Quantum Processes and Computation

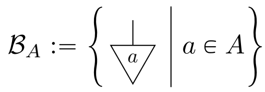
Assignment 3, Friday, 29 Oct, 17:00

Deadline: Week 4 (group 1) or week 5 (groups 2-6)

Goals: After completing these exercises you should know about (orthonormal) bases, composition of linear maps, and encoding logic gates as linear maps.
Material covered in book: Sections 5.1, 5.2, 5.3.4 and a bit of 5.3.5.

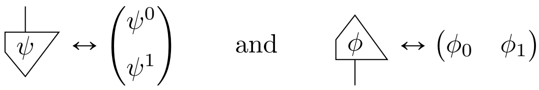
Note: Many of these exercises also appear in *Picturing Quantum Processes*, but sometimes they have been slightly modified for the problem sheet. The corresponding exercise number from the book is shown in brackets. **If you are stuck, try looking up the exercise number in the book. Usually the definitions or equations you need are nearby.**

Exercise 1 (5.4): Show that for a set A in **relations** the singletons:

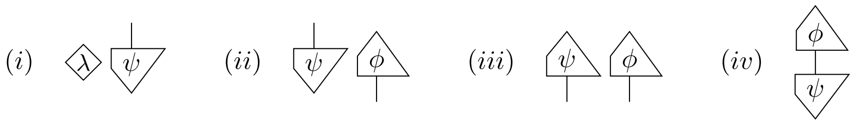


form a basis. Also show that this is the *only* basis for a system A, and consequently, that the dimension of a set A in **relations** is its number of elements.
For the next two exercises, assume we are working in a process theory with bases and sums (e.g. **relations** or **linear maps**).

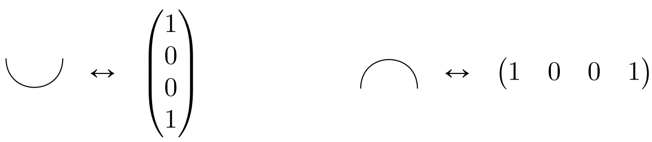
Exercise 2 (5.54): Let



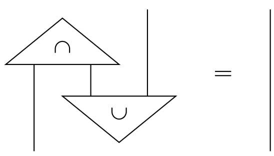
be respectively a 2-dimensional state, and 2-dimensional effect. Let λ be a number. Write the matrices for the processes



Exercise 3 (5.58): The matrices for cups and caps in 2 dimensions are:



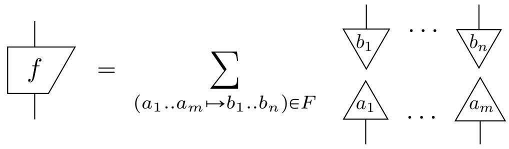
(i) First, verify the yanking equation



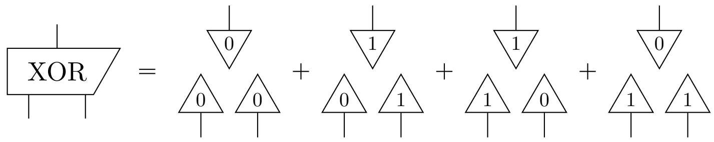
directly using the matrices of the 2-dimensional cup and cup by using the rules for sequential and parallel composition of matrices, i.e. show that $(\cap \otimes \mathrm{id}) \circ (\mathrm{id} \otimes \cup) = \mathrm{id}$ (where id is the 2×2 identity matrix).

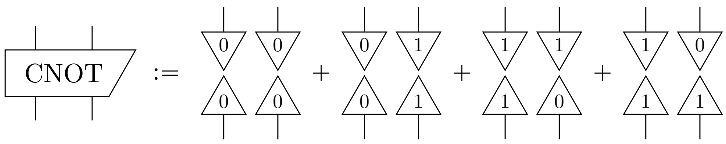
(ii) Second, give the matrices for the cup and cap in 3 dimensions.

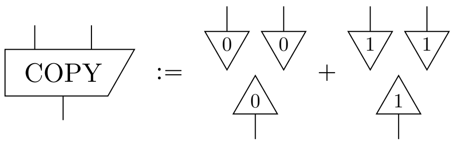
The next excercise is about encoding classical logic gates in the theory of **linear maps**, as explained in Section 5.3.4. Recall that a classical logic gate F can be encoded as a linear map via:



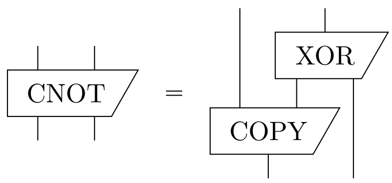
Using this encoding, we defined:





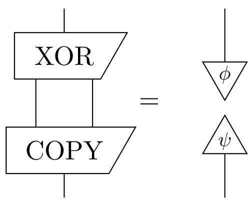


Exercise 4 (5.86): Show that



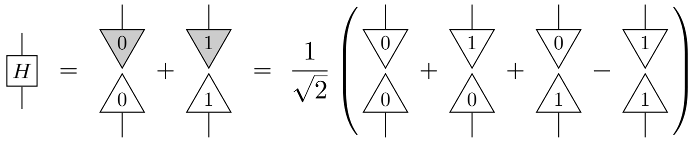
(Hint: try comparing the LHS to the RHS on all basis states, rather than writing out a big sum.)

Next, find ψ and ϕ such that the following equation holds:

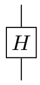


Although it might not look like much now, this equation will turn out to lie at the heart of the notion of *complementarity* which we will cover in great depth in the coming lectures.

Exercise 5 (5.93): In the proof of proposition 5.92, we see the Hadamard process written in matrix form with respect to the Z basis:



From this we can conclude the matrix of H (with respect to the Z-basis) is:



$$\leftrightarrow \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

What is the matrix of H in the X-basis?