Quantum Processes and Computation

Assignment 6 – MT 2026

Unless otherwise noted, the following exercises are taken from *Picturing Quantum Software* (see github.com/zxcalc/book). There might be slight differences between the exercises on this problem sheet and how they appear in the book. **In that case, complete the version from this problem sheet and not the book version.** For some exercises, it may nevertheless be useful to locate the exercise in the book for additional results and explanation.

Exercise 6.1. Show that $\langle X, Y, Z\rangle = \mathcal{P}_1$, but $\langle X, Z\rangle$ generates a strict subset of $\mathcal{P}_1$.

Exercise 6.2. Fix an arbitrary pair of Pauli strings $\vec{P}, \vec{Q} \in \mathcal{P}_n$, where:

$$\vec{P} := i^k P_1 \otimes \ldots \otimes P_n \qquad \vec{Q} := i^l Q_1 \otimes \ldots \otimes Q_n$$

Prove the following properties:

a) $\vec{P}$ is unitary.

b) $\vec{P}^\dagger = \vec{P}$ if k is even, otherwise $\vec{P}^\dagger = -\vec{P}$.

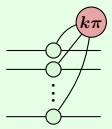
c) $\vec{P}^2 = I$ if k is even, otherwise $\vec{P}^2 = -I$.

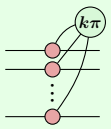
d) $\vec{P}$ and $\vec{Q}$ either commute ($\vec{P}\vec{Q} = \vec{Q}\vec{P}$) or anti-commute ($\vec{P}\vec{Q} = -\vec{Q}\vec{P}$).

Exercise 6.7. Show that the $Z \otimes \ldots \otimes Z$ and $X \otimes \ldots \otimes X$ measurements:

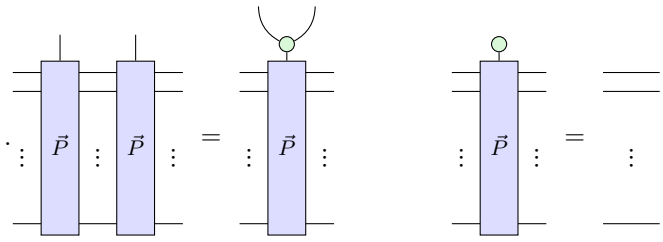
$$\mathcal{M}_{Z\ldots Z} := \{\Pi^{(k)}_{Z\ldots Z}\}_k \qquad \mathcal{M}_{X\ldots X} := \{\Pi^{(k)}_{X\ldots X}\}_k$$

are defined by the following Pauli projectors:

$\Pi^{(k)}_{Z\ldots Z} \propto$ 

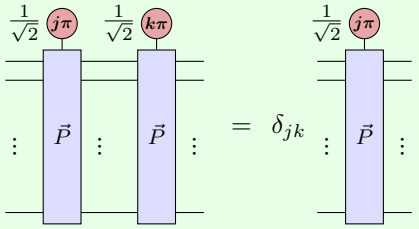
$\Pi^{(k)}_{X\ldots X} \propto$ 

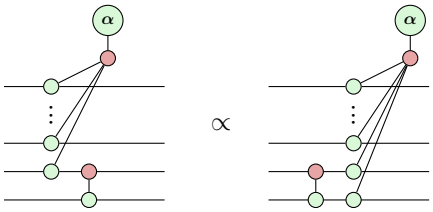
Proposition 6.2.7 in *Picturing Quantum Software* states that Pauli boxes satisfy the following two equations:



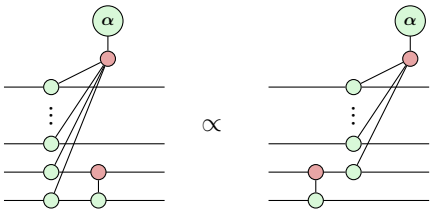
(6.1)

Exercise 6.9. Show diagrammatically that the equations in (6.1) imply that Pauli projectors on to different measurement outcomes are orthogonal:

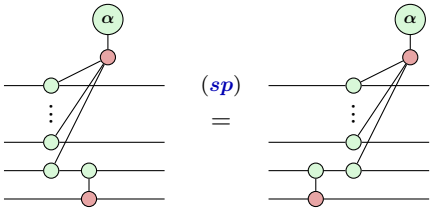




(7.2)

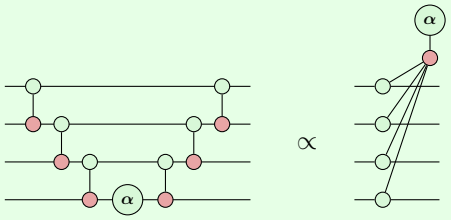


(7.3)



(7.4)

Exercise 7.2. Prove equations (7.2) and (7.3). Use the three "phase-gadget walking" equations (7.4), (7.2), and (7.3), as well as the fact that Z-phase gates are equivalent to 1-legged phase gadgets, to show that an n-legged phase gadget has the following decomposition:



(7.5)

Exercise 7.12. Show that if we have a phase gadget in a graph-like diagram with a phase that is a multiple of $\frac{\pi}{2}$, that we can remove it using a series of local complementation or pivots.

The **weight** $|\vec{P}|$ of a Pauli string is the number of non-identity single-qubit Pauli operators that appear in its tensor project. For example:

$$|I \otimes I \otimes I| = 0 \qquad |X \otimes X \otimes X| = 3 \qquad |X \otimes I \otimes Y| = 2$$

Exercise 12.2. Show that we can bound the weight of a product of two Pauli strings as follows:

$$|\vec{P}\vec{Q}| \leq |\vec{P}| + |\vec{Q}|$$

Exercise 12.8. Suppose we define two Pauli strings from the bitstrings $\vec{v}$ and $\vec{w}$ as follows:

$$\vec{X} := \bigotimes_{j=1}^{n} X^{v_j} \qquad \vec{Z} := \bigotimes_{j=1}^{n} Z^{w_j}$$

Show that $\vec{X}$ and $\vec{Z}$ commute if and only if $\vec{v}$ and $\vec{w}$ are orthogonal, i.e. $\vec{v}^T\vec{w} = 0 \pmod 2$.