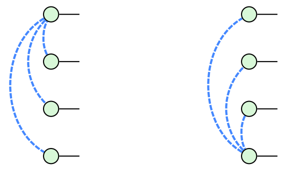
Quantum Software

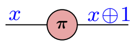
Assignment 6, Hilary 2026

Exercise 1: We say two n-qubit quantum states $|\psi_1\rangle$ and $|\psi_2\rangle$ are equivalent under local operations when $U|\psi_1\rangle = |\psi_2\rangle$ for a local quantum circuit $U = U_1 \otimes U_2 \otimes \cdots \otimes U_n$ consisting of just single-qubit gates. Show that the following two graph states are equivalent under local operations.



Hint: Use the fact that a local complementation can be done using just local unitaries.

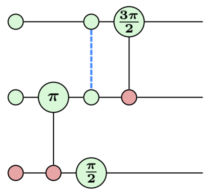
Exercise 2: Show that the phase polynomial representation also works for circuits made of CNOT, Z-phase, and X gates, using the fact that an X gate updates wire labels as follows:



More specifically, give an example circuit, compute its parity map and phase polynomial by wire-labelling, and re-synthesise a smaller circuit.

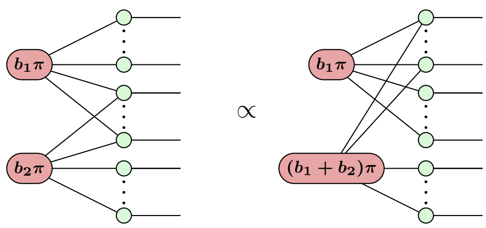
Finally, show that further simplifications are possible: namely expressions of the form $\alpha \cdot y + \beta \cdot (y \oplus 1)$, where y is some XOR of input variables, can be simplified in the phase polynomial, up to global phases.

Exercise 3: Reduce the following diagram to AP-form:



What is its parity matrix and its phase polynomial?

Exercise 4: The bit strings appearing in the superposition in an AP state are described by the solutions to the affine system of equations $M\vec{x} = \vec{b}$. When we do row operations on M and $\vec{b}$ (as in a Gaussian elimination of the linear system) this does not change the solutions, and so this transformed system $(M', \vec{b}')$ should still describe the same state. As the matrix M corresponds to the connectivity of the internal spiders to the boundary spiders, show that these row operations can be implemented diagrammatically:



Exercise 5: Reduce the following Clifford circuit to GSLC form: