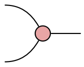
Quantum Software

Assignment 4, Hilary 2026

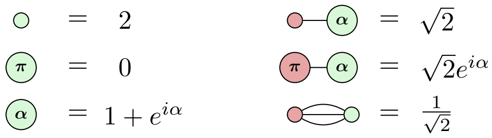
Exercise 1 (3.2): In *Picturing Quantum Software*, it was shown that the X-spider with two inputs and one output gives XOR:



$$= \frac{1}{\sqrt{2}}\left(|0\rangle\langle 00| + |0\rangle\langle 11| + |1\rangle\langle 01| + |1\rangle\langle 10|\right)$$

What classical map would we get if we instead took the X-spider with 2 inputs, 1 output and a π phase?

Exercise 2: In the lectures we often ignore scalar factors in ZX-diagrams. We can however represent any scalar we want with a ZX-diagram. For instance, we have:

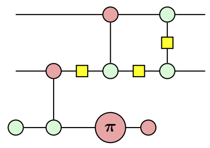


(1)

By combining the diagrams from (1), find a ZX-diagram to represent the following scalar values z:

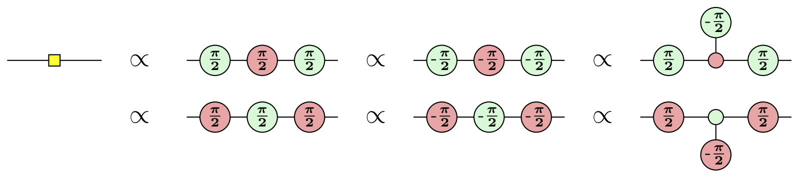
1. $z = -1$.
2. $z = e^{i\theta}$ for any θ.
3. $z = \frac{1}{2}$.
4. $z = \cos\theta$ for any value θ.
5. Find a general description or algorithm to construct the ZX-diagram for any complex number z.

Exercise 3: Using ZX-calculus rewrites, help the poor trapped π phase find it's way to an exit (i.e. an output).



Note that it might be leaving with friends.

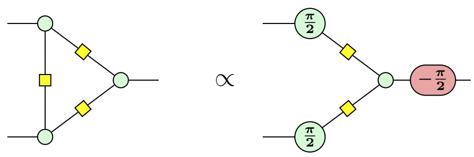
Exercise 4: The Euler decomposition from the lecture is just one possible way to write the Hadamard in terms of spiders. There is in fact an entire family of representations that will also be useful to note:



(2)

Prove that all the equations of (2) hold in the ZX-calculus, by using the top-left decomposition and the other rewrite rules of the ZX-calculus we have seen so far.

Exercise 5: Prove the following base-case of the local complementation lemma using the ZX-calculus:

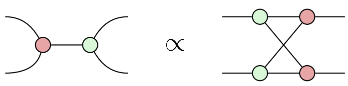


(3)

Hint: Push the top Hadamards up and decompose the middle Hadamard using one of Eq. (2) to reveal a place where you can apply strong complementarity.

How this result can be used to prove general local complementation is shown in Picturing Quantum Software.

Exercise 6 (3.7): In Section 3.2.4 of *Picturing Quantum Software*, it is shown how to prove 3 CNOT gates equals a SWAP gate, by applying the strong complementarity rule:



in the reverse direction (i.e. by replacing the RHS with the LHS). Show that there is an alternative proof, applying strong complementarity in the forward direction and applying complementarity (possibly multiple times).