IMPACT Kindergarten Quantum Theory

Bob Coecke, Selma Dundar-Coecke, Stefano Gogioso

The ultimate goal of this project is to provide empirical evidence for the use of diagrammatic representations in secondary (or even primary) school education. Specifically, the project's explicit aim is to show that diagrammatic methods can outperform traditional symbolic methods in the teaching of Quantum Theory—a subject traditionally considered intractable below university level—at high school level and above. If successful, this experiment would be a world first in education, showing that techniques developed within the Department of Computer Science of Oxford University can be used to teach an advanced mathematical topic of remarkable future technological interest to an extremely wide audience, with no mathematical background required.

1 Objectives of the Experiment

The proposed research aims to investigate whether diagrammatic methods outperform traditional symbolic methods in the context of teaching Quantum Theory. Our main reason to study such methodological effectiveness lies in the belief that Quantum Theory should be introduced to a broad audience at high school level, as opposed to being confined to a restricted community of specialists (as is the case today). Furthermore, empirical evidence of diagrammatic methods outperforming symbolic methods in the context of teaching of an advanced topic such as Quantum Theory will make a strong case for the use of diagrammatic representations to teach mathematical subjects—both elementary and advanced—in secondary (or even primary) school education, furthering and enriching the existing literature on the use of diagrammatic representations in education (Post & Cramer, 1989; Leinhardt, et al., 1991; Howe, et al., 1992; Mancosu, et al., 2005; Barmby, et al., 2012; Dundar-Coecke, 2014).

Our choice of Quantum Theory as a test case is motivated by two key observations. Firstly, Quantum Theory provides a crucial intersecting point between physics and computer science: with the impeding introduction of quantum devices in computing, communication and cryptography, it is poised to become a foundational component of engineering and computer science curricula in the years to come. Secondly, the reason why Quantum Theory is almost universally considered an advanced topic is the large amount of mathematical background required by traditional teaching methods, rather than the complexities intrinsic to the topic itself: mathematicians and physicists only acquire the necessary background in the later years of their university education, and the topic is not ordinarily offered to computer science and engineering students. Conversely, the diagrammatic methods considered in our research require little or no mathematical background, and would allow the subject to be taught at high school level and in a larger spectrum of higher education curricula.

The traditional formalisms (Dirac, 1930; Landau & Lifshitz, 1958; Feynman, et al., 1964; Sakurai, 1994; Nielsen & Chuang, 2000) used in the teaching and practice of Quantum Theory have their focus in the theory of Hilbert spaces, an advanced mathematical topic related to complex linear algebra. It is an abstract presentation of the subject, and a solid mathematical background is required to master even the simplest practical applications. To obviate this problem, the last decade has seen the introduction of diagrammatic techniques alternative to the traditional Hilbert space formalism, pioneered and developed by the Oxford Quantum Group (Coecke, et al., 2008; Coecke, 2009; Selinger, 2010; Coecke & Duncan, 2011; Horsman, 2011; Vicary, 2012; Backens, 2014; Coecke, et al., 2016; Coecke & Kissinger, 2017).

Diagrammatic techniques can be used to describe the same quantum devices that the Hilbert space formalism is traditionally used to describe. Contrary to the Hilbert space formalism, however, there is a close, intuitive correspondence between diagrammatic representations and real-world implementation of quantum circuits and experiments: the wires and boxes appearing in the diagrams provide an abstract depiction of the cables and devices appearing in quantum circuit, much in the same way that an electrical diagram provides an abstract depiction of the wires, switches and electrical devices found in an electrical circuit. But most importantly, the diagrammatic formalism does not require any pre-existing mathematical background: all the rules describing how a quantum circuit behaves can be presented and applied in a purely graphical way. The proposed research aims to investigate whether the diagrammatic formalism can be used to successfully teach Quantum Theory to high-school students and other groups not possessing the pre-requisites to learn the discipline with the traditional Hilbert space formalism. We therefore hypothesise that participants who are trained with the diagrammatic formalism will outperform participants who are trained with the Hilbert space formalism. In line with this hypothesis, the proposed research addresses the following question:

"To what extent and in what manner do diagrammatic methods provide learners of Quantum Theory with better problem-solving skills than symbolic (Hilbert space) methods?"

2 METHODOLOGY

2.1 DESIGN AND SAMPLE

The proposed study will follow a pre- and post-test design and compare groups performances to identify the relative effectiveness of training the following six categories with the diagrammatic and Hilbert space formalisms. Sample participants will be recruited across the UK via purposive sampling.

- **A-level students**: 2 groups, labelled AS1 and AS2, of 12-16 participants each;
- **A-level teachers**: 2 groups, labelled AT1 and AT2 and roughly equal in number, composed of teachers accompanying the students of groups AS1 and AS2.
- **undergraduates (scientific degrees)**: 2 groups, labelled US1 and US2, of 12-16 participants each;
- **pre-GCSE students** (around 14yo): 1 group, labelled GS, of 12-16 students;
- **pre-GCSE teachers**: 1 group, labelled GT, composed of teachers accompanying the students of group GS;
- **undergraduates (non-scientific degrees)**: 1 group, labelled UN, of 12-16 students.

A-level students, their teachers and undergraduates from scientific degrees will be taught using both the traditional Hilbert space formalism and the diagrammatic formalism: each group will be taught with both formalisms, but in different order (see tables below). Pre-GCSE students, their teachers and undergraduates from non-scientific degrees will only be taught using the diagrammatic formalism. The numbers have been chosen in such a way that each group (possibly except for teachers) will have enough participants for independent statistical analysis. In the optimal scenario where each student is accompanied by an individual teacher, we expect a total of 72-96 students (plus teachers) to participate in this experiment.

Participants for the pre-GCSE and A-level groups (together with their teachers) will by recruited by contacting a randomly chosen group of schools across the United Kingdom, and by advertising the opportunity on a selected handful of popular science magazines targeted at the pre-GCSE and A-level age bracket. Upon contact, schools will be explained the premises of the project and asked to perform the recruitment internally, according to the criteria specified below and with a pre-determined target number of applicants. Participants for the two undergraduate groups will be recruited by asking a select group of universities to advertise the opportunity internally to their students, who will be asked to apply individually using an online form. A suitable number of applicants will be selected as prospective participants, and individually contacted with details about the next stages of enrolment.

To be enrolled into the experiment, each participant will be asked for a signed consent form, confirming that they fully understand the design, duration and requirements of the experiment—including conditions of travel and accommodation in Oxford—and testifying that they satisfy the applicable selection criteria listed below. For participants under the age of 18 (pre-GCSE and A-level students), the form will need to be signed by parents or legal guardians, and will include a section acknowledging that the student will be under the legal responsibility of the accompanying teacher. All participants will furthermore be asked to sign an "enrolment and retention" form, confirming their long-term commitment to the schedule and tests that the experiment will require, and providing appropriate contact information for follow up. Finally, all participants will be required to complete a series of online tasks prior to their final enrolment, testing their working memory capacity, basic mathematical background, language processing ability and spatial ability.

2.2 PROCEDURES

The experiment itself will take place over 2 weeks (labelled W1 and W2), with a total of 4 lecturers involved: 2 lecturers (labelled D1 and D2) will deliver one tutorial each using diagrammatic methods, and 2 lecturers (labelled H1 and H2) will deliver one tutorial each using Hilbert space methods.

In order to reduce teacher effects on training outcomes, the 2 lecturers teaching Hilbert space methods will be chosen to be academics from fields closely related to quantum theory (ideally in the Physics or Materials departments), who are favouring the importance of teaching Quantum Theory with the symbolic, Hilbert space approach. The 2 lectures teaching diagrammatic methods will instead be chosen to be academics from the Oxford Quantum Group, who are favouring the importance of teaching Quantum Theory with the diagrammatic approach. The four lecturers will receive dedicated training, providing them with all the necessary details about the research process and the relevant ethical codes.

Each tutorial will be delivered within a single week. Each group of A-level students, A-level teachers and undergraduates from scientific degrees will undergo 1 diagrammatic tutorial and 1 Hilbert space tutorial across 2 weeks (although in different order); pre-GCSE students, pre-GCSE teachers and undergraduates from non-scientific degrees will undergo 1 diagrammatic tutorial only.

The following tables show the proposed allocation of tutorials to the different groups of participants. Allocation is designed to ensure that participants of similar or compatible backgrounds are taught together, as to minimise the overall duration of the experiment and the number of lecturers involved. The top table shows the schedule for high schoolers, who are to be taught by lecturers H1 and D1; the bottom table shows the schedule for teachers and university students, who are to be taught by lecturers H2 and D2. The schedule is structured in such a way as to ensure that high school students and their accompanying teachers are taught the same material in the same week.

High schoolers	W1-H1	W1-D1	W2-H1	W2-D1
AS1	X			X
AS2		X	X	
GS		X		

Adults	W1-H2	W1-D2	W2-H2	W2-D2
AT1	X			X
AT2		X	X	
GT		X		
US1	X			X
US2		X	X	
UN		X		

Each tutorial will span the 5 working days of 1 week. Days 1, 2 and 3 will entail 6 lectures lasting 2 hours each, and covering the entire material for the tutorial. The first lecture of the day will take place between 10am and 12noon, the participants will then be provided with lunch and allowed some time for discussion and some exercises, and the second lecture of the day will take place between 3pm and 5pm. Day 4 will entail private study and practice on part of the participants, together with an additional lecture—of the duration of 2 hours in the afternoon—in which the participants will be allowed to ask questions and do some supervised exercises. Day 5 will consist of an examination, lasting 3 hours, on the material covered in the tutorial. Each tutorial will therefore involve 14 hours of face time spread across 6+1 lectures, plus 3 hours of written examination.

At the beginning of each tutorial, the participants will be given a tutorial book covering the syllabus that will be taught across the first 3 days, together with some worked out exercises and some practice exercises and exam questions, which will be reviewed during the day 4 lecture.

2.3 Preparation of the tutorial books

The two tutorial books (Hilbert and diagrammatic) will be written by two academics in 2017. The syllabi for the two tutorials will cover the same range of topics in Quantum Theory, which are deemed to be representative introductory topics for the subject. The Exam Board (EB)—which includes the principal investigator, two lecturers and an educational psychologist—will review the tutorial books and provide feedback.

Additionally, the tutorial books will be reviewed by three independent academics each, to ensure that each tutorial adequately covers the intended topics and is truly representative of the formalism it purports to use. Review criteria will include range of topics, appropriateness, presentation of the material, time management and difficulty level. Six reviews will be received in total, and the tutorials will subsequently undergo one further round of revision by the author. The six independent academic reviewers will be chosen based on their expertise: three reviewers who are experts in Hilbert space approaches to Quantum Theory, and three reviewers who are expert in diagrammatic approaches to Quantum Theory.

2.4 Preparation of the assessment kit

The assessment kit will involve three kinds of assessments: psychometric tasks, to be completed at the time of enrolment; mathematical background tasks, to be completed at the beginning of the tutorial training; tutorial examination papers, to be completed at the end of the tutorial training.

The psychometric tasks will target: spatial ability, using the Purdue Spatial Visualization-Rotation Test (Bodner & Guay, 1997); working memory, using the Symbol Counter Task abilities (Garavan, et al., 2000); language processing, using the WASI Vocabulary Subtest for expressive language abilities (Wechsler, 1999). These tasks will be completed online by the participants at the time of enrolment, and their completion will be a necessary condition for participation in the experiment.

The mathematical background tasks will involve the completion of a selection of GCSE and A-level questions, selected by the Exam Board based on their difficulty level and their relevance to the understanding of the material to be taught in the tutorials. The participants will be asked to attempt the questions in a timed exam session on site, before the beginning of the tutorials, and their answers will be graded according to the relevant GCSE and A-level marking schemes.

A total of four tutorial examination papers will be prepared, two pertaining to the symbolic tutorial (one for week 1 and one for week 2), and two pertaining to the diagrammatic tutorial (one for week 1 and one for week 2). On the Friday of each week, the participants who attended the symbolic tutorial during that week will be asked to sit the symbolic examination paper, while the participants who attended the diagrammatic tutorial will be will be asked to sit the diagrammatic examination paper. Each examination paper will consist of a selection of 2-3 questions per topic in the syllabus, in a variety of formats: examples will include multiple choice, true/false, matching, and short answers. Each examination paper will also include one open-ended question, asking participants to approach a problem in their own way and explain their problem-solving strategy.

The examination papers will be prepared by the same academics who wrote the tutorial books, and will be checked by the Exam Board and by the same six independent academic reviewers who previously attended to the tutorial books. The aim of these checks will be to ensure adequate coverage of the syllabus, consistent difficulty level and appropriateness, as well as to raise any possible ethical concerns. All examination papers will carry the same number of questions, in the same formats, with the same marks: in each given week there will be an exact correspondence between the questions in the symbolic examination paper and the diagrammatic examination paper, but questions will change from week 1 to week 2, to avoid cross-contamination of the results.

2.5 Analysis

Our main research question (*cf.* end of Section 1) will be addressed by comparing the score distribution for the symbolic examination papers with that of the diagrammatic examination papers. To properly capture the full breadth of background and training effects, a variety of comparison methods will be used, and the examination results will be correlated with the psychometric and mathematical background data collected before the commencement of the tutorial teaching.

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