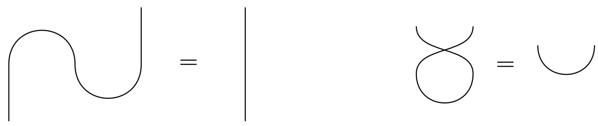
# Quantum Processes and Computation

**Assignment 2, Hilary 2026**

*Solutions are shown after each question. Note some solutions are marked* Sketch*. These are intended to be instructions on how to work out the solution yourself, rather than an example of how you should answer this question on an exam.*

**Exercise 1:** We can write the cup/cap for any dimension as a sum over ONB elements:

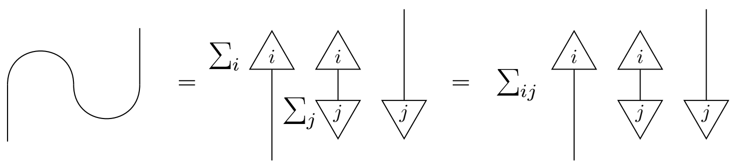
(i) Using this definition (and not the matrix form) verify the yanking equations.

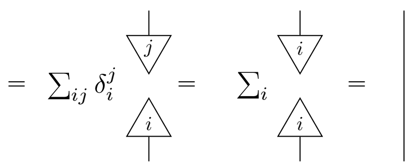


(ii) Compute the matrices for the cup and cap in 3 dimensions.

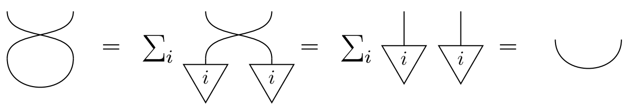
**Begin Solution:** ..........................................................................

*(i) These can be verified using the properties of ONBs and sums. For the first one:*





*For the second one:*



*(ii) This is a column vector, whose entries correspond to the basis elements $|i, j\rangle$ for $i, j \in \{0, 1, 2\}$. If $i = j$, the entry is a 1, otherwise it is a 0. This results in the following vector:*

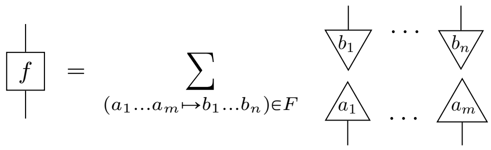
$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

*n.b. it is the same as the columns of a $3 \times 3$ identity matrix, all stacked on top of each other. Cups in all dimensions have this same pattern. The cap is the transpose of the above, which is row vector:*

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

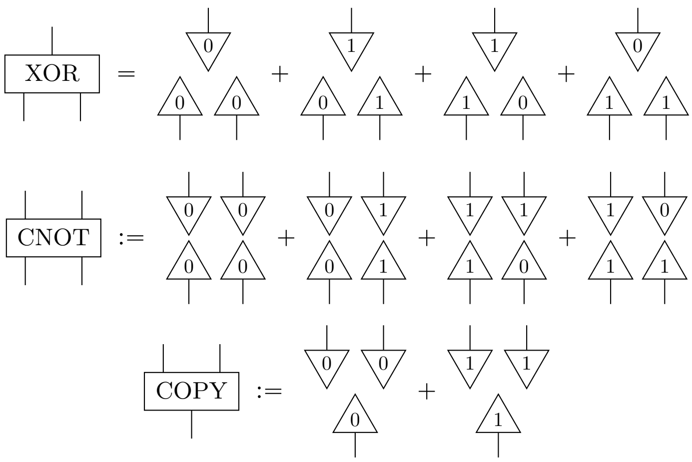
**End Solution** ..........................................................................................

**Exercise 2 (5.86):** This excercise is about encoding classical functions as linear maps using ONB states and effects, as explained in Section 5.3.4. For a function $F : \{0,1\}^m \to \{0,1\}^n$, we can define an associated linear map $f$ as follows:

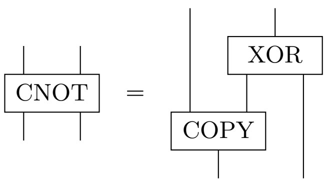



where the notation $(a_1...a_m \mapsto b_1...b_n) \in F$ means we are summing over the *graph of* $F$, i.e. the set of bitstrings $\{(a_1, \ldots, a_m, b_1, \ldots b_n) \mid F(a_1, \ldots, a_m) = (b_1, \ldots b_n)\}$.

Using this encoding, define:

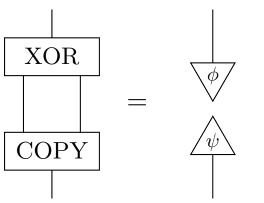



Show that




(Hint: try comparing the LHS to the RHS on all basis states, rather than writing out a big sum.)

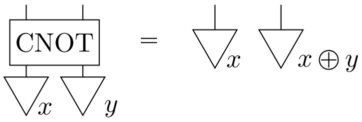
Next, find $\psi$ and $\phi$ such that the following equation holds:



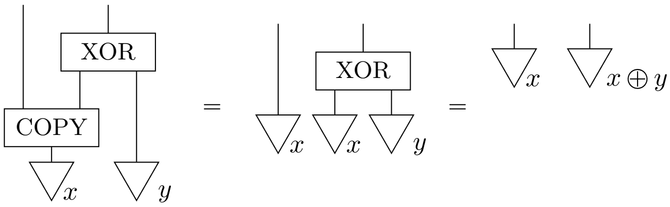
Although it might not look like much now, this equation will turn out to lie at the heart of the notion of *complementarity* which is an important part of the ZX-calculus.

**Begin Solution:** ..........................................................................................

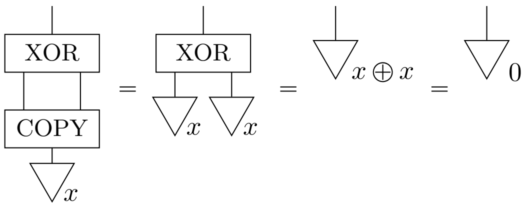
*The first part can be done by plugging in basis states, and nothing that the LHS gives:*



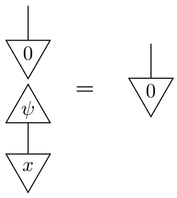

*and the RHS gives:*




*If I evaluate the second diagram at a basis state, I get:*

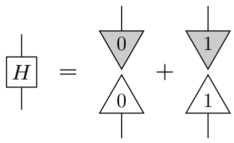



*This tells me that* $|\phi\rangle$ *should be* $|0\rangle$*. For* $\langle\psi|$*, I need the effect that "deletes" any basis state:* $\langle\psi| = \langle 0| + \langle 1| = \sum_i \langle i|$*. Then* $\langle\psi|0\rangle = \langle\psi|1\rangle = 1$*, so:*




**End Solution** ..........................................................................................

**Exercise 3:** Let the *Hadamard gate*, which sends the Z-basis to the X-basis be defined as follows:




where

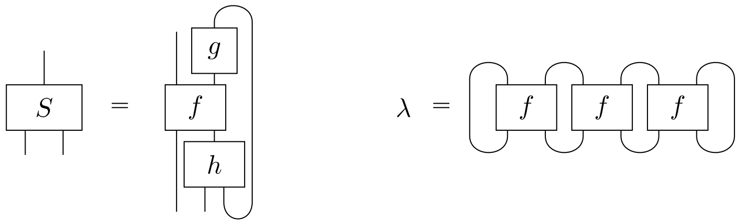

Compute the matrix of $H$. Show that $H = H^\dagger = H^T$. Using this fact (or otherwise) show that $H$ also sends the X-basis back to the Z-basis.

**Begin Solution:** ..........................................................................................

*Sketch: The matrix can be computed by plugging in each of the 4 bras and kets of the computational basis. We can see it sends X-basis elements to Z-basis elements by applying the adjoint to both sides of the definition of $H$ and using $H = H^\dagger$.*

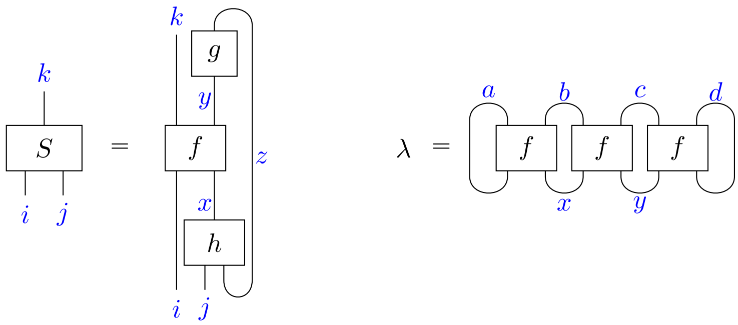
**End Solution** . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

**Exercise 4:** Write the following diagrams as tensor contractions, i.e. as sums over products of matrix elements $f_{ij}^{kl}$, etc.




**Begin Solution:** . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

*Labelling the wires with some index names:*




*...we get:*

$$S_{ij}^{k} = \sum_{xyz} f_{ix}^{ky} g_{y}^{z} h_{jz}^{x}$$

$$\lambda = \sum_{abcdxy} f_{ax}^{ab} f_{xy}^{bc} f_{yd}^{cd}$$

*Note $\lambda$ is a scalar, so all indices on the RHS are summed over.*

**End Solution** . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .