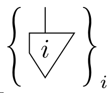
Quantum Processes and Computation

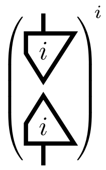
Assignment 5, Friday, 12 Nov

Deadline: Week 6 (group 1) or week 7 (groups 2-6)

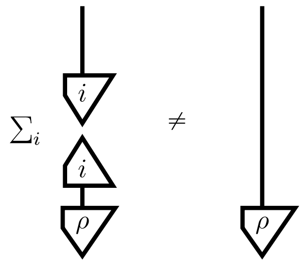
Goals: After completing these problems, you will be able to work with (non-deterministic) quantum processes, both in indexed form and with classical wires and spiders.

Note: Many of these exercises also appear in *Picturing Quantum Processes*, but sometimes they have been modified for the problem sheet. The corresponding exercise number from the book is shown in brackets. **If you are stuck, try looking up the exercise number in the book. Usually the definitions or equations you need are nearby.**

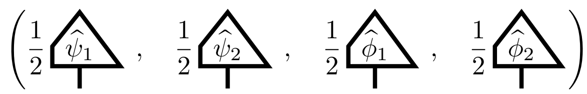
Exercise 1 (7.13): Let  form an ONB for any non-trivial system (so that it contains at least two elements). We can use this ONB to form a non-demolition ONB measurement in the following way:



If we sum over all the branches we get a process that is called *decoherence*. Show that decoherence is not equal to the identity by finding an explicit quantum state ρ such that:



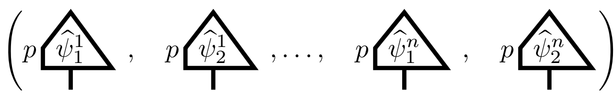
Exercise 2: Suppose we have two different ONB measurements on a qubit. Before we do anything with the quantum system we can flip a coin. If it comes up heads we can perform the first ONB measurement, and if it comes up tails we can do the other one. This is modelled by the quantum process



where $(\widehat{\psi}_1, \widehat{\psi}_2)$ and $(\widehat{\phi}_1, \widehat{\phi}_2)$ both form ONB measurements.

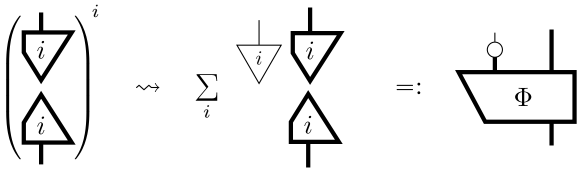
(i) Show that the above set of 4 effects indeed forms a quantum process (i.e. that their sum satisfies the causality condition).

(ii) We can generalise the above construction. Suppose we now have n different ONB measurements which we call $(\widehat{\psi}_1^j, \widehat{\psi}_2^j)$ for $j = 1, \ldots, n$. Find the number p such that

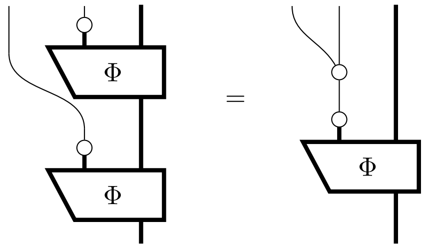


is a quantum measurement. Would the number p change if instead of ONB measurements on a qubit, we would consider ONB measurements on a bigger system?

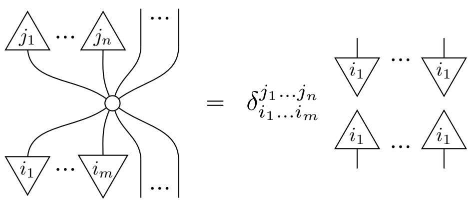
Exercise 3: Write a non-demolition ONB-measurement in the classical-quantum notation:



and show that



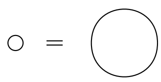
Exercise 4 (8.32): Prove the *generalised copy rule* for spiders:



where $\delta^{j_1 \dots j_n}_{i_1 \dots i_m}$ is the generalised Kronecker delta that is 1 when all the in- and outputs match and is 0 otherwise.

Exercise 5 (8.37 & 8.38):

(i) Prove using just the spider fusion law that the spider with no legs equals the 'circle' (i.e. the dimension):



(ii) Not all spiders are causal. Determine which spiders are causal, which can be made causal by rescaling by a number, and which cannot.