Developments on the ZW calculus

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The ZW calculus is a string-diagrammatic calculus for the theory of qubits, refining the ZX calculus [CD08, Bac14]. Its first version [Had15] was universal and complete for the theory of $\mathbb{Z}$-bits, that is, free abelian groups on 2^n generators, with the monoidal structure given by tensor products; this was later extended to free modules on an arbitrary commutative ring R [Had17]. When R is the field of complex numbers, this gives the first diagrammatic axiomatisation of the full theory of qubits. Recently, there has been renewed interest after the strongest extensions of ZX calculi, with relative completeness proofs, have been obtained through translations into the ZW calculus [JPV17].

The ZW calculus has specific advantages over ZX calculi, including the existence of a computationally meaningful normal form, and of a fragment whose diagrams can be interpreted physically as setups of fermionic oscillators. These make it better suited to generalisations in arbitrary dimension, and universal sets of generators have already been defined for general qudits, based on the qubit case.

In this talk, I want to review the main features of the ZW calculus, discuss the most recent developments on its fragments and extensions, and provide an overview of the remaining open questions.

References

[Bac14] M. Backens. The ZX-calculus is complete for stabilizer quantum mechanics. *New Journal of Physics*, 16(9):093021, 2014.

[CD08] B. Coecke and R. Duncan. Interacting quantum observables. In *Automata, Languages and Programming*, pages 298–310. Springer, 2008.

[Had15] A. Hadzihasanovic. A diagrammatic axiomatisation for qubit entanglement. In *Proceedings of the 30th Annual ACM/IEEE Symposium on Logic in Computer Science*, LICS '15, pages 573–584. IEEE, 2015.

[Had17] A. Hadzihasanovic. *The algebra of entanglement and the geometry of composition.* PhD thesis, University of Oxford, 2017.

[JPV17] E. Jeandel, S. Perdrix, and R. Vilmart. A complete axiomatisation of the ZX-calculus for Clifford+T quantum mechanics. *arXiv preprint arXiv:1705.11151*, 2017.