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# Generic Programming

Advanced Lectures



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# Preface

Generic programming is about making programming more effective by making it more general. This volume is about a novel form of genericity in programs, based on parameterizing programs by the structure of the data they manipulate. The material is based on lectures presented at a summer school on Generic Programming held at the University of Oxford in August 2002.

The lectures by Hinze and Jeuring introduced Generic Haskell, an extension of the Haskell programming language that allows the programmer to define a function by induction on the structure of types. The implementation of Generic Haskell provided a valuable tool for students to experiment with applications of this form of datatype genericity. The lecture material in this volume is divided into two parts. The first part (“practice and theory”) introduces Generic Haskell and the theory that underlies its design. The second part (“applications”) discusses three advanced applications of Generic Haskell in some depth.

The value of generic programming is illusory unless the nature and extent of the genericity can be described clearly and precisely. The lectures by Backhouse and Crole delve deeper into the theoretical basis for datatype genericity. Backhouse reviews the notion of parametric polymorphism (a notion well known to functional programmers) and then shows how this notion is extended to higher-order notions of parametricity. These are used to characterize what it means for a value to be stored in a datatype. Also, transformations on data structures are given precise specifications in this way. Underlying this account are certain basic notions of category theory and allegory theory. Crole presents the category theory needed for a deeper understanding of mechanisms for defining datatypes.

The final chapter, by Fiadeiro, Lopes and Wermelinger applies the mathematical “technology” of parameterization to the larger-scale architectural structure of programs. The description of a system is split into components and their interactions; architectural connectors are parameterized by components, leading to an overall system structure consisting of components and connector instances establishing the interactions between the components.

Our thanks go to all those involved in making the school a success. We are grateful to the technical support staff of the Oxford University Computing Laboratory for providing computing facilities, to Yorck Hunke, David Lacey and Silvija Seres of OUCL for assistance during the school, and to St. Anne’s College for an amenable environment for study. Thanks also go to Peter Buneman and Martin Odersky, who lectured at the school on *Semi-structured Data* and on *Object-Oriented and Functional Approaches to Compositional Programming*, respectively, but were unable to contribute to the proceedings.

June, 2003

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# Table of Contents

Chapter 1. Generic Haskell: Practice and Theory .....	1
<i>R. Hinze and J. Jeuring</i>	
1 Introduction .....	1
1.1 Type Systems .....	2
1.2 Haskell’s <b>data</b> Construct .....	3
1.3 Towards Generic Programming .....	7
1.4 Towards Generic Haskell .....	12
1.5 Stocktaking .....	17
1.6 Related Work .....	18
2 Generic Haskell—Practice .....	22
2.1 Mapping Functions .....	22
2.2 Kind-Indexed Types and Type-Indexed Values .....	25
2.3 Embedding-Projection Maps .....	26
2.4 Reductions .....	27
2.5 Pretty Printing .....	30
2.6 Running Generic Haskell .....	33
3 Generic Haskell—Theory .....	35
3.1 The Simply Typed Lambda Calculus as a Type Language .....	35
3.2 The Polymorphic Lambda Calculus .....	40
3.3 Specializing Type-Indexed Values .....	42
3.4 Bridging the Gap .....	45
4 Conclusion .....	52
Chapter 2. Generic Haskell: Applications .....	57
<i>R. Hinze and J. Jeuring</i>	
1 Introduction .....	57
2 Generic Dictionaries .....	62
2.1 Introduction .....	63
2.2 Signature .....	65
2.3 Properties .....	66
2.4 Type-Indexed Tries .....	67
2.5 Empty Tries .....	70
2.6 Singleton Tries .....	71
2.7 Look up .....	72
2.8 Inserting and Merging .....	73
2.9 Deleting .....	75
2.10 Related Work .....	76
3 XComprez: A Generic XML Compressor .....	77
3.1 Implementing an XML Compressor as a Generic Program .....	78
3.2 Analysis .....	83
3.3 Conclusions .....	84

## VIII Table of Contents

4	The Zipper . . . . .	85
4.1	The Basic Idea . . . . .	85
4.2	Data Types as Fixed Points of Pattern Functors . . . . .	86
4.3	Type Indices of Higher Kinds . . . . .	86
4.4	Locations . . . . .	87
4.5	Navigation Functions . . . . .	88
5	Conclusions . . . . .	93
Chapter 3. Generic Properties of Datatypes . . . . .		97
<i>R. Backhouse and P. Hoogendoijk</i>		
1	Introduction . . . . .	97
2	Theorems for Free . . . . .	98
2.1	Verifiable Genericity . . . . .	102
3	Commuting Datatypes — Introduction . . . . .	102
3.1	Structure Multiplication . . . . .	104
3.2	Broadcasts . . . . .	106
4	Allegories and Relators . . . . .	109
4.1	Allegories . . . . .	109
4.2	Relators . . . . .	110
4.3	Composition and Relators Are Parametric . . . . .	112
4.4	Division and Tabulation . . . . .	113
4.5	Domains . . . . .	114
5	Datatype = Relator + Membership . . . . .	114
5.1	Pointwise Closure . . . . .	114
5.2	Regular Relators . . . . .	116
5.3	Natural Transformations . . . . .	119
5.4	Membership and Fans . . . . .	121
6	Commuting Datatypes — Formal Specification . . . . .	125
6.1	Naturality Requirements . . . . .	126
6.2	Composition . . . . .	128
6.3	Half Zips and Commuting Relators . . . . .	128
7	Consequences . . . . .	129
7.1	Shape Preservation . . . . .	129
7.2	All Regular Datatypes Commute . . . . .	130
8	Conclusion . . . . .	131
Chapter 4. Basic Category Theory for Models of Syntax . . . . .		133
<i>R.L. Crole</i>		
1	Introduction . . . . .	133
1.1	Prerequisites . . . . .	133
1.2	The Aims . . . . .	134
1.3	Learning Outcomes . . . . .	134
2	Syntax Defined from Datatypes . . . . .	134
2.1	An Example with Distinguished Variables and without Binding . . . . .	135
2.2	An Example with Distinguished Variables and Binding . . . . .	136
2.3	An Example with Arbitrary Variables and Binding . . . . .	137
2.4	An Example without Variables but with Binding . . . . .	138

3	Category Theory . . . . .	139
3.1	Categories . . . . .	139
3.2	Functors . . . . .	143
3.3	Natural Transformations . . . . .	144
3.4	Products . . . . .	146
3.5	Coproducts . . . . .	150
3.6	Algebras . . . . .	151
3.7	The Functor $1 + (-): \mathcal{S}et \rightarrow \mathcal{S}et$ . . . . .	152
3.8	The Functor $A + (-): \mathcal{S}et \rightarrow \mathcal{S}et$ . . . . .	153
3.9	The Functor $1 + (A \times -): \mathcal{S}et \rightarrow \mathcal{S}et$ . . . . .	154
4	Models of Syntax . . . . .	154
4.1	A Model of Syntax with Distinguished Variables and without Binding . . . . .	157
4.2	A Model of Syntax with Distinguished Variables and with Binding . . . . .	162
4.3	A Model of Syntax with Arbitrary Variables and Binding . . . . .	165
4.4	A Model of Syntax without Variables but with Binding . . . . .	168
4.5	Where to Now? . . . . .	168
5	Appendix . . . . .	170
5.1	Lists . . . . .	170
5.2	Abstract Syntax Trees . . . . .	170
5.3	Inductively Defined Sets . . . . .	170
5.4	Rule Induction . . . . .	173
5.5	Recursively Defined Functions . . . . .	175
Chapter 5. A Mathematical Semantics for Architectural Connectors . . . . .		178
<i>J.L. Fiadeiro, A. Lopes and M. Wermelinger</i>		
1	Introduction . . . . .	178
2	System Configuration in CommUnity . . . . .	180
2.1	Component Design . . . . .	180
2.2	Configurations . . . . .	184
3	Architectural Description in CommUnity . . . . .	191
3.1	Architectural Connectors . . . . .	191
3.2	Examples . . . . .	196
4	An ADL-Independent Notion of Connector . . . . .	201
4.1	Architectural Schools . . . . .	201
4.2	Adding Abstraction to Architectural Connectors . . . . .	207
5	Towards an Algebra of Connectors . . . . .	213
5.1	Role Refinement . . . . .	214
5.2	Role Encapsulation . . . . .	216
5.3	Role Overlay . . . . .	218
6	Concluding Remarks . . . . .	219
Author Index . . . . .		223