Causal categories: a backbone for a quantum-relativistic universe of interacting processes

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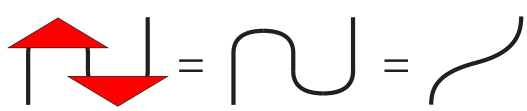
Abstract

We encode causal space-time structure within categorical process structure, by restricting the tensor to space-like separated entities, i.e. between which there is no causal flow of information. In such a causal category, a privileged set of morphisms captures the idea of an event horizon. This structure enables us to derive statements independent of specific models and detailed descriptions of processes, for example, that for a teleportation-like configuration from which the classical channel is removed, information flow from Alice to Bob cannot occur. We show that causal categories with compact structures or a dagger collapse, and define a process projector which recovers the full power of categorical quantum mechanics.

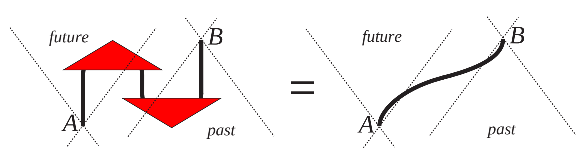
1 Introduction

Categorical quantum mechanics, as initiated in [2], and in particular its diagrammatic calculus [6, 13], enables one to reason in very intuitive and yet very abstract terms about quantum phenomena, e.g. the diagrammatic derivation within (strict) dagger compact symmetric monoidal categories of post-selected quantum teleportation:



(1)

One thinks of the objects in this category as physical systems, and of the morphisms as physical processes. Composition stands for 'after' and the tensor for 'while'. Unfortunately, Pic. (1) may be 'abused' to show that quantum theory can be used to signal:



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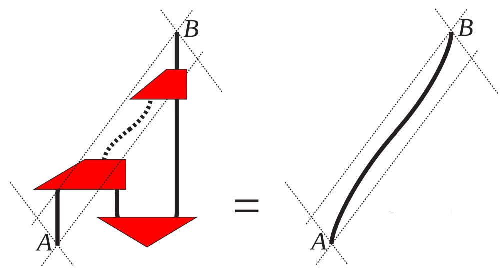
The origin of this apparent ability to signal is *post-selection*, i.e. conditioning on the outcome of the quantum measurement, which is easily seen to be a (virtual) resource that enables signaling. Hence 'quantum processes in a causal universe' cannot form a dagger or a compact symmetric monoidal category, since both the dagger and the transpose (induced by the compact structure) of a state yield a post-selected effect.

In order to retain compatibility of quantum mechanics with relativity one needs to exclude post-selection and only consider processes with a certain 'overall' probability; that is, one always needs to consider all possible measurement outcomes together. Formally, still in terms of monoidal categories, this can be achieved by using internal dagger Frobenius structures to index over all possible outcome scenarios as in [8]. In that case, an 'index-channel' prevents signaling in the teleportation protocol:



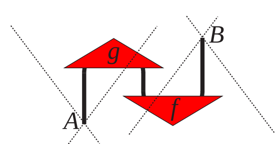
So this requires restricting admissible operations to a certain class, e.g. states, projective measurements, completely positive maps, classical communication.

What we want to do here is to implement the constraints imposed by relativity at a more basic structural level, independently of a detailed description of the specific nature of operations involved. To do this we will devise a hybrid of categorical quantum mechanics and abstract models of causality, most notably (a generalization of) causal sets [4, 14]. In the causal sets approach [4], causal structure is a partially ordered structure where $A \leq B$ stands for A being in the causal past of B. Here, rather than expressing that there *is* a causal connection, we will assert *what type* of process can take place starting in A, and ending in B.

Proof theory has seen a similar passage, from expressing that there *is* a proof which derives predicate B from predicate A, to an explicit account of the *space* of proofs which establish this, these proofs then being the morphisms in some category (see e.g. [11]). The paradigm connecting the ordered structure and the categorical is

$$A \leq B \iff \mathbf{C}(A, B) \neq \emptyset\,.$$

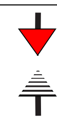
But this paradigm cannot be retained here. When performing a physical scenario:



where f and g are arbitrary physical processes, although relativity theory tells us that no information can flow from A to B, as a whole this still is a physical process. But the proof-theory paradigm tells us $A \not\leq B \Rightarrow \mathbf{C}(A,B) = \emptyset$, i.e. such a process doesn't even exist! Hence the crucial thing to do is to formally make a clear distinction between:

- the existence of a physical process ; and
- the flow of information enabled by such a process.

While the existence of a process $A \to B$ will indeed imply that $\mathbf{C}(A,B) \neq \emptyset$, the causality assertion $A \leq B$ will stand for the fact that there is a non-zero flow of information from A to B. In the diagrammatic language of symmetric monoidal categories this means that A and B are *connected*, while in the absence of information flow they will be *disconnected* :

information flow	no information flow
	

There are many motivations for this work; we mention in particular the recently emerging study of quantum computation and information in relativistic space-times [1, 3].

In fact, much of the conceptual motivation for the structures introduced in this paper was already part of the 'informal practice' in categorical quantum mechanics. So what we do here is turn these intuitions into rigorous mathematics.

2 Definition and interpretation

We use $[-]$ to denote pointwise application.

Definition 1. A *partial functor* $F : \mathbf{B} \to \mathbf{C}$ is a functor $\hat{F} : \mathbf{A} \to \mathbf{C}$, where $\mathbf{A}$ is a subcategory of $\mathbf{B}$; $\mathbf{A}$ is called the *domain of definition* of F, written $\text{dd}(F) = \mathbf{A}$, and $\mathbf{B}$ is called the *domain* of F, written $\text{dom}(F) = \mathbf{B}$. A *partial bifunctor* is a partial functor whose domain is a product category.

Definition 2. A *strict partial monoidal category* is a category $\mathbf{C}$, together with a partial bifunctor $\otimes : \mathbf{C} \times \mathbf{C} \to \mathbf{C}$, for which $\text{dd}(\otimes)$ is a full subcategory of $\text{dom}(\otimes)$, and such that there exists a *unit object* I, which is the unit of a partial monoid $(|\mathbf{C}|, \otimes, I)$:

(u1) $\forall A \in |\mathbf{C}|$, both $(A, I) \in \text{dd}(\otimes)$ and $(I, A) \in \text{dd}(\otimes)$, and

(u2) $A \otimes I = A = I \otimes A$;

(a1) $\forall A, B, C \in |\mathbf{C}|$, $(A, B), (A \otimes B, C) \in \text{dd}(\otimes)$ iff $(B, C), (A, B \otimes C) \in \text{dd}(\otimes)$,

(a2) when they exist, $A \otimes (B \otimes C) = (A \otimes B) \otimes C$, and

(a3) for any morphisms f, g, h in $\mathbf{C}$, $(f \otimes g) \otimes h = f \otimes (g \otimes h)$ when they exist.

Definition 3. A *causal category* (or *caucat*) $\mathbf{CC}$ is a partial monoidal category whose unit object I is terminal, i.e. for each object $A \in |\mathbf{CC}|$ there is a privileged morphism $\top_A : A \to I$, and for which the monoidal product, $A \otimes B$, exists iff

$$\mathbf{CC}(A, B) = [\mathbf{CC}(I, B)] \circ \top_A \quad \text{and} \quad \mathbf{CC}(B, A) = [\mathbf{CC}(I, A)] \circ \top_B\,. \tag{3}$$

We also require that each object has at least one element, i.e. $\forall A \in |\mathbf{CC}| : \mathbf{CC}(I, A) \neq \emptyset$.

Proposition 4. (i) *In a caucat morphisms* $f : A \to B$ *are 'normalized', i.e.* $\top_B \circ f = \top_A$. **(ii)** *In a caucat* $\top_I = 1_I$ *and* $\top_{A\otimes B} = \top_A \otimes \top_B$ *whenever* $A \otimes B$ *exists.*

Definition 5. A morphism $f : A \to B$ is *disconnected* if it decomposes as $f = p \circ e$ for some $e : A \to I$ and $p : I \to A$, and a hom-set $\mathbf{C}(A, B)$ is *disconnected* if it contains only disconnected morphisms.

Proposition 6. (i) *In Definition 3, Eq. (3) is equivalent to both* $\mathbf{CC}(A, B)$ *and* $\mathbf{CC}(B, A)$ *being disconnected.* **(ii)** *In a caucat,* $A \otimes B$ *exists iff* $B \otimes A$ *exists.* **(iii)** *Conditions (u1) and (a1) in the definition of partial monoidal category are implied by Eq. (3) together with the condition that if* $A \otimes B$, $A \otimes C$, $B \otimes C$ *exist then also* $A \otimes (B \otimes C)$ *exists.*

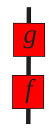
Definition 7. In a causal category:

- If $A \otimes B$ exists then we call A and B *space-like separated*.
- If $\mathbf{CC}(A, B)$ is connected while $\mathbf{CC}(A, B)$ isn't then A *causally precedes* B.
- If $\mathbf{CC}(A, B)$ and $\mathbf{CC}(B, A)$ are connected then A and B are *causally intertwined*.

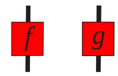
Remark 1. Since in non-degenerate situations identities are connected, the tensor of A with itself will typically not exist.

In the graphical representation of monoidal categories, objects are represented by wires, and morphisms by boxes with the input and output wires determined by its type. Physically, we think of objects/wires as systems, and of morphisms/boxes as processes, those of type $I \to A$ being states. There are two manners of composing processes:

- The *dependent*, or *causal*, or *connected* composition of processes $f : A \to B$ and $g : A \to B$ is $g \circ f : A \to C$, which as usual can be depicted as:



- The *independent*, or *acausal*, or *disconnected* composition of processes $f : A \to B$ and $g : C \to D$ is $f \otimes g : A \otimes C \to B \otimes D$, and is depicted as:



A disconnected morphism $f = p \circ \top_A : A \to B$ is depicted as:

So by requiring Eq. 3 to hold in a caucat, the graphical calculus translates causal dependencies to topological connectedness on-the-nose.

3 Examples and constructions

Example 1. Each category induces a caucat by freely adjoining a monoidal unit; we could call such a degenerate caucat *purely temporal*. Each monoid induces also another caucat with the monoid as the tensor by freely adjoining a unique morphism for each ordered pair of objects; we could call such a degenerate caucat *purely spatial*.

Example 2. Given a poset P and a symmetric strict monoidal category $\mathbf{C}$,we construct a caucat as follows. Denote the restriction of $CPM(\mathbf{C})$ [13] to normalized morphisms by $CPM_\perp(\mathbf{C})$. In this category I is terminal: if $\pi : A \to I$, then $\pi = 1_I \circ \pi = \top_I \circ \pi = \top_A$. Define $R(P) \subseteq 2^R$ to consist of those subsets $a \subseteq P$ satisfying

$$x, y \in a \Rightarrow (x \not\leq y \wedge y \not\leq x)\,,$$

which we can call *spatial slices*. When tensoring we need to keep track of the space-time point an object in $\mathbf{C}$ is assigned to, and we therefore define our objects as follows:

- Objects are either sets of pairs $\{(A_i, x_i)\}_{i\in\mathcal{I}}$ with $A_i \in |\mathbf{C}| \setminus \{I\}$ for all $i \in \mathcal{I}$ and $\{x_i\}_{i\in\mathcal{I}} \in R(P)$, or $(I, \emptyset)$. For $\{(A_i, x_i)\}_{i\in\mathcal{I}}$ and $\{(B_j, y_j)\}_{j\in\mathcal{J}}$ the tensor is the union and exists provided that:

  $$\{x_i\}_{i\in\mathcal{I}} \cap \{y_j\}_{j\in\mathcal{J}} = \emptyset \qquad \{x_i\}_{i\in\mathcal{I}} \cup \{y_j\}_{j\in\mathcal{J}} \in R(P)\,,$$

  and we set $\{(A_i, x_i)\}_{i\in\mathcal{I}} \otimes (I, \emptyset) := \{(A_i, x_i)\}_{i\in\mathcal{I}}$.

We can now define hom-sets as follows. For states we set:

$$\mathbf{CC}(\mathbf{C}, P)\big((I, \emptyset), \{(A_i, x_i)\}_{i\in\mathcal{I}}\big) := CPM_\perp(\mathbf{C})(I, \otimes_{i\in\mathcal{I}} A_i)$$

where due to the fact that $\mathbf{C}$ is symmetric the order of tensoring is not essential, just a matter of bookkeeping. For general morphisms we set:

$$\mathbf{CC}(\mathbf{C}, P)\big(\{(A_i, x_i)\}_{i\in\mathcal{I}}, \{(B_j, y_j)\}_{j\in\mathcal{J}}\big) :=$$

$$\left\{ \sigma' \circ \Big(f \otimes (p \circ \top_{\otimes_{i\in\mathcal{I}\setminus\mathcal{I}'} A_i}) \Big) \circ \sigma \;\middle|\; \begin{array}{c} \mathcal{I}' \subseteq \mathcal{I}, \mathcal{J}' \subseteq \mathcal{J} \\ \{x_i\}_{i\in\mathcal{I}'} \sqsubseteq \{y_j\}_{j\in\mathcal{J}'} \\ p \in CPM_\perp(\mathbf{C})(I, \otimes_{j\in\mathcal{J}\setminus\mathcal{J}'} B_j) \\ f \in CPM_\perp(\mathbf{C})(\otimes_{i\in\mathcal{I}} A_i, \otimes_{j\in\mathcal{J}} B_j) \end{array} \right\}$$

where σ and σ' are the unique symmetry isomorphisms that re-order the objects of $\mathbf{C}$ to match the ordering of $\mathcal{I}$ and $\mathcal{J}$, and $X \sqsubseteq Y$ means:

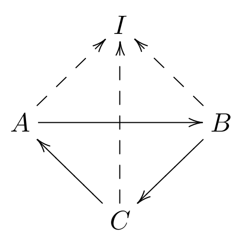
$$\forall x \in X, \forall y \in Y : x \leq y\,.$$

Finally, we close under tensoring, that is, for all $\{(A_i, x_i)\}_{i\in\mathcal{I}}$ and $\{(A'_i, x'_i)\}_{i\in\mathcal{I}'}$ for which the tensor exists, and all $\{(B_j, y_j)\}_{j\in\mathcal{J}}$ and all $\{(B'_j, y'_j)\}_{j\in\mathcal{J}'}$ for which the tensor exists, if $f \in \mathbf{CC}(\{(A_i, x_i)\}_{i\in\mathcal{I}}, \{(B_j, y_j)\}_{j\in\mathcal{J}})$ and $f' \in \mathbf{CC}(\{(A'_i, x'_i)\}_{i\in\mathcal{I}'}, \{(B'_j, y'_j)\}_{j\in\mathcal{J}'})$ then $f \otimes f' \in \mathbf{CC}(\{(A_i, x_i)\}_{i\in\mathcal{I}} \cup \{(A'_i, x'_i)\}_{i\in\mathcal{I}'}, \{(B_j, y_j)\}_{j\in\mathcal{J}} \cup \{(B'_j, y'_j)\}_{j\in\mathcal{J}'})$.

4 Causal structure

Definition 8. The *causal structure* of a caucat is a directed graph $\mathcal{G}$, whose vertices G are the objects of $\mathbf{CC}$, and an edge $(A, B) \in G \times G$ exists iff $\mathbf{CC}(A, B)$ is connected.

Example 3. We can define a caucat $\mathbf{CC}$, shown in the diagram below, whose causal structure is the directed graph of a '3-loop'. The caucat can be obtained from the graph by freely adjoining the monoidal unit.



In this caucat, the only pairs of objects for which $\otimes$ exists are $A \otimes I$, $I \otimes A$ and $I \otimes I$. The restrictions on the morphisms are as follows. Firstly, for related pairs (A, B), we have $\mathbf{CC}(A, B) \neq [\mathbf{CC}(I, B)] \circ \top_A$. Secondly, we must ensure that any pair of composable connected morphisms f, g, the composite $g \circ f$ is disconnected. This is allowed, since there is nothing in the definition of a caucat that forces the composition of connected morphisms to be a connected morphisms. That is, connectedness of hom-sets is not transitive.

Although such causal structures are not studied in, e.g. the causal set programme, they have recently been used to gain insights into the nature of quantum computation, and its relation to causality [1, 3], so it is advantageous to be able to accommodate them.

Example 4. The causal structure of the caucat $\mathbf{CC}(\mathbf{C}, P)$ in Example 2 is simply the set $R(P)$. As discussed above, the relation of causal precedence on $R(P)$ is reflexive, but not necessarily transitive, although the underlying relation on P is transitive.

5 Incompatibilities

In this section we show that some basic aspects of categorical quantum mechanics, involving identical and isomorphic objects (which allows to identify systems of the same kind), compactness and dagger structure, are incompatible with the caucat structure! (But we will re-instate the expressiveness of categorical quantum mechanics in the next section.)

Proposition 9. *Given a caucat* $\mathbf{CC}$*, suppose that* A *causally precedes* B*, or that* A *and* B *are space-like separated. Then if* $A \cong B$*, it follows that* $A \cong I \cong B$*.*

Proposition 10. *If* $A \otimes A$ *exists then* 1_A *is disconnected.*

Proposition 11. *For compact objects* A *in caucats* 1_A *is disconnected, and morphisms between compact objects are disconnected. Hence for a compact subcategory of a caucat all morphisms are disconnected.*

Proposition 12. *In a caucat with a dagger functor every object has only one state.*

6 Process structure

Recall that a *section* for a functor $F : \mathbf{D} \to \mathbf{C}$ is a functor $G : \mathbf{C} \to \mathbf{D}$ such that $F \circ G = 1_{\mathbf{C}}$. By a *partial section* for F we mean a partial functor $G : \mathbf{C} \to \mathbf{D}$ such that $F \circ G = 1_{\mathrm{dom}(G)}$. Below, by a monoidal functor we will mean one which (strictly) preserves the tensor whenever it exits, and let $\mathbf{C}$ and $\mathbf{D}$ be strict partial monoidal categories.

Definition 13. A *partial monoidal section* for a monoidal functor $F : \mathbf{D} \to \mathbf{C}$ is a partial section $G : \mathbf{C} \to \mathbf{D}$ which is a monoidal functor on its domain of definition, and this domain of definition may itself only inherit part of the monoidal structure of $\mathbf{C}$.

Definition 14. A *process projector* F for a caucat $\mathbf{CC}$ is a faithful monoidal functor $F : \mathbf{CC} \to \mathbf{C}$, for which $F^{-1}(I) = \{I\}$, where $\mathbf{C}$ is a monoidal category, called the *process category*. A *process embedding* $G : \mathbf{C} \to \mathbf{CC}$ is any partial monoidal section for F.

Proposition 15. *A process projector preserves normalization, a process embedding embeds only normalized processes, a process embedding preserves the monoidal unit, and both process embeddings and process projectors preserve (dis)connectedness.*

If we think of a morphism in a caucat as a process ‘embedded in space-time’, then a process projector forgets the space-time structure, and returns the morphism to the arena of categorical quantum mechanics, a category $\mathbf{C}$. In other words, the process projector forgets the causal structure, which amounts to forgetting the partiality of the tensor.

Dually, the process embeddings formalize the idea of placing processes in space-time. There will be many such embeddings, corresponding to different placements in space-time. Note here that the monoidal structure of the domain of definition $\mathcal{D}$ of a process embedding will in general not be total; totality would force all objects in $\mathcal{D}$ to be spacelike separated when embedded in the caucat, which is impossible for any connected morphism.

Example 5. A canonical process projector for the caucat $\mathbf{CC}(\mathbf{C}, P)$ in Example 2 is the monoidal functor $F : \mathbf{CC}(\mathbf{C}, P) \to \mathbf{C}$ which forgets causal structure.

While, as we saw in Section 5, a caucat $\mathbf{CC}$ can’t be dagger compact nor accommodate isomorphic non-trivial objects, the category can $\mathbf{C}$ can, and hence it retains full expressiveness of categorical quantum mechanics, enabling one to define concepts such as classical data, measurement, unitarity, complementarity, ... as in [6].

Definition 16. Let $\mathbf{CC}$ be a caucat together with a process projector $F : \mathbf{CC} \to \mathbf{C}$. A *protocol* is a strict partial monoidal subcategory $\mathcal{G}$ of $\mathbf{C}$ for which there exists at least one process embedding with $\mathcal{G}$ as its domain of definition.

For a given a process category $\mathbf{C}$ and caucat $\mathbf{CC}$, any process embedding $G : \mathbf{C} \to \mathbf{CC}$ defines a protocol, simply as its domain of definition. The usual conception of a protocol is indeed a set of operations taken from a 'library', together with a specification of how they are to be composed together. As a whole this protocol may be realised at different locations and times, and these different realisations correspond to different possible embeddings.

Example 6. Consider the simple protocol of a system evolving 'unchanged'. When embedding a morphism $f : A_1 \to A_2$ in the caucat using a process embedding G, if $A_1 = A_2$, then G will embed both objects on a single object in the caucat, so f does not correspond to a single system moving through space-time. Therefore, rather than equal, we need to take $A_1$to be isomorphic to A_2 in $\mathbf{C}$. Isomorphisms in the process category can also be used to state that two systems are of the same kind.

Example 7. For a process projector $F : \mathbf{CC} \to \mathbf{C}$ with $\mathbf{C}$ dagger compact, it is natural to put certain requirements on how the dagger compact structure of $\mathbf{C}$ interacts with

$$F\top_A : FA \to I\,.$$

Such a correspondence has already been proposed earlier by one of the authors [5, 7], as an axiomatization of Selinger's CPM-construction [13], namely:

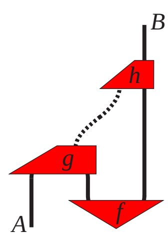
$$\top_B \circ f_{pure} = \top_{B'} \circ g_{pure} \iff f_{pure}^\dagger \circ f_{pure} = g_{pure}^\dagger \circ g_{pure}\,, \tag{4}$$

where $f_{pure} : A \to B$ and $g_{pure} : A \to B'$ live in a subcategory of 'pure morphisms' and from which in particular the following fact on normalization follows:

$$\top_B \circ f_{pure} = \top_A \iff f_{pure}^\dagger \circ f_{pure} = 1_A\,, \tag{5}$$

7 Teleportation requires classical communication

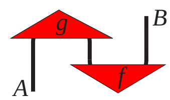
A T-*protocol* is a protocol $\mathcal{G}_T$ for a given process category $\mathbf{C}$ that contains all the ingredients (i.e. f, g, h, isomorphisms, composition, tensor) that make up following morphism:

$s =$  $: A \to B\,,$

and we moreover ask this protocol to be minimal, i.e. there is no other protocol contained in $\mathcal{G}_T$ that also contains all these ingredients. A standard teleportation protocol is an example of this, where g is a measurement and h is the unitary correction.

We now define a similar protocol, but from which we removed the wire that in teleportation corresponds to classical communication, or equivalently, we take it to be a disconnected morphism. Since the morphism h is now not 'causally constrained' by the classical wire, we can use bifunctoriality and 'absorb' h within f as $(1_C \otimes h) \circ f$.

Concretely, a T^-*-protocol* is a minimal protocol $\mathcal{G}_{T^-}$ of $\mathbf{C}$ containing that makes up:

$t =$  $: A \to B$.

where the types are:

$$A = A \otimes I \to A' \otimes C \otimes B' \to I \otimes B = B\,,$$

and the compositions are:

$$t = (g \otimes 1_{B',B}) \circ (1_{A,A'} \otimes f)\,. \tag{6}$$

with $1_{B',B} : A \cong A'$ and $1_{A,A'} : B \cong B'$ proper isomorphisms in $\mathbf{C}$ (cf. Example 6).

Theorem 17. *For a T^--protocol $\mathcal{G}_{T^-}$ the morphism t is always disconnected.*

Proof. By bifunctoriality in $\mathcal{G}_{T^-}$, we write Eq. 6 as follows:

$$t = (1_I \otimes 1_{B',B}) \circ (g \otimes 1_{B'}) \circ (1_{A'} \otimes f) \circ (1_{A,A'} \otimes 1_I) = 1_{B',B} \circ t' \circ 1_{A,A'}$$

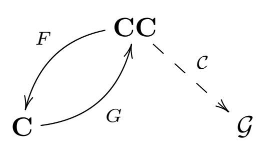
Since any embedding $G : \mathbf{C} \to \mathbf{CC}$ is monoidal, $GA' \otimes GC \otimes GB'$ exists in $\mathbf{CC}$; hence $GA' \otimes GB'$ also exists. Therefore $Gt' : GA' \to GB'$ is disconnected; and by definition of the process projector F, we have that $FGt' = t'$ is also disconnected: $t' = p' \circ e'$, for some morphisms $p' : I \to B'$ and $e' : A' \to I$. So

$$t = 1_{B',B} \circ t' \circ 1_{A,A'} = 1_{B',B} \circ p' \circ e' \circ 1_{A,A'} = p \circ e$$

for some morphisms $p : I \to B$ and $e : A \to I$; hence t is disconnected. □

8 Conclusion

The following diagram shows how caucats relate to the key ancillary structures:



For a caucat $\mathbf{CC}$, a process projector $F : \mathbf{CC} \to \mathbf{C}$ restores categorical quantum mechanics for a caucat; a process embedding $G : \mathbf{C} \to \mathbf{CC}$ places a protocol in space-time; and a graph isomorphism $\mathcal{C} : \mathbf{CC} \to \mathcal{G}$ extracts the causal structure as a directed graph.

A possible future direction would be to explore information-processing when closed time-like curves are available [1, 3]. In a more foundational direction, various authors have recently proposed operational frameworks for quantum theory, often motivated by the aim of better reconciling quantum theory and relativity [10, 9]. One aspect of this could be to isolate quantum theory in the space of no-signaling probabilistic theories, and the framework of caucats would seem to be well suited for this.

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