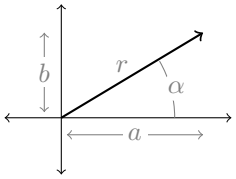
Quantum Processes and Computation

Assignment 1 – MT 2026

Unless otherwise noted, the following exercises are taken from *Picturing Quantum Software* (see github.com/zxcalc/book). There might be slight differences between the exercises on this problem sheet and how they appear in the book. **In that case, complete the version from this problem sheet and not the book version.** For some exercises, it may be useful to locate the exercise in the book for additional results and explanation.



(2.1)

Exercise 2.2. Write $\cos\alpha$ and $\sin\alpha$ as a sum of complex phases with rational coefficients.

Exercise 2.4. Let $M : H \to H$ be a linear map such that $M^\dagger M = MM^\dagger$. Then $M = \sum_j \lambda_j |\phi_j\rangle\langle\phi_j|$ for some set of complex numbers $\{\lambda_j\}_j$ and ONB $\{|\phi_j\rangle\}_j$. Show that:

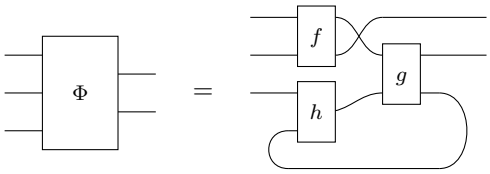
a) M is self-adjoint iff all $\lambda_j \in \mathbb{R}$

b) M is positive iff all $\lambda_j \in \mathbb{R}_{\geq 0}$

c) M is a projector iff all $\lambda_j \in \{0, 1\}$

d) M is a unitary iff all $\lambda_j \in U(1)$

where $\mathbb{R}_{\geq 0}$ is the set of all real numbers ≥ 0 and $U(1) := \{e^{i\alpha} \mid \alpha \in [0, 2\pi)\}$ is the set of all **phases**.



(2.2)

$$\Phi^{de}_{abc} = \sum_{xyz} f^{xd}_{ab} g^{ez}_{xy} h^{y}_{cz} \tag{2.3}$$

Exercise 2.6. Write an expression for (2.2) using $\otimes, \circ$, SWAP, and $\mathrm{Tr}(-)$.

Exercise 2.7. Show that for a normalised state $|\psi\rangle$, the operator $|\psi\rangle\langle\psi|$ is a projector with image $S := \{\, \lambda|\psi\rangle \mid \lambda \in \mathbb{C} \,\}$.

Exercise 2.8. Show that, for the following antipodal states on the Bloch sphere:

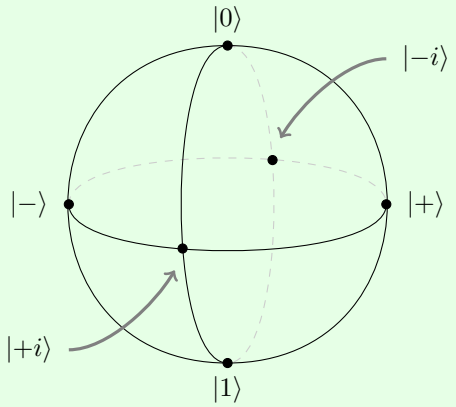
$$|\phi_0\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\alpha}|1\rangle$$
$$|\phi_1\rangle = \cos\frac{\theta+\pi}{2}|0\rangle + \sin\frac{\theta+\pi}{2}e^{i\alpha}|1\rangle$$

we have $\langle\phi_0|\phi_1\rangle = 0$.

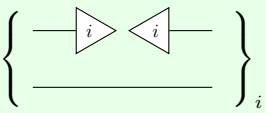
Exercise 2.9. The standard basis $|0\rangle$, $|1\rangle$ corresponds to the Z axis of the Bloch sphere. Show that the following states

$$|+\rangle := \frac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)$$
$$|-\rangle := \frac{1}{\sqrt{2}}\big(|0\rangle - |1\rangle\big)$$
$$|+i\rangle := \frac{1}{\sqrt{2}}\big(|0\rangle + i|1\rangle\big)$$
$$|-i\rangle := \frac{1}{\sqrt{2}}\big(|0\rangle - i|1\rangle\big)$$

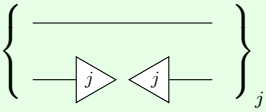
correspond to the X and Y axes, respectively. That is, show they are located on the Bloch sphere as follows:



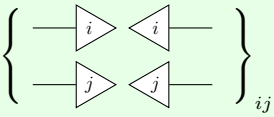
Exercise 2.12. Define the following 3 measurements on a pair of qubits:

$\mathcal{A} =$ 

$_i$

$\mathcal{B} =$ 

$_j$

$\mathcal{C} =$ 

$_{ij}$

Show that the probability of getting outcome i for $\mathcal{A}$ then getting outcome j when measuring $\mathcal{B}$ on the resulting state is the same as the probability of getting outcome (i, j) for $\mathcal{C}$.