$ $NI_m^N \text{NodeLabel}_m)$ $\text{Edges} (S_1^1 D_1^1 \text{EdgeLabel}_1 , \dots ,$ $S_\ell^1 D_\ell^1 \text{EdgeLabel}_\ell , \dots ,$ $S_\ell^N D_\ell^N \text{EdgeLabel}_\ell)))$
ClassName AND Property AT MOST ZN Predicative	$BA (\text{ClassName } \text{AND } \text{NOT } \text{AuxClassName})$ and also add the following rule $\text{NER} (\text{begin } \text{AuxClassName } \text{SuperClassOf}$ $\text{Property } \text{AT LEAST } ZN + 1 \text{ Predicative}$ $\text{end})$
Property EXACTLY N Predicative	$BA (\text{Property } \text{AT LEAST } N \text{ Predicative}$ $\text{AND } \text{NOT } \text{AuxClassName})$ and also add the following rule $\text{NER} (\text{begin } \text{AuxClassName } \text{SuperClassOf}$ $\text{Property } \text{AT LEAST } ZN + 1 \text{ Predicative}$ $\text{end})$

Table A.9: Mappings to NER for **CardinalityRestriction** expressions; we have $k \geq 0$, $m \geq 1$ and $\ell \geq 0$ where applicable, **AuxClassName** is a fresh **ClassName** and $ZN + 1$ is ZN increased by 1

Element E_{def} of the BNF grammar
<pre> begin ClassName EquivalentTo PropertyName SOME Graph (Nodes (N_{1_1} NodeLabel₁¹ ... NodeLabel_{k_1}¹ , ..., N_{1_ℓ} NodeLabel₁^{ℓ} ... NodeLabel_{k_ℓ}^{ℓ}) Edges (S_1 D_1 EdgeLabel₁¹ ... EdgeLabel_{m_1}¹ , ..., S_n D_n EdgeLabel₁^{n} ... EdgeLabel_{m_n}^{n})) AND ClassName₁ AND ... AND ClassName_{h} end </pre>

Table A.10: **ImpliesAndImpliedBy** expression; we have $\ell \geq 1$, $n \geq 0$, $k_i \geq 0$ for $1 \leq i \leq \ell$, $m_i \geq 1$ for $1 \leq i \leq n$ and $h \geq 0$

Logical expression $\text{NER}(E_{\text{def}})$
<pre> ClassName (x) \wedge not RecClassName (x) $\rightarrow \exists_{i=1}^{\ell} y_i \cdot \bigwedge_{i=1}^{\ell} \mathbf{PropertyName} (x, y_i) \wedge \bigwedge_{i=1}^{\ell} \left[\bigwedge_{j=1}^{k_i} \mathbf{NodeLabel}_j^i (y_i) \right] \wedge$ $\bigwedge_{i=1}^n \left[\bigwedge_{j=1}^{m_i} \mathbf{EdgeLabel}_j^i (y_{S_i}, y_{D_i}) \right] \wedge \bigwedge_{i=1}^h \mathbf{ClassName}_i (x) \wedge$ $\bigwedge_{i=1}^{\ell} \mathbf{NewClassName} (y_i)$ $\bigwedge_{i=1}^{\ell} \mathbf{PropertyName} (x, z_i) \wedge \bigwedge_{i=1}^{\ell} \left[\bigwedge_{j=1}^{k_i} \mathbf{NodeLabel}_j^i (z_i) \right] \wedge$ $\bigwedge_{i=1}^n \left[\bigwedge_{j=1}^{m_i} \mathbf{EdgeLabel}_j^i (z_{S_i}, z_{D_i}) \right] \wedge \bigwedge_{i=1}^h \mathbf{ClassName}_i (x) \wedge$ $\bigwedge_{i=1}^{\ell} \mathbf{not} \mathbf{NewClassName} (z_i)$ $\rightarrow \mathbf{ClassName} (x) \wedge \mathbf{RecClassName} (x)$ </pre>

Table A.11: Mappings to NER for **ImpliesAndImpliedBy** expressions; we have $\ell \geq 1$, $n \geq 0$, $k_i \geq 0$ for $1 \leq i \leq \ell$, $m_i \geq 1$ for $1 \leq i \leq n$ and $h \geq 0$; **RecClassName** and **NewClassName** are fresh unary predicates unique for each **ClassName**

Element $E_{\text{def-inv}}$ of the BNF grammar
<pre> begin ClassName EquivalentTo Inverse (PropertyName) SOME Graph (Nodes (N_{i_1} NodeLabel₁¹ ... NodeLabel_{k_1}¹ , ... , N_{i_ℓ} NodeLabel₁^{ℓ} ... NodeLabel_{k_ℓ}^{ℓ}) Edges (S_1 D_1 EdgeLabel₁¹ ... EdgeLabel_{m_1}¹ , ... , S_n D_n EdgeLabel₁^{n} ... EdgeLabel_{m_n}^{n})) AND ClassName₁ AND ... AND ClassName_{n} end </pre>

Table A.12: **ImpliesAndImpliedBy** expression with inverse property; we have $\ell \geq 1$, $n \geq 0$, $k_i \geq 0$ for $1 \leq i \leq \ell$, $m_i \geq 1$ for $1 \leq i \leq n$ and $h \geq 0$

Logical expression $\text{NER}(E_{\text{def-inv}})$
<pre> ClassName (x) \wedge not RecClassName (x) $\rightarrow \exists_{i=1}^\ell y_i \cdot \bigwedge_{i=1}^\ell \text{PropertyName} (y_i, x) \wedge \bigwedge_{i=1}^\ell \left[\bigwedge_{j=1}^{k_i} \text{NodeLabel}_j^i (y_i) \right] \wedge$ $\bigwedge_{i=1}^n \left[\bigwedge_{j=1}^{m_i} \text{EdgeLabel}_j^i (y_{S_i}, y_{D_i}) \right] \wedge \bigwedge_{i=1}^h \text{ClassName}_i (x) \wedge$ $\bigwedge_{i=1}^\ell \text{NewClassName} (y_i)$ $\bigwedge_{i=1}^\ell \text{PropertyName} (z_i, x) \wedge \bigwedge_{i=1}^\ell \left[\bigwedge_{j=1}^{k_i} \text{NodeLabel}_j^i (z_i) \right] \wedge$ $\bigwedge_{i=1}^n \left[\bigwedge_{j=1}^{m_i} \text{EdgeLabel}_j^i (z_{S_i}, z_{D_i}) \right] \wedge \bigwedge_{i=1}^h \text{ClassName}_i (x) \wedge$ $\bigwedge_{i=1}^\ell \text{not } \text{NewClassName} (z_i)$ $\rightarrow \text{ClassName} (x) \wedge \text{RecClassName} (x)$ </pre>

Table A.13: Mappings to NER for **ImpliesAndImpliedBy** expressions with inverse property; we have $\ell \geq 1$, $n \geq 0$, $k_i \geq 0$ for $1 \leq i \leq \ell$, $m_i \geq 1$ for $1 \leq i \leq n$ and $h \geq 0$; where **RecClassName** and **NewClassName** are fresh unary predicates unique for each **ClassName**

