Item 4: Detailed research decription

Where quantum meets classical: foundational structures and their ramifications

Bob Coecke

Oxford University Computing Laboratory

August 2006

1a. Physical context

Quantum mechanics (QM) is arguably the most successful physical theory ever in terms of experimental predictions. However, its conceptual implications, sometimes referred to as *quantum weirdness*, have been an ongoing source of distress to the Foundations of Physics community. At the core of this distress is the notion of quantum measurement, which describes the process of *extracting classical knowledge from the quantum state* i.e. establishing a *flow of information* from the *quantum* to the *classical domain*. While many books and many decades of intense research have been devoted to the different aspects of quantum measurement, the strikingly intense disagreements about the "measurement problem" within the Foundations of Physics community shows that compelling conceptual progress has yet to be made.

On the other hand, the emergence of the field of *Quantum Information and Computation* (QIC) was mainly a consequence of embracing this quantum weirdness not as a bug or paradox, but as one of nature's most intriguing features, and moreover, capable of being exploited to great effect for several information-processing purposes. This new attitude has indeed shown some remarkable physical potential, such as [46]: provably secure quantum public key distribution (devices which establish this can in fact already be purchased on-line); substantial algorithmic speed-up for the important factoring (Shor) and search (Grover) algorithms; *quantum teleportation* [9], a protocol which enables transmission of continuous quantum data by only using a finitary classical communication channel; and *dense coding* [10], a protocol which enables transmission of two classical bits by only using one quantum bit. All of these have important potential applications within *Quantum Information Technology* (QIT).

Key to quantum teleportation and dense coding, aside from the flow of information from the quantum to the classical domain, is the *flow of information* from the *classical* (cf. measurement outcome) to the *quantum domain*. For example, in teleportation the outcome of the measurement in the Bell basis determines the subsequent unitary correction. More elaborate physical phenomena, such as the *one-way quantum computational model* [49], and *measurement-based quantum computation* in general, also involve highly non-trivial interactions between the classical and the quantum domains. However, *a profound logical understanding of how these domains interact has yet to be obtained*. Such an insight would contribute to a deeper understanding of QM itself, and moreover is vital for QIC, and hence also for the further development of QIT. A key obstruction to achieving such a deep logical insight is the fact that *classical data lives outside the quantum mechanical formalism*, e.g. in quantum teleportation one treats classical data-flow in a purely informal manner. As such, work exposing the structure of the classical-quantum interaction needs to start by recasting the canonical quantum mechanical formalism itself, which is by now about seventy years old.

1b. Logical and mathematical context

Elsewhere in the scientific spectrum, *Programming Language Semantics, Concurrency Theory, Proof Theory, and Category Theory*, have seen a significant number of developments:

1. Linear Logic (LL), a resource-sensitive logic [28, 1, 6, 26],
2. particular kinds of Monoidal Categories (MC) which provide semantics for LL-style logics [8],
3. diagrammatic calculi (tracing back to Penrose's work [48]), which provide graphical representations for general MCs, and give rise to proof systems for LL and similar logics [40, 27, 25, 38].

LL allows explicit accounting for computational resources, by dropping the structural rules of Contraction and Weakening in Gentzen-style logical sequent calculus, and hence restricting the ability to copy and delete premisses [28]. Bearing in mind the No-Cloning and No-Deleting theorems [54, 47] which apply to quantum data, as opposed to the delete-ability and unlimited copy-ability of classical data, LL is perfectly suited to incorporate these fundamental informatic constraints of QM.

Semantics in terms of MCs have been extremely successful in Computer Science because of their generality, their ability to find common structure in many different situations, and their support for compositional modelling – analysing complex systems in terms of how they are built up, using a stock of basic operations of wide applicability, from (simpler) sub-systems. This leads to an algebraic view of systems which is both elegant, and extremely effective in allowing concise descriptions of complex systems, algebraic manipulation of these descriptions, and which, in particular, provides a basis for distinguishing between different types of systems, e.g. quantum vs. classical, or more general, "types reflecting kinds", *contra* the Hilbert space formalism, where an operator of type $\mathcal{H} \to \mathcal{H}$ can be a mixed state, a unitary transformation or a measurement. A compositional structure would be highly desirable as a foundation for QM and hence QIC, since many results in QIC-theory show that the same basic components are being combined over and over again to form various protocols cf. [22]. In fact, at a far more basic level, compositionality (i.e. performing one operation after another one, or, applying one operation to part of a joint system and another operation to another part) and types (i.e. distinguishing between different kinds of systems such as distinct elementary particles) constitute the most basic structure of the operations carried out by the physicist practising in his lab. This basic structure turns out to be an MC [17]. Therefore we can conclude that for *operational theories* like QIC, which deals with composable entities such as unitaries, channels, measurements and subsystems, categorical algebra is the most obvious candidate for an axiomatic framework.

1c. Categorical quantum mechanics-logic-informatics

The precise connection between LL, MCs, and QM, was recently established by the applicants. They recast the usual quantum mechanical formalism in terms of *strongly compact closed categories* [4, 5, 20, 51, 19] (the latter two respectively introduce generalized operations or CPMs and generalized measurements or POVMs), for which the corresponding graphical calculus turns out to be a *two-dimensional extension of Dirac's calculus* [16]. Moreover, the categorical semantics are flexible enough to jointly accommodate both the quantum and the classical structure, as exemplified by a fully formal description and very intuitive derivation of quantum teleportation [4, 20] and dense coding [20].

Interestingly, the particular monoidal categorical structures which we will discuss below, namely *compact closed categories* [40, and elaborations thereon] and *categorical Frobenius algebras* [41, and references therein], have not only emerged in the context of our logical understanding of the quantum structure, but turn out to be extremely *natural*, having also sprung up within *general relativity*, *quantum field theory*, *braid and knot theory*, *statistical physics* etc. [3, 7, 44].

2. Objectives

Categorical quantum mechanics-logic-informatics of [4, 5, 20, 51, 19] provides an appropriate starting point for a thorough study of the logical structure of the quantum-classical interaction. Such a study will obviously have important ramifications both for QIC and FQM. In particular, high-level and structural methods for QIC will provide the same benefits as the use of logic has done in CS, providing the programmer with intuitive user-friendly languages as opposed to the hacking with 0's and 1's that took place in the early days of computing [2]. We provide some examples of the important applications which could stem from this line of research:

- There are four components to the information flow in elaborate quantum phenomena and protocols: quantum-classical (cf. measurement), classical-quantum (cf. control operations), classical-classical (e.g. in one-way quantum computing), and quantum-quantum, which, besides capturing unitary operations, is also implicitly encoded in the complex behavioural properties of entanglement, and entails the capability to transmit continuous data while effectively only sending discrete classical data in quantum teleportation. Through the categorical quantum formalism, which makes available LL-methods and tools, the power of categorical semantics and the corresponding intuitive graphical calculi, we expect to achieve a structural understanding of each of these information flows, their distinct capabilities and relative importance for certain tasks, and how they interact with each other.

- We aim to achieve a logical understanding of how the distinct abilities for copying and deleting quantum vs. classical data exposes itself at the level of the relative structure of these information flows, relying on the results in [20] where it was shown that measurement can be defined purely in terms of this distinction, and that the classical world re-emerges by weakening informatic constraints. We expect to obtain a precise description of how the quantum-logical structure transforms in classical logical structure when considering a gradual passage from quantum structure to classical structure e.g. by allowing for restricted forms of either cloning or deleting.

- We wish to understand the logical status of *decoherence*, which includes a profound understanding of purity versus mixedness, and unraveling the measurement process in distinct logical components as initiated in [20], where it was shown that the process of decoherence is structurally derivable in the presence of a classical context which comes with a copying operation.

- We expect that all this will also yield a structural account of the many QIC-quantities which are currently being considered in the literature, and use all this in establishing a compositional categorical general theory of QIC-resources, which should extend the recent proposal in [22].

These results would entail a profound step forward in the logical understanding of the quantum-classical interaction, both from the perspective of QIC-applications and in terms of a fundamental conceptual understanding, including a unprecedented logical understanding of the quantum measurement process.

3. Approach

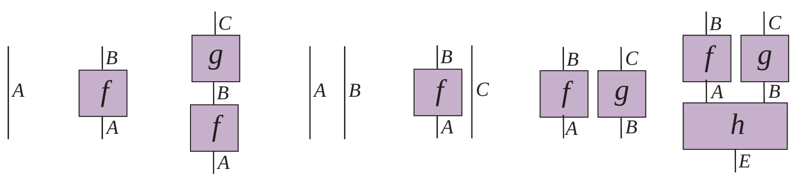
We discuss the relevant aspects of the categorical quantum mechanics-logic-informatics developed by [4, 5, 20, 51, 19] in more detail, present some recent progress concerning quantum-classical interaction, and stipulate, in terms of enumerated "tasks", how we wish to elaborate on this as part of the research project.

3.1. Categorical semantics and graphical calculus

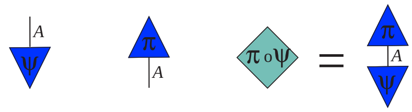
After refining compact closure [40] to *strong compact closure* [4, 5] we were able to recover many key quantum mechanical notions such as *scalar, inner-products, unitarity, full and partial traces, the Hilbert-Schmidt inner-product and map-state duality, projection, positivity, measurement*, and *the Born-rule* (which provides the *probabilities*), axiomatically with a high level of abstraction and generality. While at this level of abstraction there is no underlying field of complex numbers, there *is* still an intrinsic notion of 'scalar', and we could still make sense of *transposition vs. adjoint* [4, 5], *global phase and elimination thereof*, *vectorial vs. projective formalism* [16], *mixed state, complete positivity*, *Jamiolkowski map-state duality* [51, Selinger], *decoherence, generalized measurements* and *Naimark's theorem* [20, 19, resp. with Pavlovic and Paquette]. The corresponding "strongly compact closed graphical calculus" is not merely an illustration, but one can show that an equational statement is derivable in the graphical calculus if and only if it is derivable from categorical algebra [51]. We depict physical processes by boxes, where the inputs and outputs are labelled by *types* that indicate on which kind of system these boxes act cf. one qubit, several qubits, classical data etc. Sequential composition (in time) is depicted by connecting matching outputs and inputs by wires, and parallel composition (cf. tensor) by locating entities side by side e.g.

$$1_A : A \to A \qquad f : A \to B \qquad g \circ f \qquad 1_A \otimes 1_B \qquad f \otimes 1_C \qquad f \otimes g \qquad (f \otimes g) \circ h$$

for $g : B \to C$ and $h : E \to A \otimes B$ are respectively depicted as:



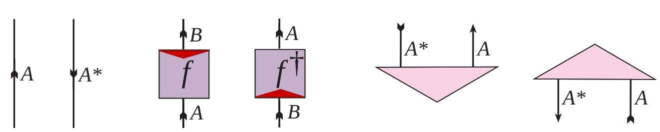
— i.e. the 'upward' vertical direction represents progress of time. A special role is played by boxes with either no input or no output, respectively called *states* and *costates* (cf. Dirac's kets and bras) which we depict by triangles. Finally, we also need to consider diamonds which arise by post-composing a state with a matching costate (cf. inner-product or Dirac's bra-ket):



that is, syntactically,

$$\psi : \mathrm{I} \to A \qquad \pi : A \to \mathrm{I} \qquad \pi \circ \psi : \mathrm{I} \to \mathrm{I}$$

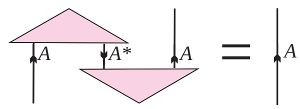
where I is the tensor unit i.e. $A \otimes \mathrm{I} \simeq A \simeq \mathrm{I} \otimes A$. Extra structure is represented by (i) assigning a direction to the wires, and reversal of this direction is denoted by $A \mapsto A^*$, (ii) allowing reversal of boxes (cf. the *adjoint* for vector spaces), and, (iii) assuming that for each type A there exists a special bipartite *Bell-state* and its adjoint *Bell-costate*:



that is, syntactically,

$$A \qquad A^* \qquad f : A \to B \quad f^\dagger : B \to A \qquad \eta_A : \mathrm{I} \to A^* \otimes A \qquad \eta_A^\dagger : A^* \otimes A \to \mathrm{I}.$$

Hence, bras and kets are adjoint and the inner product has the form $(-)^\dagger \circ (-)$ on states. The *axiom* :

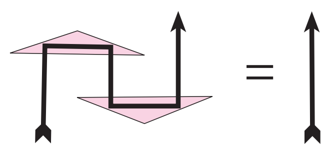


that is, syntactically,

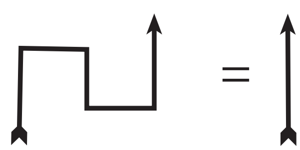
$$(\eta_{A^*}^\dagger \otimes 1_A) \circ (1_A \otimes \eta_A) = 1_A\,.$$

is the only one we impose. If we extend the graphical notation of Bell-(co)states to:

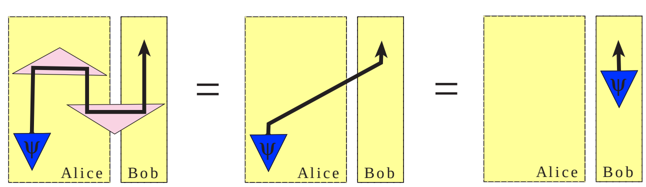
we obtain a far more lucid interpretation for the axiom:



which now tells us that we are allowed to *yank* the black line:



We called this line the *quantum information flow* [4, 15]. It is this quantum information flow which is responsible for teleporting continuous data:

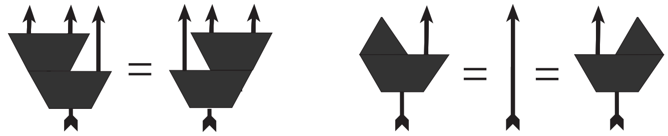


3.2. Classical data, copying and deleting

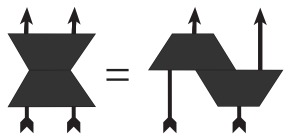
The *type* of a non-demolition quantum measurement is

$$A \to X \otimes A$$

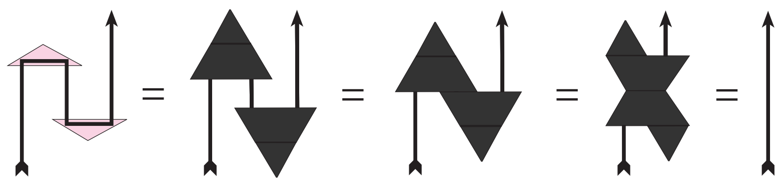
expressing that we have as input a quantum state of type A, and as output a measurement outcome of type X together with the collapsed quantum state still of type A. Following [20], we distinguish between *quantum data* A and *classical data* X by our ability to freely copy and delete the latter. Hence a *classical object* (X, δ, ϵ) is defined to be an object X together with a *copying operation* $\delta : X \to X \otimes X$ and a *deleting operation* $\epsilon : X \to I$, which satisfy some obvious behavioural constraints that capture the particular nature of these operations. Clearly they constitute a *comonoid structure*:



expressing that there are two equivalent methods to obtain three copies (=*co-associativity*), and that deleting after copying yields the identity. A more refined analysis, focussing on the fact that relative phases have no counterpart at the classical level, yields the *Frobenius identity*:



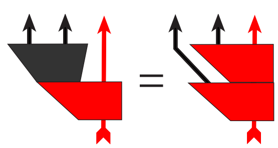
In turn, the Frobenius identity guarantees that Bell-states arise by post-composing copying with the adjoint to deletion when extended by superposition within the quantum domain:



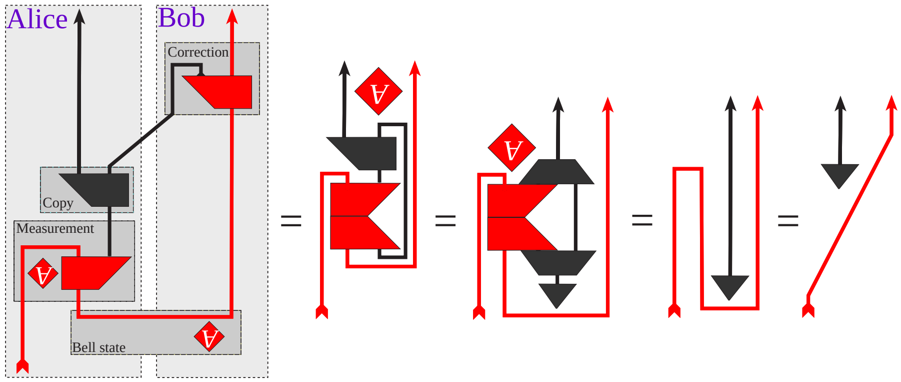
Hence a classical object structure refines the notion of strong compact closure, in that the Bell-states admit non-trivial conceptually meaningful factors. In Hilbert space terms, a classical object exactly captures a canonical (=computational) *orthonormal base* i.e. a *classical context*. In fact, both the copying operation as defined here, and its adjoint, when embedded in the category of quantum operations (i.e. extend by allowing for superposition), have appeared in mainstream QIC-literature, respectively as Harrow's *coherent bits* [33] and the *fusion operation* [52, 13] used for *cluster state preparation*.

3.3. Measurement and control structure

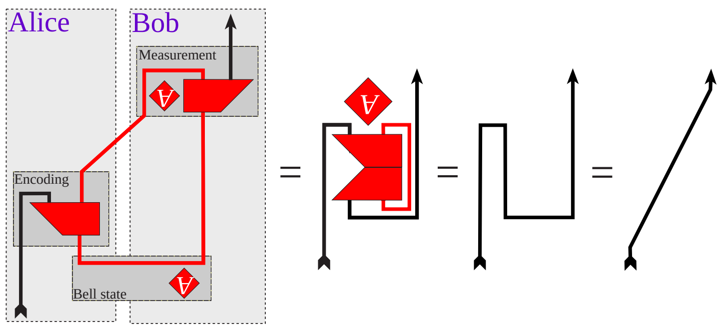
Given this *classical object type* we can then derive a notion of quantum measurement along the same lines as we derived the behavioural properties of copying and deleting. In particular, the analogue to co-associativity of the copying operation exactly turns out to be a *resource-sensitive* generalisation of *von Neumann's projection postulate* for repeated measurements (black:=classical & red:=quantum):



stating that *repeating a measurement is equivalent to copying the data obtained in its first execution*. This then allows us to define a *generalized Bell-base measurement* in abstract categorical terms by its ability to establish, for example, either *perfect teleportation*:



(where we copy the classical data obtained in the generalized Bell-base measurement before the unitary correction *consumes* one copy of it) or *dense coding*:



Surprisingly, these two tasks seem not to be equivalent in terms of the *structural resources* which they require, contrasting the usual view that these two protocols are somehow mutually dual e.g. [14, 12, 53].

TASK 1a. *We wish to explore and compare the minimal structural requirements on Bell-bases and other components in a variety of protocols, including variations on the teleportation and dense coding theme, measurement-based computational schemes etc. This will bring to bear how the distinct classical-quantum interactions translate in axiomatic terms, and will provide crucial insights and paradigmatic test-cases for the further analysis of the quantum-classical interaction outlined below.*

Making explicit the copying and deleting of data in the classical world has striking consequences: when proving theorems such as Naimark's, which involves the abstract POVMs of [19], it turns out that the categorical semantics allow the straightforward mathematical manipulations of quantum data while classical data manipulations become quite involved and seemingly ad hoc [19].

TASK 1b. *In* [12] *Braunstein et al. showed that both teleportation and dense coding can be described as "rather special" POVMs, with the role played by classical and quantum channels "interchanged". We expect that our rigid axiomatic setting can shed more light on the exact significance of "rather special", the exact structural implications of "interchanged", and we expect that our analysis would then extrapolate to a wider range of situations involving quantum-classical interaction, again providing insights and test-cases for the further analysis of the quantum-classical interaction outlined below.*

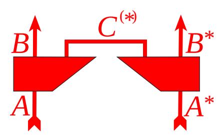
TASK 2. *Relying on standard methods from Computer Science Logic we wish to obtain a better understanding of the complexity of classical data-manipulations within the above axiomatic scheme. This* *could possibly include results on existence and properties of normal forms, confluence etc. This would contribute to a better understanding of hybrid (i.e. mixed quantum classical) computational settings.*

The copying and deleting operations of the classical object structure considered above clearly only copy and delete the classical data implicitly defined by that classical object structure, since otherwise we would be in conflict with the No-Cloning and No-Deleting theorems [54, 47] which apply to the Hilbert space formalism. Recently we showed that a strongly compact closed category must be trivial if it has an operation which either copies or deletes all quantum data too. This is a first example, drawing on ideas from Categorical Logic, of a new vein of results showing the axiomatic relationships between fundamental ideas in QM and QIC.

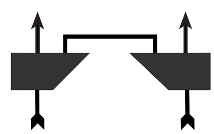
TASK 3. *We wish to refine the strongly compact closed structure even further, to see which ingredients exactly obstruct either copying and deleting, and for which structures admissibility of these two operations diverges. Natural candidate components are* $*$*-autonomy* [8], *trace structure* [38] *etc.*

3.4. Classical structure from quantum structure

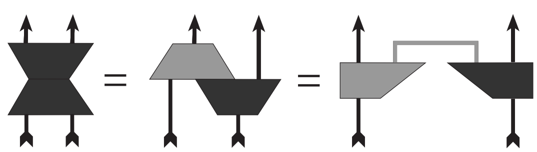
Surprisingly, given a classical object, and in particular, a copying operation, we can construct a special decoherence-operation. Following Selinger [51], given a category of pure states and pure operations, the corresponding category of mixed states and CPMs arises by taking operations of type $A \to B$ to be



and the decoherence-operation then arises as



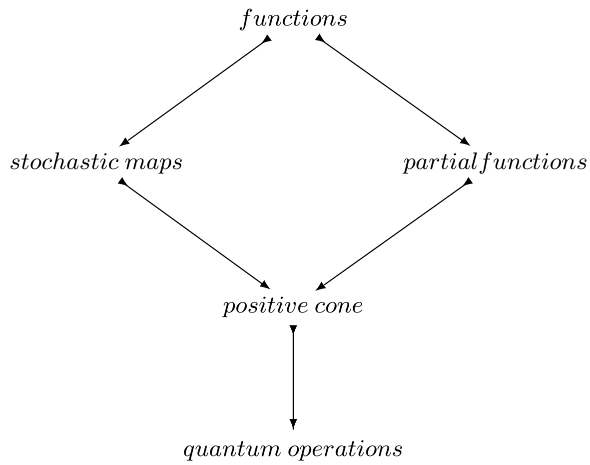
For the specific case of Hilbert space quantum mechanics this operation indeed erases the non-diagonal elements when the matrix is expressed in the base captured by the classical object structure [20]. It's *structural canonicity* is exhibited by the following provable equalities [20]:



We have the following remarkable facts concerning copying, deleting, and the decoherence-operation:

- The category of *sets and functions*, the canonical model of classical categorical logic [43], arises when we ask for operations to be preserved both under copying and deleting [20].
- We recently showed that *partial functions* arise when only requiring preservation under copying, that *stochastic maps* arise by post-composition with the decoherence-operation and requiring preservation under deletion, and that the *maps of the positive cone* arise by post-composition with the decoherence-operation.

Hence, we obtain a structural account on the passage from quantum to classical structure for arbitrary strongly compact closed categories with a classical object fine-structure:



TASK 4. *Our general setting allows for such a refined analysis of which "structural resources" distinguish the quantum structure from several instances of classical structure such as: sets and functions, relations, stochastic maps, partial functions, maps in the positive cone etc. We wish to achieve a crisp structural account on these distinct instances of classical structure as subcategories or derived categories from a given strongly compact closed category. Along the same line of thought, we wish to further unravel the structural passage from quantum to classical. For example, where does the distinction between real and complex Hilbert spaces come in, where do non-orthogonal states come in, what is the precise connection between copying, deleting, their adjoints, and the* decoherence*-operation with respect to a particular classical context, which phenomena and protocols do these enable, and where does the distinction beween relative and global phases structurally emerges etc.*

In fact, we obtain a refined account of the passage from classical (categorical) logic to a new kind of quantum logic, on which we will now go into more detail.

3.5. Quantum "hyper"-logicality

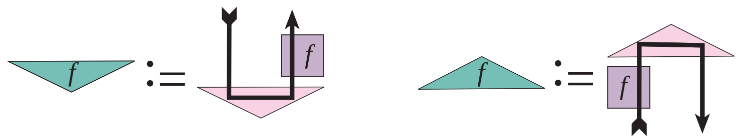
The term *quantum logic* is usually understood in connection with the 1936 Birkhoff-von Neumann proposal [11, 18, 50] to consider the (closed) linear subspaces of a Hilbert space ordered by inclusion as the formal representative for the logical distinction between quantum and classical physics. While in classical logic we have deduction, the linear subspaces of a Hilbert space form a non-distributive lattice and hence there is no obvious notion of implication or deduction. Therefore quantum logic was always seen as logically very weak, or even a non-logic. In addition, it has never given a satisfactory account of compound systems and entanglement. On the other hand, *compact closed logic* in a sense goes beyond ordinary logic in the principles it admits. Indeed, while in ordinary categorical logic "logical deduction" implies that *morphisms internalize as elements* (which above we referred to as *states*) i.e.

$$B \xrightarrow{f} C \quad \stackrel{\simeq}{\longleftrightarrow} \quad \mathrm{I} \xrightarrow{\ulcorner f \urcorner} B \Rightarrow C$$

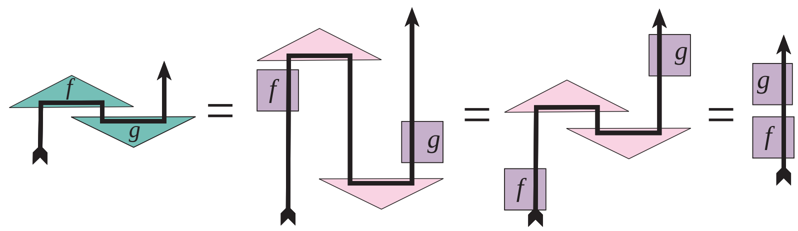
(where I is the $\otimes$-unit), in *compact closed logic* they internalize *both* as states *and* as costates i.e.

$$B \otimes C^* \xrightarrow{\llcorner f \lrcorner} \mathrm{I} \quad \stackrel{\simeq}{\longleftrightarrow} \quad B \xrightarrow{f} C \quad \stackrel{\simeq}{\longleftrightarrow} \quad \mathrm{I} \xrightarrow{\ulcorner f \urcorner} B^* \otimes C.$$

It is exactly this dual internalization which allows the defining axiom of the strongly compact closed structure to be expressed. Graphically this is witnessed by the fact that we can define states and costates:



for each operation f. While our categorical semantics are obviously compositional, both with respect to sequential composition of operations and parallel composition of types and operations, we also have some additional level of compositionality:



i.e. composition of operations *internalizes* in the behavior of entangled states and costates, and note in particular the interesting phenomenon of "apparant reversal of the causal order" which is the source of many quite mystical interpretations of quantum teleportation in terms of "traveling backward in time" — cf. [10, 42]. Indeed, while on the left, physically, we first prepare the state labeled g and then apply the costate labeled f, the global effect is *as if* we first applied f itself, and only then g. Important QIC-protocols such as logic-gate teleportation [29], an instance of general measurement-based quantum computation, are a consequence of this highly particular logical behaviour.

3.6. Breakdown of strong compact closed logic

By *closedness* we refer to the fact that operations internalize as states, by *coclosedness* we refer to the fact that they internalize as costates. One can show the following two striking results:

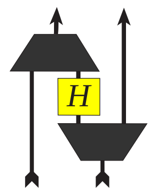
- If in the passage from quantum operations to stochastic maps (see above) closedness is preserved then the category of quantum operation has to be trivial.
- If in the passage from quantum operations to partial maps (see above) coclosedness is preserved then the category of quantum operations has to be trivial.

This means that some quantum logical properties necessarily will get lost in the passage to classical structure, and this is due to some very general abstract reasons.

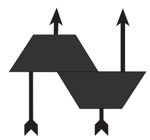
TASK 5. *We wish to track down how the logical structure evolves when passing from the quantum to the classical structure, and what the general abstract, conceptual and operational reasons are for this. We wish to relate this "change of logic" to operational capabilities: Which capabilities in terms of information processing protocols and other quantum phenomena correspond to which logical structure?*

3.7. Structural resources for quantum behavior

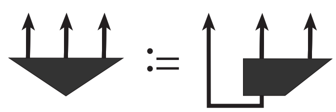
While the refinement of strong compact closure in terms of classical objects enables to stipulate notions such as Bell-bases, quantum teleportation and dense coding, it does not guaranty the existence of Bell-bases, and hence, nor does it guaranty the capability of quantum teleportation and dense coding. This initiates a general strand of research for: *What are sufficient/necessary structural resources enabling certain quantum theoretic phenomena and quantum informatic tasks?* To illustrate this idea we present some recently obtained new result. For generalised quantum teleportation as described in §3.3 it suffices that there exists a non trivial factorization of the identities $1_X = H^\dagger \circ H$ which is such that



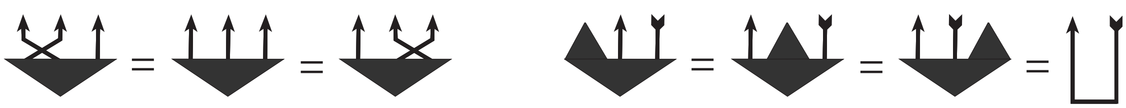
is unitary — note here that due to the Frobenius identity we have that



itself is typically not unitary but singular. In the case of the Hilbert space setting the *Hadamard-gate* is such a gate, and the resulting unitary turns out to be the *control-Z-gate* [46]. Conceptually, what the Hadamard-gate implicitly provides is the $\{|+\rangle, |-\rangle\}$-base, besides the $\{|0\rangle, |1\rangle\}$-base which is guaranteed by the classical object structure. Hence it introduces distinct bases of non-orthogonal states. Such distinct bases are, for example, also essential for the GHZ-argument [30] and secure key distribution (cf. BB84 and Ekert91). Note in this context that existence of a GHZ-state is guaranteed by the classical object structure as



for which the typical symmetric properties



arise from those of copying and deleting. It is only at this stage that we need to bring in the notion of the existence of *incompatible measurements*, while we have already unveiled quite a substantial portion of the quantum mechanical structure. c.f. the field of study called the *logic-algebraic approach* to quantum mechanics [35, 36, 18], which comprised much of the foundational quantum structural research in the previous century, typically takes the existence of non-compatible measurements as its starting point, and abstracts over most of the structure we have been considering thus far.

TASK 6. *Identify and study the necessary and sufficient structural resources enabling certain quantum theoretic phenomena and quantum informatic tasks, e.g. the GHZ-argument, secure key distribution, distinct measurement-based quantum computational schemes etc. Identify equivalence classes of such resources, and determine which ones compare (cf. weaker/stronger). Build a general (functorial) theory which relates structural resources to quantum theoretic phenomena and quantum informatic tasks.*

3.8. Physical resources for quantum informatics

Classical Shannon-style information theory has strongly influenced the research on the quantitative understanding of quantum information exchanges, resulting in a host of seemingly incomparable resources:

"qubits", "ebits" (i.e. our Bell-states) and "cbits" (i.e. our copying operation) — cf. the work by Hayden-Jozsa-Winter, Lo-Popescu Devetak-Winter and Harrow [45, 34, 23, 33]. Devetak, Harrow and Winter [22] recently initiated a symbolic calculus of *resource inequalities* to describe relations between different mechanisms for information exchanges and systematically organized the many different types of quantum resources. The simplest such inequality is $[qq] + 2[c \to c] \geq [q \to q]$ capturing teleportation: an entangled pair $[qq]$ and a channel for two classical bits is *at least as powerful as* a channel for quantum bits, and this holds *in compositional contexts*, that is, embedded situations. While this formalism is quite powerful, it is limited to describing two-party protocols at this time, and a more intuitive formalism would be highly desirable. The natural mathematical context to study compositionality and process- or protocol-equivalence is again a categorical setting, and we believe in particular that the work described above on structural resources would be pivotal for such a formalism.

TASK 7. *Develop a typed compositional, and in particular, more intuitive account of quantum informatic resources, using the results obtained on structural resources for quantum theoretic phenomena and quantum informatic task. Study its logical properties.*

4. Other related approaches

As already mentioned above, most of the quantum structural research in the previous century [35, 36, 18, and references therein] abstracts over much of the structure we aim to expose (e.g. decoherence, projection postulate, information-flow). In particular, one of the main failures of that line of research was the inability to find an abstract counterpart to the Hilbert space tensor product, while in our approach, the tensor product is the prime ingredient in terms of MCs, and our degrees of axiomatic freedom seem to account for all considerations which justified these other approaches. There are however some interesting recent developments, which are kin to our approach: D'Ariano [21] also relies on the Hilbert-Schmidt correspondence (which constitutes the core of strong compact closure), Griffiths *et al.* [31, 32] use quite similar graphical calculi, and Isham *et al.* [24, 37] rely on complementary categorical structures such as *topoi* and *M-sets* to address complementary conceptual issues.

TASK 8. *Study the connections, differences and possible cross-fertilizations between our approach and other structural approaches to the foundations of quantum mechanics.*

5. Dissemination

The results will be disseminated by means of papers co-authored by the applicants and the named researcher, and also by means of lectures, either through submitted papers or during invited addresses. Applicants Abramsky and Coecke each give, on average, about 20 invited talks a year, at Physics, Mathematics, Logic, Computer Science and Interdisciplinary-events, and both are also actively involved in the organisation of several international events. The applicants Abramsky and Coecke co-ordinate a FP6-STREP European Network "Foundational Structures for Quantum Information and Computation" involving both leading Computer Science Logic and leading QIC groups, which provides an appropriate environment both to give talks upon, and disseminate, the proposed work.

References

[1] Abramsky, S. (1993). *Computational interpretations of linear logic*. Theoretical Computer Science **111**, 3–57.

[2] Abramsky, S. (2004) *High-level methods in quantum computation and information*. Proceedings of the 19th Annual IEEE Symposium on Logic in Computer Science, pp. 410–414, IEEE Computer Science Press.

[3] Abramsky, S. (2006) *Temperley-Lieb algebra: From knot theory to logic and computation via quantum mechanics*. To appear in: Mathematics of Quantum Computing and Technology. Taylor and Francis.

[4] Abramsky, S. and Coecke, B. (2004) *A categorical semantics of quantum protocols*. Proceedings of the 19th Annual IEEE Symposium on Logic in Computer Science, pp. 415–425, IEEE Computer Science Press. `arXiv:quant-ph/0402130`

[5] Abramsky, S. and Coecke, B. (2005) *Abstract physical traces*. Theory and Applications of Categories **14**, 111–124. `www.tac.mta.ca/tac/volumes/14/6/14-06abs.html`

[6] Abramsky, S. and Jagadeesan, R. (1994) *New foundations for the geometry of interaction*. Information and Computation **111**, 53-119.

[7] Baez, J. (2004) *Quantum quandaries: a category-theoretic perspective*. In: *Structural Foundations of Quantum Gravity*, Oxford University Press. `arXiv:quant-ph/0404040`

[8] Barr, M. (1979) *$*$-Autonomous Categories*. Lecture Notes in Mathematics **752**, Springer-Verlag.

[9] Bennett, C. H., Brassard, C., Crépeau, C., Jozsa, R., Peres, A. and Wootters, W. K. (1993) *Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels*. Physical Review Letters **70**, 1895–1899.

[10] Bennett, C. H. and Wiesner, S. J. (1992) *Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states*. Physical Review Letters **69**, 2881–2884.

[11] Birkhoff, G. and von Neumann, J. (1936) *The logic of quantum mechanics*. Annals of Mathematics **37**, 823–843.

[12] Braunstein, S. L., D'Ariano, G. M., Milburn, G. J. and Sacchi, M. S. (2000). *Universal teleportation with a twist*. Physical Review Letters **84**, 3486–3489.

[13] Browne, D. E. and Rudolph, T. (2005) *Resource-efficient linear optical quantum computation*. Physical Review Letters **95**, 010501.

[14] Cerf, N. J., and Adami, C. (1997) *Negative entropy and information in quantum mechanics*. Physical review Letters **79**, 5195–5198. `arXiv: quant-ph/9512022`

[15] Coecke, B. (2005) *Quantum information-flow, concretely, and axiomatically.* In: *Proceedings of Quantum Informatics 2004*, pp. 15–29, Proceedings of SPIE Vol. 5833. `arXiv: quant-ph/0506132`

[16] Coecke, B. (2005) *Kindergarten quantum mechanics — lecture notes*. In: *Quantum Theory: Reconsiderations of the Foundations* III, pp. 81–98, AIP Press. `arXiv:quant-ph/0510032`

[17] Coecke, B. (2006) *Introducing categories to the practising physicist*. In: *What is Category Theory?* Advanced Studies in Mathematics and Logic **30**, pp.45–74, Polimetrica Publishing.

[18] Coecke, B., Moore, D.J. and Wilce, A. (2000) *Current Research in Operational Quantum Logic: Algebras, Categories, Languages*, Kluwer. Introduction at `arXiv:quant-ph/0008019`

[19] Coecke, B. and Paquette, E. O. (2006) *POVMs and Naimark's theorem without sums*. Electronic Notes in Theoretical Computer Science, to appear. `arXiv:quant-ph/0608072`

[20] Coecke, B. and Pavlovic, D. (2006) *Quantum measurements without sums*. To appear in: Mathematics of Quantum Computing and Technology. Taylor and Francis. `arXiv:quant-ph/ 0608035`

[21] D'Ariano, G. M. (2006) *How to derive the Hilber-space formulation of quantum mechanics from purely operational axioms*. `arXiv:quant-ph/0603011`

[22] Devetak, I., Harrow, A. W. and Winter, A. (2004) *A family of quantum protocols*. Physical Review Letters **93**, 230504. `arXiv:quant-ph/0307091`

[23] Devetak, I. and Winter, A. (2003) *Classical data compression with quantum side information*. Physical Review A **68**, 042301.

[24] Doering, A. and Isham, C. J. (2006) Talks at Oxford University in the Philosophy of Physics Seminar (June 15) and at Cats, Kets and Cloisters (July 21).

[25] Joyal, A. and Street, R. (1991) *The Geometry of tensor calculus* I. Advances in Mathematics **88**, 55–112.

[26] Ehrhard, T., Girard, J.-Y, Ruet, P. and Scott, Ph. (eds.) (2004) *Linear Logic in Computer Science*. Cambridge University Press.

[27] Freyd, P. and Yetter, D. (1989) *Braided compact closed categories with applications to low-dimensional topology*. Advances in Mathematics **77**, pp. 156–182.

[28] Girard, J.-Y. (1987) *Linear logic*. Theoretical Computer Science **50**, 1–102.

[29] Gottesman, D. and Chuang, I. L. (1999) *Quantum teleportation is a universal computational primitive*. Nature **402**, 390–393. `arXiv:quant-ph/9908010`

[30] Greenberger, D. M., Horn, M. A., Shimony, A. and Zeilinger, A. (1990) *Bell's theorem without inequalities.* American Journal of Physics **58**, 1131–1143.

[31] Griffiths, R. B., Wu, S., Yu, L. and Cohen, S. M. (2006) *Atemporal diagrams for quantum circuits*. Physical Review A **73**, 052309. `arXiv:quant-ph/0507215`

[32] Griffiths, R. B. (2006) *Channel kets, entangled states, and the location of quantum information*. Physical Review A **71**, 042337. `arXiv:quant-ph/0409106`

[33] Harrow, A. (2004) *Coherent communication of classical messages*. Physical Review letters **92**, 097902. `arXiv:quant-ph/0307091`

[34] Hayden, P., Jozsa, R. and Winter, A. (2002) *Trading quantum for classical resources in quantum data compression*. Journal of Mathematical Physics **43**, 4404-4444.

[35] Hooker, C. A. (1975) *The Logico-Algebraic Approach to Quantum Mechanics I: Historical Evolution*. Reidel Publishing Company.

[36] Hooker, C. A. (1979) *The Logico-Algebraic Approach to Quantum Mechanics II: Contemporary Consolidation*. Reidel Publishing Company.

[37] Isham, C. J. (2005) *A topos perspective on state-vector reduction*. `arXiv:quant-ph/0508225`

[38] Joyal, A., Street, R. and Verity, D. (1996) *Traced monoidal categories*. Proceedings of the Cambridge Philosophical Society **119**, 447–468.

[39] Kauffman, L. H. (2005) *Teleportation topology*. Optics and Spectroscopy **99**, 227–232. `arXiv:quant-ph/0407224`

[40] Kelly, G. M. and Laplaza, M. L. (1980) *Coherence for compact closed categories*. Journal of Pure and Applied Algebra **19**, 193–213.

[41] Kock, J. (2003) *Frobenius Algebras and 2D Topological Quantum Field Theories*. Cambridge University Press.

[42] Laforest, M., Laflamme, R. and Baugh, J. (2005) *Time-reversal formalism applied to maximal bipartite entanglement: Theoretical and experimental exploration*. `arXiv:quant-ph/0510048`

[43] Lambek, J. and Scott, Ph. J. (1986) *Introduction to Higher Order Categorical Logic*. Cambridge University Press.

[44] Lauda, A. T. (2005) *Frobenius algebras and planar open string topological field theories*. `arXiv:math.QA/0508349`

[45] Lo, H.-K., Popescu, S. (1999) *The classical communication cost of entanglement manipulation: Is entanglement an inter-convertible resource?* Physical Review Letters **83**, 1459–1462.

[46] Nielsen, M. A. and Chuang, L. (2000) *Quantum Computation and Quantum Information*. Cambridge University Press.

[47] Pati, A. K. and Braunstein, S. L. (2000) *Impossibility of deleting an unknown quantum state*. Nature **404**, 164–165.

[48] Penrose, R. (1971) *Applications of negative dimensional tensors*. In: *Combinatorial Mathematics and its Applications*, pp. 221–244, Academic Press.

[49] Raussendorf, R., Browne, D. E. and Briegel, H.-J. (2003) *Measurement-based quantum computation on cluster states*. Physical Review A **68**, 022312. `arXiv:quant-ph/0301052`

[50] Rédei, M. (1997) *Why John von Neumann did not like the Hilbert space formalism of quantum mechanics (and what he liked instead)*. Studies in History and Philosophy of Modern Physics **27**, 493–510.

[51] Selinger, P. (2005) *Dagger compact closed categories and completely positive maps*. Electronic Notes in Theoretical Computer Science, to appear.

[52] Verstraete, F. and Cirac, J. I. (2004) *Valence-bond states for quantum computation*. Physical Review A **70**, 060302(R).

[53] Werner, R. F. (2001) *All teleportation and dense coding schemes.* Journal of Physics A **35**, 7081–7094.

[54] Wootters, W. and Zurek, W. (1982) *A single quantum cannot be cloned*. Nature **299**, 802–803.