# Quantum Processes and Computation

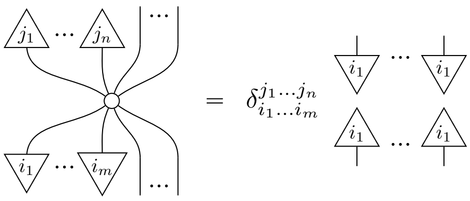
**Assignment 5, Friday, 11 Nov**

**Deadline:** Fri Week 6 (Groups 3 and 4), Weds Week 7 (Groups 1 and 2)

**Goals:** After completing these problems, you will be able to work with spiders, unbiased/phase states, and the phase group.

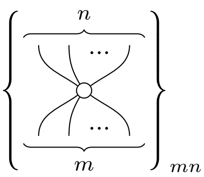
**Note:** Many of these exercises also appear in *Picturing Quantum Processes*, but sometimes they have been modified for the problem sheet. The corresponding exercise number from the book is shown in brackets. **If you are stuck, try looking up the exercise number in the book. Usually the definitions or equations you need are nearby.**

**Exercise 1 (8.32):** Prove the *generalised copy rule* for spiders:

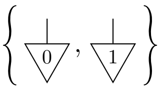



where $\delta^{j_1 \cdots j_n}_{i_1 \cdots i_m}$ is the generalised Kronecker delta that is 1 when all the in- and outputs match and is 0 otherwise.

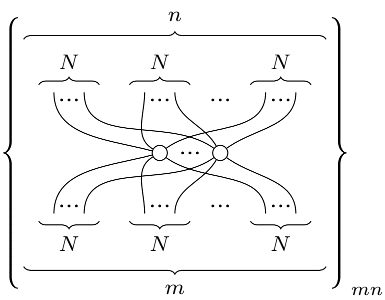
**Exercise 2 (8.39):** The spiders:



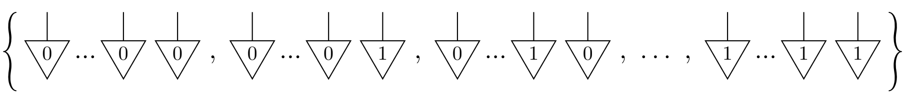

for the 2D basis:




are associated with *bits*. Show that the following family of classical maps:

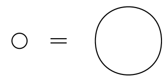



is also a family of spiders (i.e. that it fits into Definition 8.31) and that it is associated with the ONB of $N$-*bitstrings*:



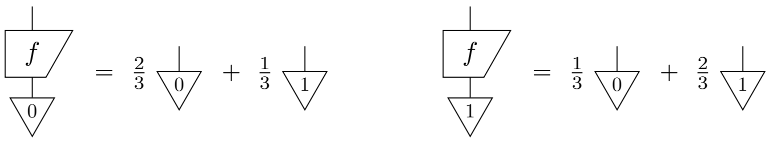
**Exercise 3 (8.37 & 8.38):**

(i) Prove using just the spider fusion law that the spider with no legs equals the 'circle' (i.e. the dimension):

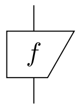


(ii) Not all spiders are causal classical maps. Determine which spiders are causal, which can be made causal by rescaling by a number, and which cannot.

**Exercise 4 (8.15):** We saw in the lecture that a classical map has a matrix with only positive numbers. For instance the classical map characterised by



has the matrix

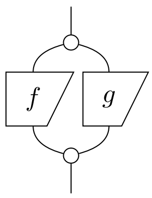


$$\leftrightarrow \quad \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

However, if we change the basis the numbers in the matrix no longer have to be positive. Find a basis such that the matrix of $f$ with respect to this basis contains some negative values.

**Exercise 5 (8.26 & 8.55 & 8.69):**

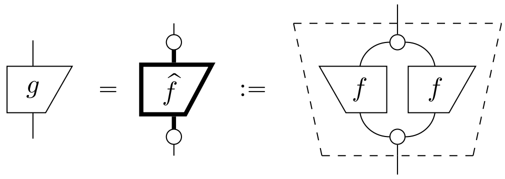
(i) Show that for any two linear maps $f$ and $g$ of the same type, the diagram:



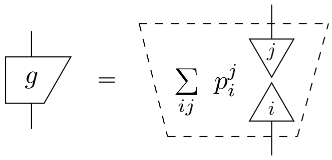
yields the Hadamard product of matrices:

$$\begin{pmatrix} f_1^1 & \cdots & f_D^1 \\ \vdots & \ddots & \vdots \\ f_1^D & \cdots & f_D^D \end{pmatrix} \star \begin{pmatrix} g_1^1 & \cdots & g_D^1 \\ \vdots & \ddots & \vdots \\ g_1^D & \cdots & g_D^D \end{pmatrix} = \begin{pmatrix} f_1^1 g_1^1 & \cdots & f_D^1 g_D^1 \\ \vdots & \ddots & \vdots \\ f_1^D g_1^D & \cdots & f_D^D g_D^D \end{pmatrix}$$

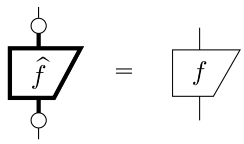
(ii) For a classical map $g$, find a pure quantum map $\widehat{f}$ such that



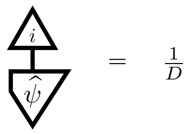
Hint: Use the representation



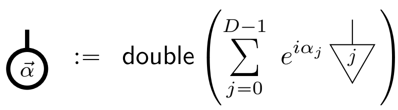
(iii) Show that a function map $f$ (i.e. a deterministic causal classical map) satisfies



**Exercise 6 (9.2):** Show that a normalised pure state is *unbiased* for an ONB-measurement if and only if for all $i$ we have:



**Exercise 7 (9.21):** In section 9.1, it was shown that any phase state of dimension $D$ is of the form:




and furthermore that we can assume, up to a global phase, that $\alpha_0 = 0$. Hence, a $D$-dimensional phase state is labelled by a vector of $D-1$ phases $\vec{\alpha} := (\alpha_2, \ldots, \alpha_D)$.

Show that the phase group unit, addition, and inverse are defined in terms of these vectors as follows:

$$\vec{0} := (0, \ldots, 0)$$
$$\vec{\alpha} + \vec{\beta} = (\alpha_1 + \beta_1, \ldots, \alpha_{D-1} + \beta_{D-1})$$
$$-\vec{\alpha} := (-\alpha_1, \ldots, -\alpha_{D-1})$$