# Quantum Processes and Computation

**Assignment 2, Friday, 22 Oct, 17:00**

**Deadline:** Week 3 (group 1) or week 4 (groups 2-6)

**Goals:** After completing these exercises you should know how to reason with the transpose, adjoints, and the conjugate, and work with projections, unitaries and isometries. Material covered in book: Chapter 4.

**Note:** Many of these exercises also appear in *Picturing Quantum Processes*, but sometimes they have been slightly modified for the problem sheet. The corresponding exercise number from the book is shown in brackets.

**Exercise 1 (4.10 & 4.16):**

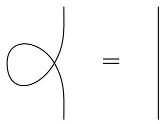
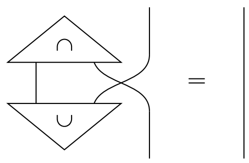
(a) Prove that in **relations**, the following relations on a set $A$:

 $:: * \mapsto \{(a, a) \mid a \in A\}$  $:: \forall a \in A : (a, a) \mapsto *$

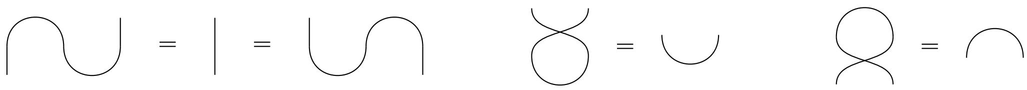
satisfy the *yanking equations* (eq. 4.11 in the book), and thus that **relations** has process-state duality.

(b) Show that process-state duality does not hold for **functions**.

**Exercise 2 (4.12):** Prove that

 or written differently: 

follows from the following 3 equations:

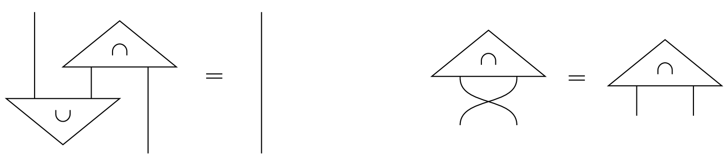


**Hint:** Use the second notation (with the boxes) as it prevents you from accidentally cheating.

**Exercise 3 (4.14 in online version of PQP):** Show that the following are equivalent:

(i) a state and an effect satisfying:

(ii) a state and an effect satisfying:

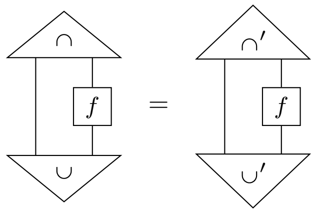


So in particular, if either eqs. (i) or eqs. (ii) hold, then all equations hold.

**Exercise 4 (4.59):** An *inverse* for a process $f : A \to B$ is a process $f^{-1} : B \to A$ such that $f^{-1} \circ f = \mathrm{id}_A$ and $f \circ f^{-1} = \mathrm{id}_B$. Show that for a process $f$ the following are equivalent:

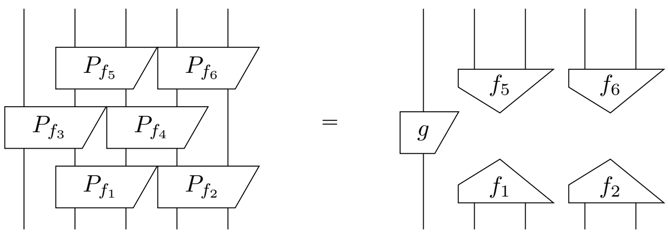
- $f$ is unitary.
- $f$ is an isometry and has an inverse.
- $f^\dagger$ is an isometry and has an inverse.

**Exercise 5 (4.37):** Show that the trace of a process is independent of the particular choice of cup and cap, i.e. that if $\cup$ and $\cap$ satisfy the yanking equations, but $\cup'$ and $\cap'$ also satisfy it that then:



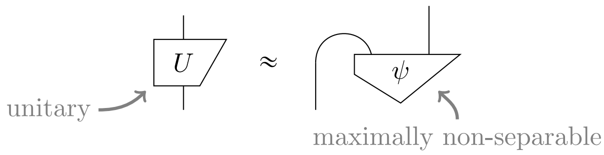
For a process $f : A \to A$ we define its *separable projector* by


**Exercise 6 (4.73):** Given processes $f_i : A \to A$ find the process $g$ such that:

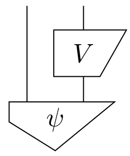


Write $g$ as a sequential composition of the conjugates, transposes and adjoints of the $f_i$'s.
**Hint:** Doing exercise 4.73 from the book first might reveal whether you understand the concept.

**Exercise 7 (4.82):** A state $\psi$ is *maximally non-separable* if it corresponds to a unitary $U$ by process-state duality, up to a number:



Show (i) that if one applies a unitary $V$ to one side of a maximally non-separable state:




that one again obtains a maximally non-separable state, and (ii) that this unitary can always be chosen such that the resulting state is the cup (up to a number).