Quantum Processes and Computation

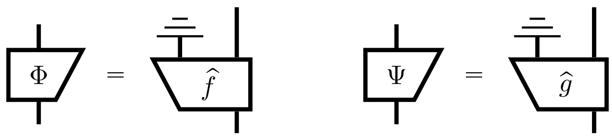
Assignment 4, Friday, 4 Nov

Deadline: Fri Week 5 (Groups 3 and 4), Weds Week 6 (Groups 1 and 2)

Goals: After completing these exercises you should know how to work with (non-deterministic) quantum processes, both in indexed form and with classical wires. You should understand what it means for a process to be causal and what it means to preserve this property.

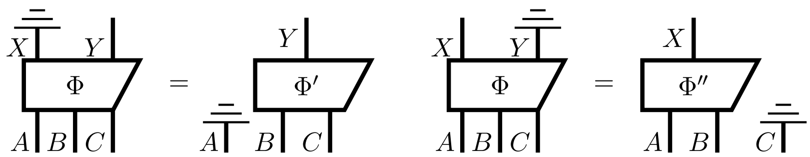
Note: Many of these exercises also appear in *Picturing Quantum Processes*, but sometimes they have been modified for the problem sheet. The corresponding exercise number from the book is shown in brackets. **If you are stuck, try looking up the exercise number in the book. Usually the definitions or equations you need are nearby.**

Exercise 1: Let Φ and Ψ be quantum maps with purifications f, respectively g:

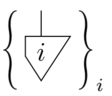


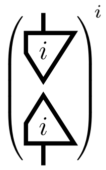
Give purifications of $\Phi \circ \Phi$ and $\Psi \circ \Phi$ in terms of f and g.

Exercise 2: Construct a causal quantum map $\Phi : A \otimes B \otimes C \to X \otimes Y$ by making a connected circuit diagram involving exactly 3 causal quantum maps Φ_1, Φ_2, Φ_3 such that

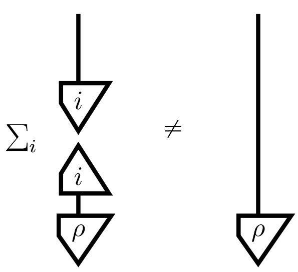


Hint: Read Section 6.3.

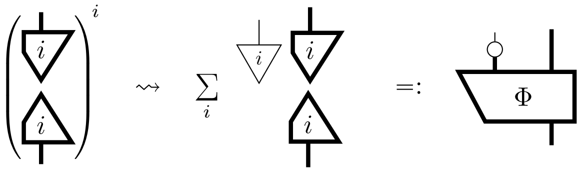
Exercise 3 (7.13): Let  form an ONB for any non-trivial system (so that it contains at least two elements). We can use this ONB to form a non-demolition ONB measurement in the following way:



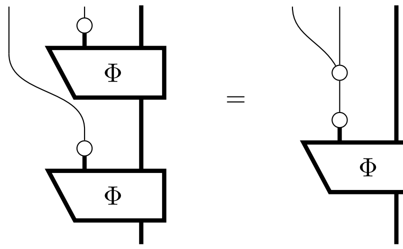
If we sum over all the branches we get a process that is called *decoherence*. Show that decoherence is not equal to the identity by finding an explicit quantum state ρ such that:



Exercise 4: Write a non-demolition ONB-measurement in the classical-quantum notation:

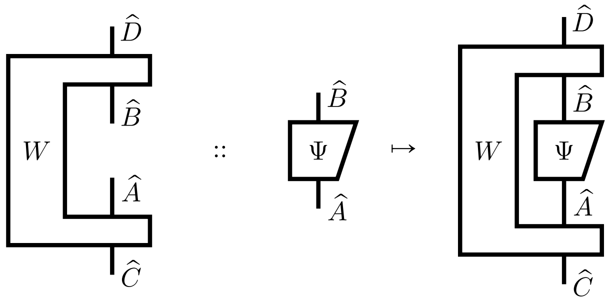


and show that

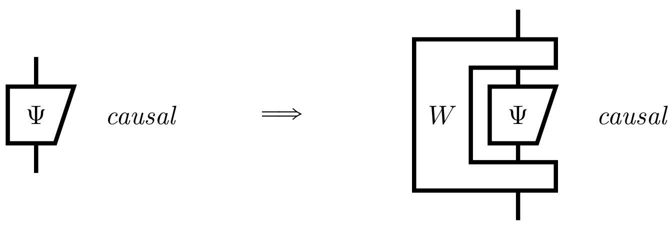


Exercise 5: This exercise is part of one of the questions on a previous QPC exam.

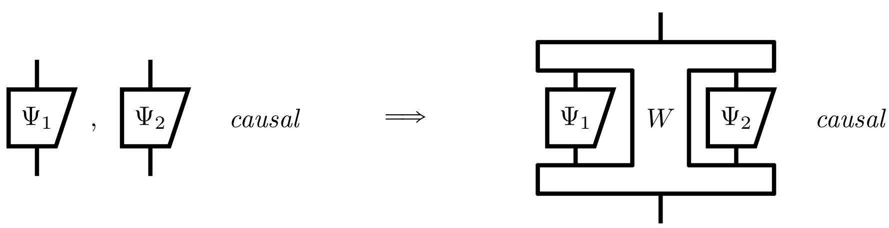
Second-order processes are "processes of processes". That is, they take one or more processes as input and produce a process as output. We depict them as boxes with 'notches' cut out, allowing us to plug in other processes. For example, the following second-order process W takes a process $\Psi : \widehat{A} \to \widehat{B}$ as input, and produces a new process from $\widehat{C}$ to $\widehat{D}$ as an output:



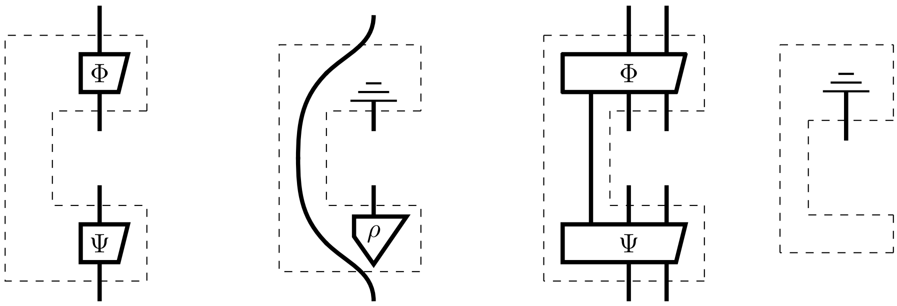
A **second-order causal** process has the property that if a causal process is input, then the output will be a causal process:



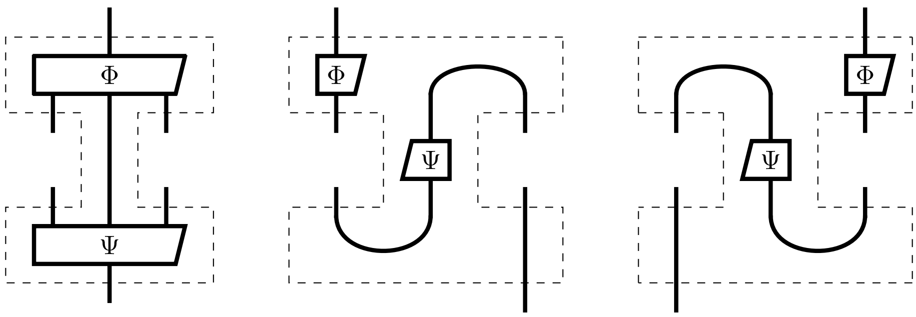
Similarly, a **second-order** 2**-causal** process has the property that if any *two* causal processes are input, then the output will be a causal process:



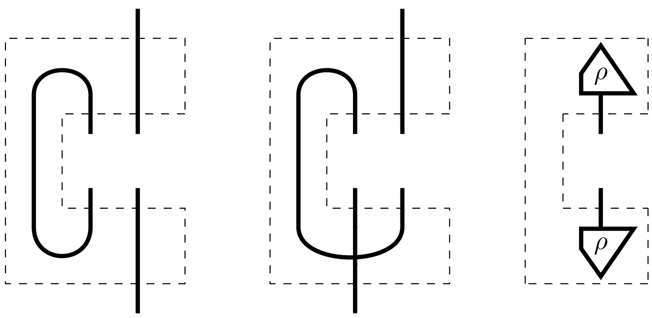
(a) Show that the following are second-order causal processes, for any causal processes Ψ, Φ, ρ:



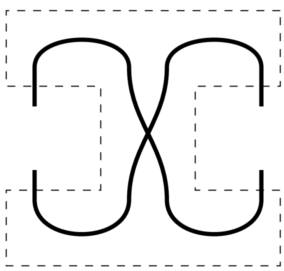
and show that the following are second-order 2-causal processes, for any causal processes Ψ, Φ:



(b) Show that the following are *not* second-order causal processes for any causal state ρ:

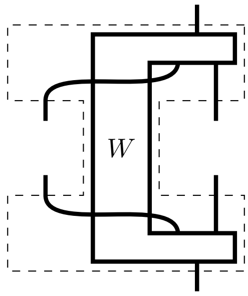


and show that the following is *not* a second-order 2-causal process:



In all cases, you may assume the wires correspond to non-trivial systems (i.e. dimension > 1).

(c) Show that being second-order 2-causal is strictly weaker than being second-order causal. That is, show that if W is second-order causal, then we can treat it as a second-order 2-causal process as follows:



but if W is is second-order 2-causal, the following process is not necessarily second-order causal: