Algebraic (and Diagrammatic) Structures in Quantum Theory

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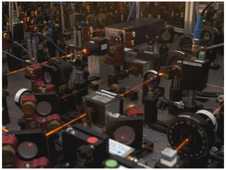
Quantum Software in NL 2017

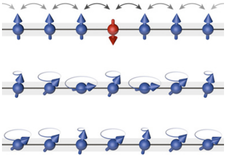
Software = components + composition

- Effective **software** comes from understanding relevant *components* and how they *compose*
- Effective **quantum software** will be exactly the same
- But now the "components" could mean:

devices



physical processes



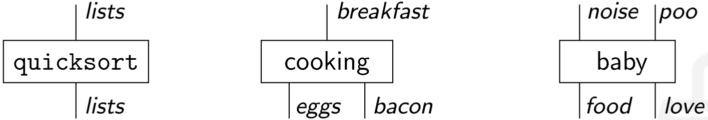
mathematical processes

$\rho \mapsto \sum_i B_i \rho B_i^\dagger$

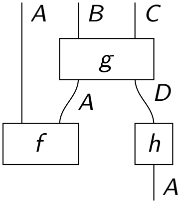
- ...each with its own (related) notion of composition

Process theories

- The moral: take *processes* as primitive
- A process is just a box with inputs and outputs:

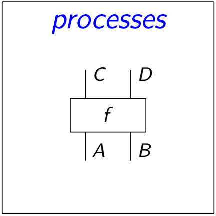


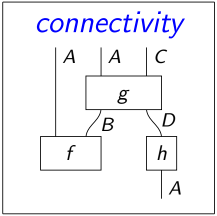
- Composition means forming *diagrams* of processes:

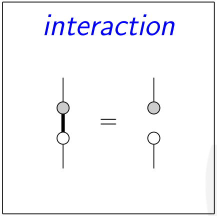


- A collection of processes that make sense to ‘plug together’ is called a *process theory*

The idea: Describe quantum theory entirely in terms of:



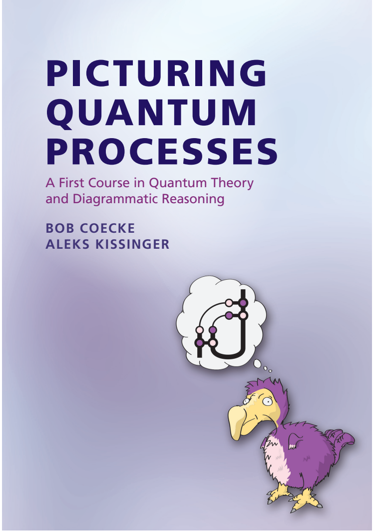




Not in terms of:

- Hilbert space
- self-adjoint operators, unitary transformations
- calculations with matrices/complex numbers
- ....

(though some may be emergent notions)



Cambridge University Press

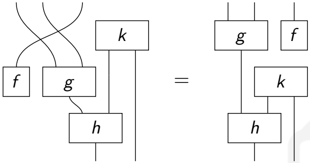
March 2017 (pre-order)

Why?

- Simpler!

$$(1 \otimes \sigma \otimes k) \circ (\sigma \otimes 1 \otimes 1 \otimes 1) \circ$$
$$(f \otimes g \otimes 1 \otimes 1) \circ (h \otimes 1) =$$
$$(g \otimes f) \circ (1 \otimes k) \circ (h \otimes 1)$$

vs.



- New perspective $=$ new insights
- Deriving QT from first principles $\Leftarrow$ 'diagrammatic backbone' $+$ a bit of information theory

  e.g. **Pavia 2010** and **Hardy 2011**

  Hardy (2010): "we join the quantum picturalism revolution"
- A 'theory playground'

  e.g. QT vs. real/boolean-valued/modal QT,
  local QT (indefinite causal structure),
  Spekken's toy theory, ...
- New calculational tools, applications in quantum info/computation

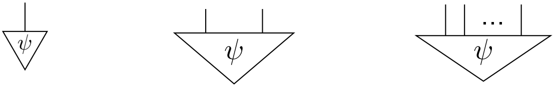
Just diagrams

Q: What kinds of behaviour can we study using just diagrams, and nothing else?

A: (Non-)separability

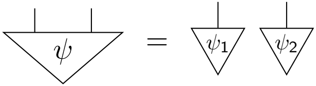
Separability for states

- States are processes with no input:



Interpret as: 'preparing a system (or some systems)'

- Separable states:

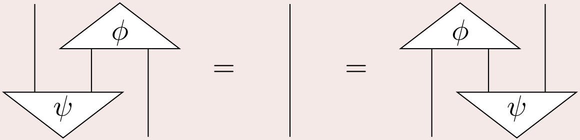


- ...are boring!
- Non-separable := 'no such ψ_1, ψ_2 exist', but this isn't very helpful

'Non-separable' isn't very helpful, but 'really non-separable' is:

Definition

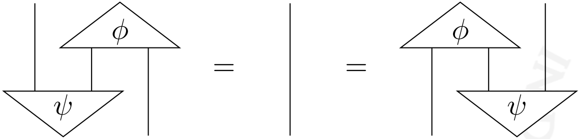
A process ψ is called *cup-state* if there exists a process ϕ, called a *cap-effect*, such that:



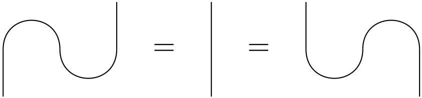
ψ looks like a state, but it *acts* like a wire.

- By introducing some clever notation:

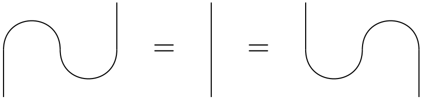
- Then these equations:



- ...look like this:

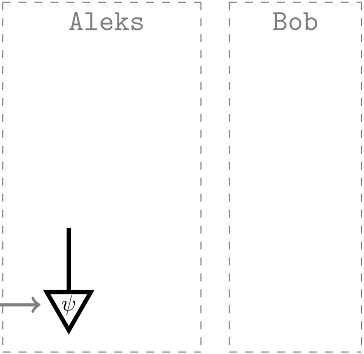


Yank the wire!

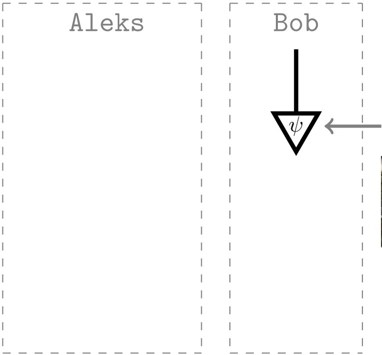


Quantum teleportation

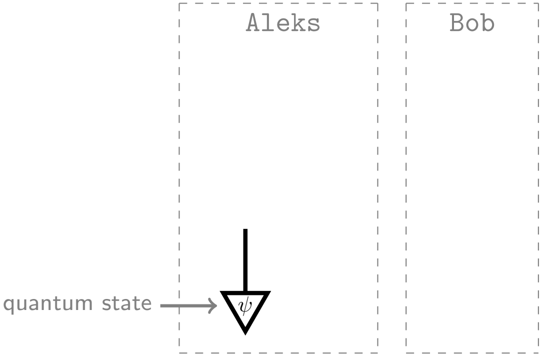
quantum state



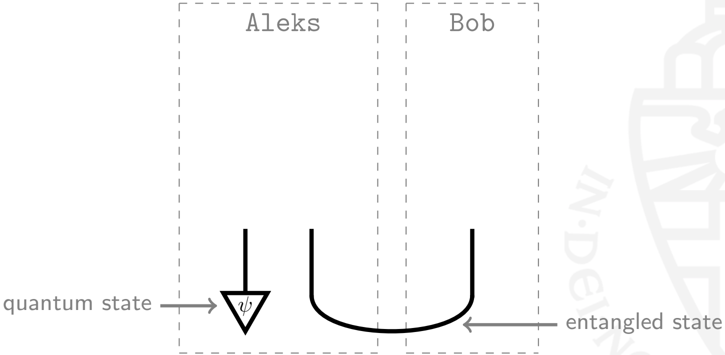
Quantum teleportation



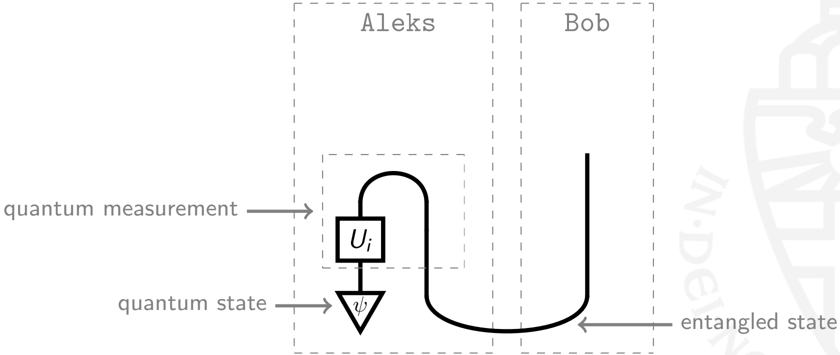
Quantum teleportation



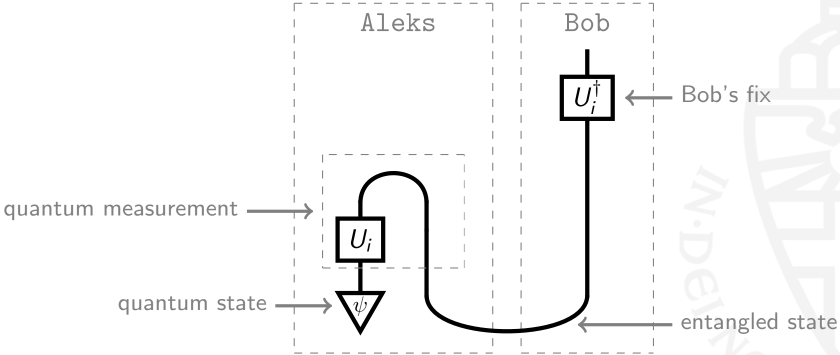
Quantum teleportation



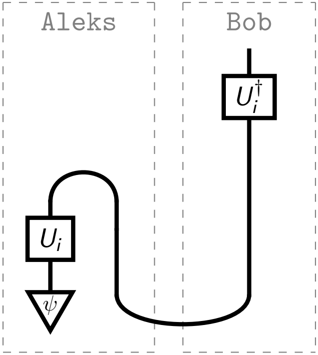
Quantum teleportation



Quantum teleportation



...and it works



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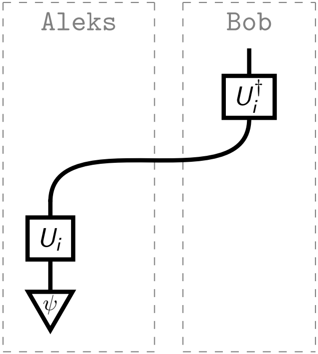

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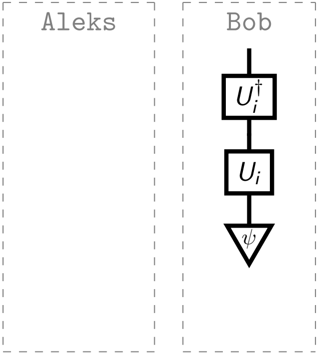

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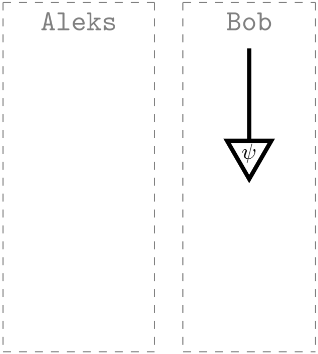
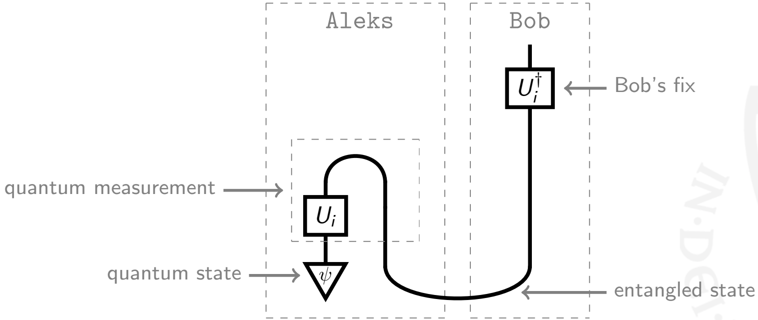


If we change the process theory...



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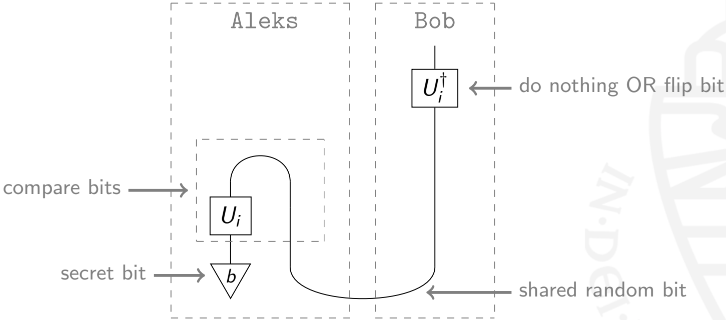

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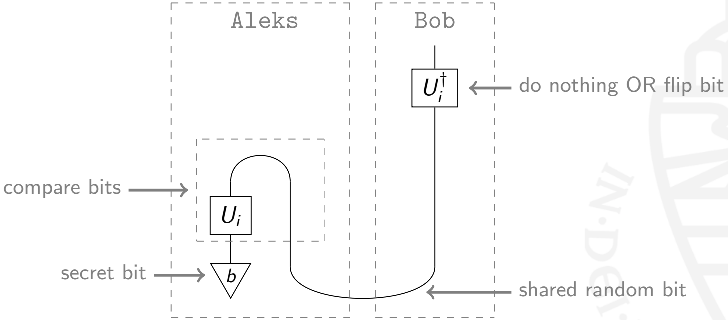
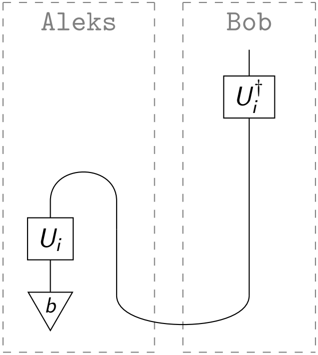
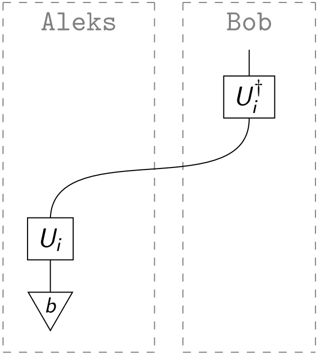


...'classical teleportation' is one-time-pad crypto

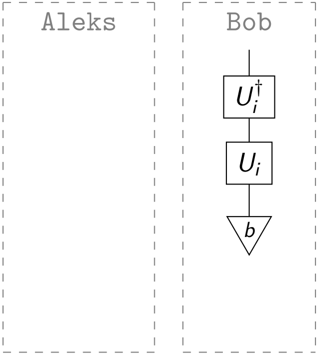
...and it works



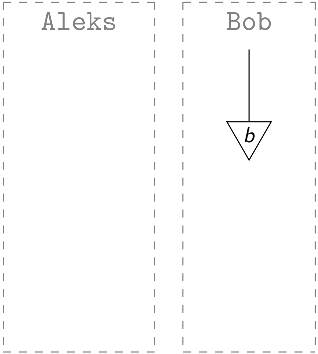
...and it works



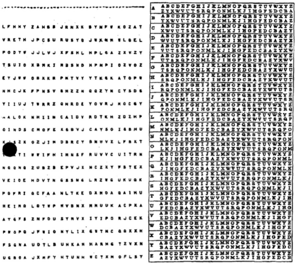
...and it works



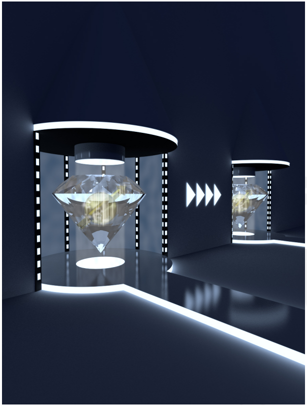
...and it works



One and the same

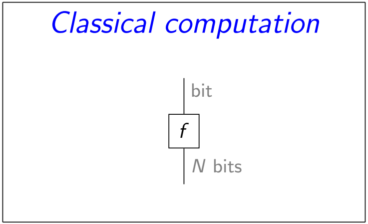


OTP cryptography
1882

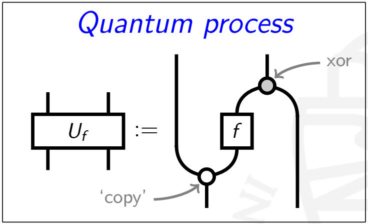


Quantum teleportation
111 years later
Image: Hanson lab@TUDelft

Quantum algorithms

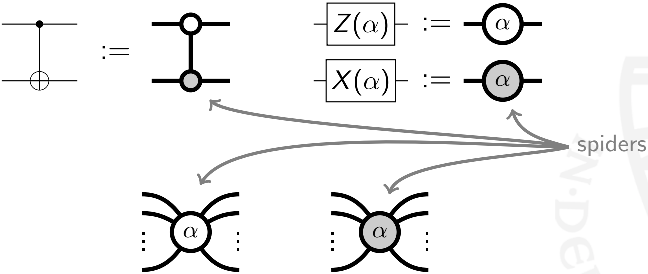


$\Rightarrow$

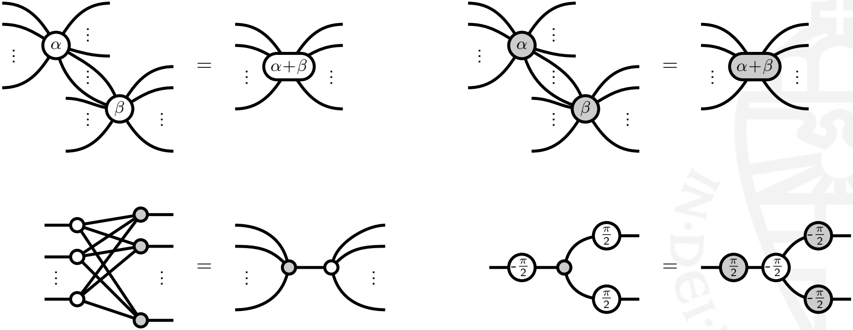


$\Rightarrow$ simple (diagrammatic) derivations of **Deutsch-Jozsa**, **Bernstein-Vazirani**, **quantum seach**, and **hidden subgroup** algorithms.

Quantum circuits + algebraic structure = **ZX-calculus**

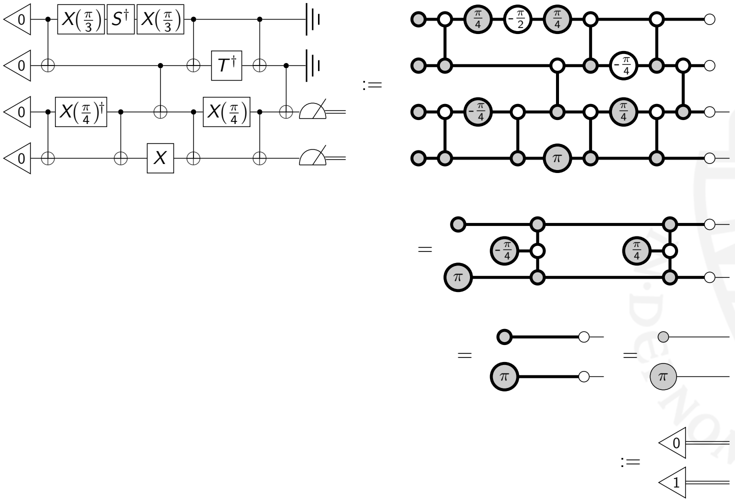


ZX-calculus has 4 equations:

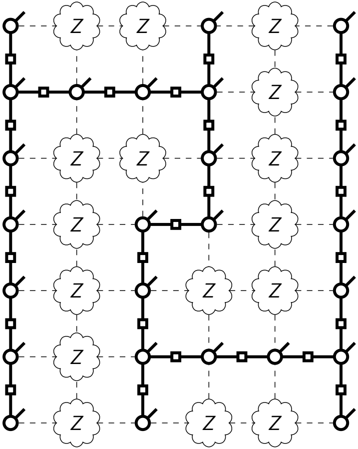


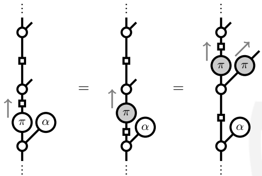
...which can prove any equality between Clifford circuits (and a bit more).

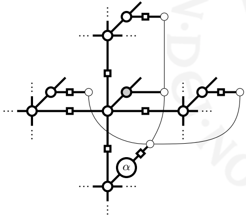
Quantum circuit simplification



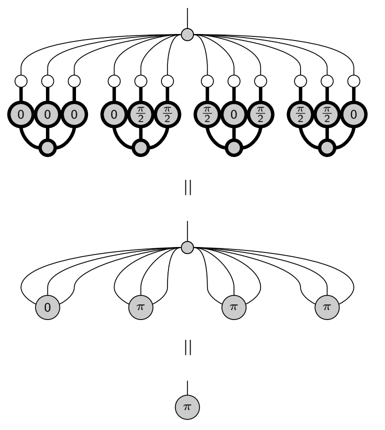
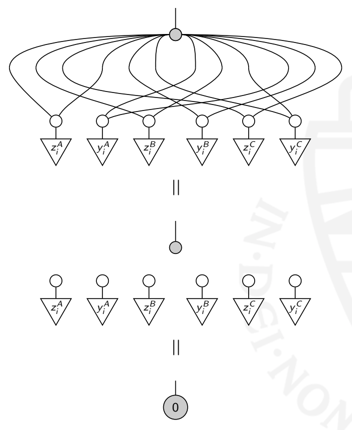
Measurement-based quantum computation



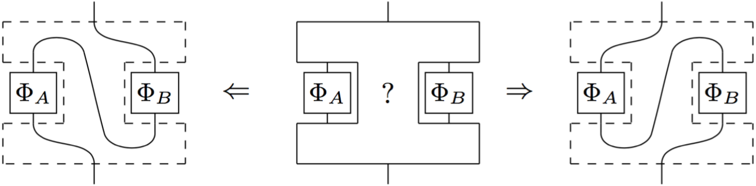




GHZ/Mermin non-locality

quantum theory	any local theory
	

Causal structures and indeterminism

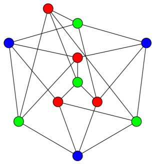


Quantum error correction

Stabiliser code

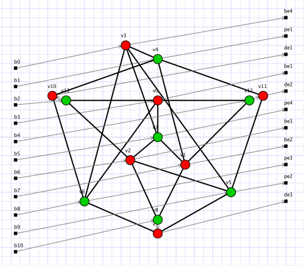
$$
\begin{array}{l}
Z_{d_1} Z_{d_2} Z_{b_1} Z_{p_2} Z_{p_4} \\
Z_{d_1} Z_{d_3} Z_{b_3} Z_{p_1} Z_{p_4} \\
Z_{d_2} Z_{d_3} Z_{b_2} Z_{p_3} Z_{p_4} \\
Z_{b_4} Z_{p_1} Z_{p_2} Z_{p_3} Z_{p_4} \\
X_{d_1} X_{d_2} X_{b_2} X_{b_4} X_{p_1} \\
X_{d_1} X_{d_3} X_{b_1} X_{b_4} X_{p_3} \\
X_{d_2} X_{d_3} X_{b_3} X_{b_4} X_{p_2} \\
X_{b_1} X_{b_2} X_{b_3} X_{b_4} X_{p_4}
\end{array}
\quad \leftrightarrow
$$

Quantum parity check graph

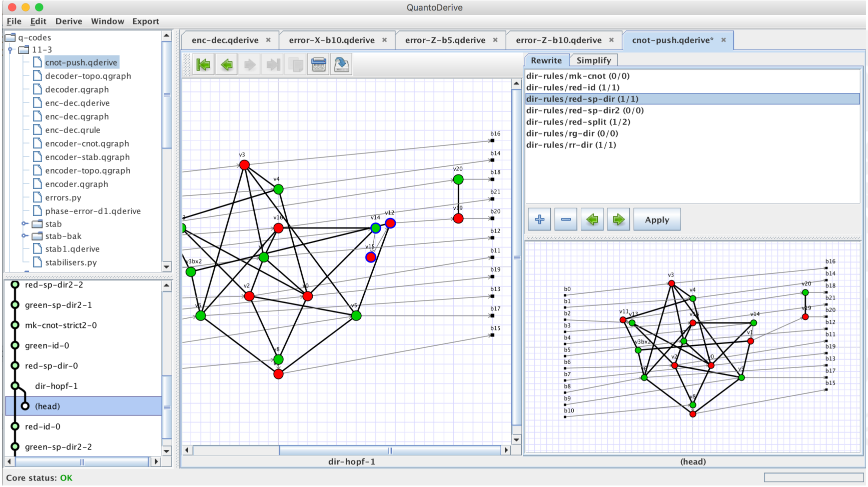


$\leftrightarrow$

Encoding circuit



Quantomatic:



Thanks!

- Picturing Quantum Processes. CUP (March 2017)
- Categorical Quantum Mechanics I: Causal Quantum Processes. Coecke and Kissinger. arXiv:1510.05468
- Categorical Quantum Mechanics II: Classical-Quantum Interaction. Coecke and Kissinger. arXiv:1605.08617
- Coherent Parity Check Construction for Quantum Error Correction. Chancellor, Kissinger, Zohren, Horsman. (arXiv on Monday!)

http://quantomatic.github.io