Quantum Processes and Computation
Assignment 2, Friday, 21 Oct, 12:00

Deadline: Fri Week 3 (Groups 3 and 4), Weds Week 4 (Groups 1 and 2)

Goals: After completing these exercises you should know how to reason with the transpose, adjoints, and the conjugate, and work with projections, unitaries and isometries. Material covered in book: Chapter 4.

Note: Many of these exercises also appear in *Picturing Quantum Processes*, but sometimes they have been slightly modified for the problem sheet. The corresponding exercise number from the book is shown in brackets.

Exercise 1 (4.10 & 4.16):

(a) Prove that in **relations**, the following relations on a set A:

$$\cup \quad :: * \mapsto \{(a,a) \mid a \in A\} \qquad \cap \quad :: \forall a \in A : (a,a) \mapsto *$$

satisfy the *yanking equations* (eq. 4.11 in the book), and thus that **relations** has process-state duality.

(b) Show that process-state duality does not hold for **functions**.

Exercise 2 (4.12): Prove that

follows from the following 3 equations:

Hint: Use the second notation (with the boxes) as it prevents you from accidentally cheating.

Exercise 3 (4.14 in online version of PQP): Show that the following are equivalent:

(i) a state and an effect satisfying:

(ii) a state and an effect satisfying:

So in particular, if either eqs. (i) or eqs. (ii) hold, then all equations hold.

Exercise 4 (4.59): An *inverse* for a process $f : A \to B$ is a process $f^{-1} : B \to A$ such that $f^{-1} \circ f = \mathrm{id}_A$ and $f \circ f^{-1} = \mathrm{id}_B$. Show that for a process f the following are equivalent:

- f is unitary.

- f is an isometry and has an inverse.

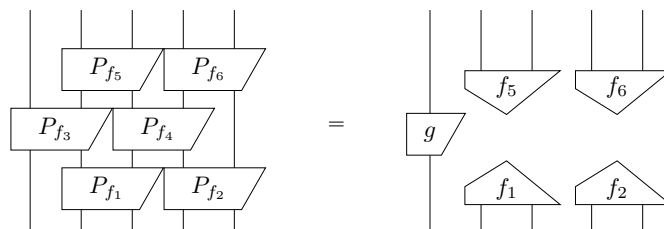- $f^\dagger$ is an isometry and has an inverse.

Exercise 5 (4.37): Show that the trace of a process is independent of the particular choice of cup and cap, i.e. that if $\cup$ and $\cap$ satisfy the yanking equations, but $\cup'$ and $\cap'$ also satisfy it that then:
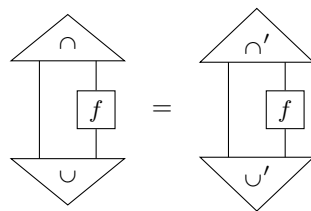
For a process $f : A \to A$ we define its *separable projector* by

**Exercise 6 (4.73):** Given processes $f_i : A \to A$ find the process g such that:

Write g as a sequential composition of the conjugates, transposes and adjoints of the f_i's.
Hint: Doing exercise 4.73 from the book first might reveal whether you understand the concept.

Exercise 7 (4.82): A state ψ is *maximally non-separable* if it corresponds to a unitary U by process-state duality, up to a number:
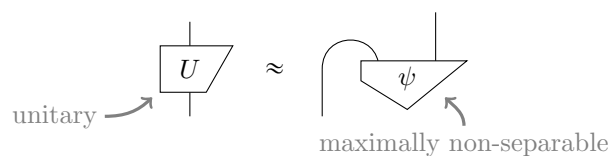
Show (i) that if one applies a unitary V to one side of a maximally non-separable state:
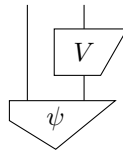
that one again obtains a maximally non-separable state, and (ii) that this unitary can always be chosen such that the resulting state is the cup (up to a number).