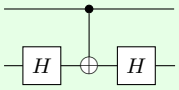
Quantum Processes and Computation

Assignment 2 – MT 2026

Unless otherwise noted, the following exercises are taken from *Picturing Quantum Software* (see github.com/zxcalc/book). There might be slight differences between the exercises on this problem sheet and how they appear in the book. **In that case, complete the version from this problem sheet and not the book version.** For some exercises, it may be useful to locate the exercise in the book for additional results and explanation.

Exercise 2.13. The CNOT gate is also often called the CX gate, because it applies an X gate to the target if the control is $|1\rangle$. We similarly also have a CZ gate that applies a Z instead. We can construct this using a CNOT and Hadamard gates as follows:



Find the matrix of the CZ by directly calculating the matrices involved in the circuit above.

$$U_f :: |\vec{x}, \vec{y}\rangle \mapsto |\vec{x}, f(\vec{x}) \oplus \vec{y}\rangle$$

Exercise 2.15. Show that $U_f(I \otimes |-\rangle) = V_f \otimes |-\rangle$, where $V_f|\vec{x}\rangle := (-1)^{f(\vec{x})}|\vec{x}\rangle$.

Exercise 3.1. Show that for $m > 0, n > 0$, Z-spiders always correspond to matrices with rank 2. What are the other possibilities for the rank of a Z-spider if m or n is 0?

Exercise 3.2. What classical map would we get if we instead took the X-spider with 2 inputs, 1 output and a π phase?

The next exercise is about this equation:

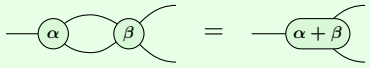
(3.1)

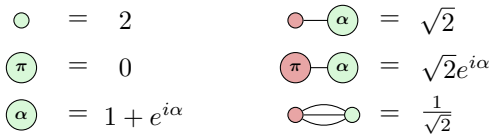
In *Picturing Quantum Software*, the LHS is computed explicitly as follows:

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&1&0&0&0&0\\0&1&1&0&0&0&0&0\\0&0&0&0&1&0&0&1\\0&0&0&0&0&1&1&0\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix} \tag{3.2}$$

Exercise 3.3. Prove Eq. (3.1) by directly calculating the matrix the right-hand side represents and showing it agrees with Eq. (3.2).

Exercise 3.4. Show that the following equality is true, by writing down the left-hand side in bra-ket notation and simplifying it:





(3.3)

Exercise 3.5. By combining the diagrams from (3.3), find a ZX-diagram to represent the following scalar values z:

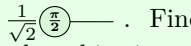
a) $z = -1$.

b) $z = e^{i\theta}$ for any θ.

c) $z = \frac{1}{2}$.

d) $z = \cos\theta$ for any value θ.

e) Find a general description or algorithm to construct the ZX-diagram for any complex number z.

Exercise 3.6. Consider the following state: 

. Find its transpose and its adjoint. Compose the state with its transpose and show that this gives the scalar zero. Now compose it with its adjoint and show that this gives the scalar 1.