Kinematic Noise

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Abstract

In this paper we introduce the notion of *kinematic noise* for quantum communications. That is, the noise introduced by the relative state of motion of the transmitter (Alice) and the receiver (Bob). We show that, as seen by inertial observers, relative motion introduces rotations in the quantum state used to represent a qubit. Depending on the relative motion between Alice and Bob, kinematic noise will tend to decrement the capacity of a simple send/receive quantum communication channel. We also show that there exists a privileged basis to encode classical information that is resilient to the effects of kinematic noise. Furthermore, even though it can be proved that kinematic noise does not affect the degree of entanglement, it does affect the capacity of the teleportation channel. However, in this case, there appears to be no privileged basis or entangled state that helps to shield the effects of kinematic noise.

1 Introduction

In this paper we will explore the noise that is induced in quantum channels due to the relative state of motion of the transmitter (Alice) and the receiver (Bob). As such, we define *kinematic noise* as those relativistic effects that affect the capacity of a quantum channel. Some simplifying assumptions have been taken into consideration. First of all, we presume that Alice and Bob are in inertial frames without acceleration or gravitational fields. Second, we assume that Alice and Bob exclusively use massive spin-1/2 particles to encode qubits.

2 Relativistic Quantum Information

Let us assume the simple inertial frame for Alice and Bob that is shown in Figure 1. Alice is moving with speed $\mathbf{v}_x$ with respect to Bob. At some point in time, Alice sends a qubit to Bob with speed $\mathbf{v}_z$. The qubit in question is represented by a massive spin-1/2 particle. The Dirac state of a massive spin-1/2 particle with momentum p and spin up is represented by $u(p, +1/2)$, while the same particle with spin down is given by $u(p, -1/2)$. We make strong emphasis that these Dirac states are momentum-spin eigenstates. The momentum of

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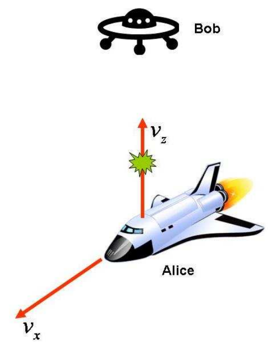

Figure 1: Inertial frames for Alice and Bob. Alice is moving with speed v_x with respect to Bob. At some point in time she sends to Bob a massive spin-1/2 particle with relative speed v_z.

the particle as measured by Bob is found by applying the Lorentz transformation defined by:

$$p \longrightarrow \Lambda p \tag{1}$$

where p is a 4-vector with spatial components on the "z" direction and Λ is a boost on the "x" direction.

The transformation of the Dirac states under a Lorentz transformation is well understood [1, 2]. The unitary transformation $\hat{R}(\Lambda p)$ on the Dirac states associated to the Lorentz transformation Λ is found to be:

$$\hat{R}(\Lambda)u(p,1/2) = \cos\left(\frac{\Omega_p}{2}\right)u(\Lambda p,1/2) + \sin\left(\frac{\Omega_p}{2}\right)u(\Lambda p,-1/2) \tag{2}$$

That is, in Bob's frame the particle has momentum Λp and the spin projection is rotated by an angle Ω_p. And a similar result is found if the particle sent by Alice has the spin down:

$$\hat{R}(\Lambda)u(p,-1/2) = -\sin\left(\frac{\Omega_p}{2}\right)u(\Lambda p,1/2) + \cos\left(\frac{\Omega_p}{2}\right)u(\Lambda p,-1/2) \tag{3}$$

If we define:

$$\tanh(-\eta_i) = v_i/c \tag{4}$$

then the magnitude of Ω_p is given by:

$$\tan\Omega_p = \frac{\sinh\eta_x\ \sinh\eta_z}{\cosh\eta_x + \cosh\eta_z} = \frac{\sqrt{(v_z/c)^2}\ \sqrt{(v_x/c)^2}}{\sqrt{1-(v_z/c)^2}\ +\ \sqrt{1-(v_x/c)^2}} \tag{5}$$

If both relevant speeds are of comparable magnitude, $v_x \approx v_z \approx v$, then we can estimate the magnitude of Ω_p as:

$$\tan\Omega_p \approx \frac{1}{2}\ \left(\frac{v}{c}\right)^2 \frac{1}{\sqrt{1-(v/c)^2}} \tag{6}$$

In the case where Alice sends Bob a particle with the spin in the up position, Bob will observe the following according to the magnitude of the relative speeds. In the non-relativistic case:

$$v \ll c \Rightarrow \tan\Omega_p \approx 0 \Rightarrow \Omega_p \approx 0 \tag{7}$$

which means that Bob will see the particle with momentum Λp and with a negligible change in the spin projection:

$$u_{Bob}^{v\ll c} = \hat{R}(\Lambda)u(p,1/2) \approx u(\Lambda p, 1/2) \tag{8}$$

On the other hand, in the ultra-relativistic limit:

$$v \approx c \Rightarrow \tan\Omega_p \approx \infty \Rightarrow \Omega_p \approx \pi/2 \tag{9}$$

and Bob will observe the particle with momentum Λp and a spin projection state with nearly equal probability of being up or down:

$$u_{Bob}^{v\approx c} = \hat{R}(\Lambda)u(p,1/2) \approx \frac{1}{\sqrt{2}}\left(u(\Lambda p, 1/2) + u(\Lambda p, -1/2)\right) \tag{10}$$

One important observation stems from Equation (5). The non-trivial spin rotation is the result of two non-parallel Lorentz transformations. If both Lorentz transformations are parallel to each other, then the rotation angle is zero regardless of the magnitude of the speeds[1].

3 Send and Receive Quantum Channel

Alice and Bob can use qubits to steganographically transmit classical information. To this end they choose an orthonormal basis of the form:

$$|\tilde{0}\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \qquad\qquad |\tilde{1}\rangle = -\beta^*\,|0\rangle + \alpha^*\,|1\rangle \tag{11}$$

where the "tilde" in $|\tilde{0}\rangle$ and $|\tilde{1}\rangle$ is used to represent the *logical states* "0" and "1" respectively, while $|0\rangle$ and $|1\rangle$ represent the *physical states* of the particle.

Suppose Alice sends a logical "0" encoded in $|\tilde{0}\rangle$, and Bob receives the state $\hat{U}\,|\tilde{0}\rangle$. The operator $\hat{U}$ may represent a noisy environment, relativistic corrections, or any other factor that may affect the measurement result. Bob performs a measurement in the steganographic basis and will obtain either a "0" or a "1". Then, the probability that Alice sends state "i" and Bob measures state "j" is given by:

$$P(i|j) = \left|\langle\tilde{i}|\hat{U}\,|\tilde{j}\rangle\right|^2 \tag{12}$$

The capacity of the channel is given by:

$$\mathcal{C} = 1 + \frac{1}{2}\sum_{i,j=0}^{1} P(i|j)\log P(i|j) \tag{13}$$

[1]This behavior is due to the fact that a single Lorentz transformation merely describes the dynamics of the particle [3]. In this case, the application of a single Lorentz transformation will lead to the Dirac equation.

For binary symmetric channels the success probability P and the error probability ϵ are:

$$P \equiv P(0|0) = P(1|1) \qquad \epsilon \equiv P(0|1) = P(1|0) \qquad \Longrightarrow P + \epsilon = 1 \tag{14}$$

and the channel capacity is given by:

$$\mathcal{C} = 1 + P \log P + \epsilon \log \epsilon \tag{15}$$

3.1 Relativistic Communications

Let us suppose now that Alice and Bob are on the inertial frames described in Figure 1. For simplicity, we can also assume that there is no environmental noise affecting the state of the qubits. In matrix form, the effect of the Lorentz transformation on a qubit is given by:

$$|0\rangle \longrightarrow \hat{R}\,|0\rangle = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} = a\,|0\rangle + b\,|1\rangle \tag{16}$$

and:

$$|1\rangle \longrightarrow \hat{R}\,|1\rangle = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -b \\ a \end{pmatrix} = -b\,|0\rangle + a\,|1\rangle \tag{17}$$

where we have defined:

$$a \equiv \cos(\Omega_p/2) \qquad b \equiv \sin(\Omega_p/2) \qquad \Longrightarrow a^2 + b^2 = 1 \tag{18}$$

In Alice's frame of reference, she prepares either $|\tilde{0}\rangle_a$ or $|\tilde{1}\rangle_a$. However, Bob will measure states rotated by an angle Ω_p:

$$\begin{aligned} |\tilde{0}\rangle_b = \hat{R}\,|\tilde{0}\rangle_a &= \alpha(a\,|0\rangle + b\,|1\rangle) + \beta(-b\,|0\rangle + a\,|1\rangle) \\ &= (\alpha a - \beta b)\,|0\rangle + (\alpha b + a\beta)\,|1\rangle \\ |\tilde{1}\rangle_b = \hat{R}\,|\tilde{1}\rangle_a &= -\beta^*(a\,|0\rangle + b\,|1\rangle) + \alpha^*(-b\,|0\rangle + a\,|1\rangle) \\ &= -(\beta^* a + \alpha^* b)\,|0\rangle - (\beta^* b - \alpha^* a)\,|1\rangle \end{aligned} \tag{19}$$

Furthermore, if we assume that both boosts have the same magnitude in the speed parameter (but different direction), $v = v_x = v_z$, then the rotation angle Ω_p is defined by:

$$\tan \Omega_p = \frac{1}{2}\,\left(\frac{v}{c}\right)^2 \frac{1}{\sqrt{1 - (v/c)^2}} \tag{20}$$

As made evident in the above expressions, the states measured by Bob will carry *kinematic noise* which makes them different from the states sent by Alice. We use the term "kinematic" because it is produced exclusively by the kinematic relative state of motion of Alice and Bob. This kinematic noise is parameterized by a and b. However, there is only one free parameter when we consider the normalization condition $a^2 + b^2 = 1$.

The probability that Bob measures "0" when Alice sends a "0" is:

$$P(0|0) = |_a\langle\tilde{0}|\tilde{0}\rangle_b|^2 = |\alpha^*(\alpha a - \beta b) + \beta^*(\alpha b + a\beta)|^2 = a^2 + 4b^2\delta^2 \tag{21}$$

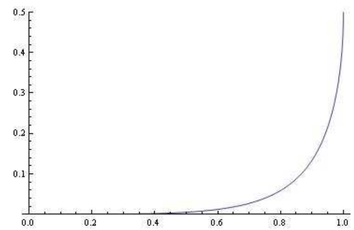

Figure 2: Plot of error probability vs. the speed ratio v/c

and the probability that Bob measures "1" when Alice sends a "1" is:

$$P(1|1) = |_a\langle\tilde{1}|\tilde{1}\rangle_b|^2 = |\beta(\beta^* a + \alpha^* b) - \alpha(\beta^* b - \alpha^* a)|^2 = a^2 + 4b^2\delta^2 \quad (22)$$

where we have defined:

$$\delta \equiv Im(\alpha^*\beta) \quad (23)$$

The error probabilities are given by:

$$P(0|1) = P(1|0) = 1 - a^2 - 4b^2\delta^2 = b^2 - 4b^2\delta^2 = b^2(1 - 4\delta^2) \quad (24)$$

Thus, this is a binary symmetric channel. Furthermore, ϵ, the error probability due to kinematic noise can be expressed as:

$$\epsilon = b^2(1 - 4\delta^2) = \left(1 - 4\, Im^2(\alpha^*\beta)\right)\left(\frac{1}{2} - \frac{\sqrt{1-(v/c)^2}}{2-(v/c)^2}\right) \quad (25)$$

Therefore, the total amount of error due to kinematic noise depends not only on the kinematic parameter v, but also on (α, β), the basis used to represent the classical information.

As an example, suppose that α and β are real numbers. Then, the probability of error is given by:

$$\epsilon = \frac{1}{2} - \frac{\sqrt{1-(v/c)^2}}{2-(v/c)^2} \quad (26)$$

As it can be seen in Figure 2, the error probability is extremely small unless we consider the ultra-relativistic case where $v \approx c$. For example, if the relative speed $v \approx 0.9c$, then we get an error probability of about 0.13. If the relative speed between Bob and Alice is about 100 times the speed of sound ($3 \times 10^2\ m/s$), then $(v/c) \approx 10^{-4}$, and the associated error probability would be $\approx 10^{-17}$.

Therefore, in the case where α and β are real numbers, in the non-relativistic regime $v \ll c$ and as a consequence $\epsilon \approx 0$, the change to the capacity is negligible. On the other hand, in the ultra-relativistic regime $v \approx c$ and as a consequence $\epsilon \approx 1/2$, the channel capacity is reduced to zero.

3.2 Relativistic Fixed Points

Because the error probability depends on the basis parameters α and β, as well as the speed v, in this section we consider the possibility of finding a basis that is resilient to the effects of kinematic noise.

Because of the normalization condition between a and b, Equation (16), it is clear that the probability of error, Equation (23), is equal to zero when:

$$\delta = \pm\frac{1}{2} \Rightarrow Im(\alpha^*\beta) = \pm\frac{1}{2} \tag{27}$$

In particular, this clearly happens when:

$$(\alpha, \beta) = \frac{1}{\sqrt{2}}(1, i) \tag{28}$$

and as a consequence, this basis does not feel the effects of kinematic noise.

The previous result can be better understood by studying the eigenvectors and eigenvalues of the Lorentz transformation. The two eigenvalues of $\hat{R}$ are given by:

$$\lambda = a \pm ib \tag{29}$$

with associated eigenvectors:

$$\begin{aligned} \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} &= (a - ib)\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{\lambda_0}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \\ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \end{pmatrix} &= (a + ib)\frac{1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \end{pmatrix} = \frac{\lambda_1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \end{pmatrix} \end{aligned} \tag{30}$$

However:

$$|a \pm ib|^2 = a^2 + b^2 = 1 \tag{31}$$

and therefore the eigenvalue is just a global phase of unit length which does not affect calculations of probabilities. Indeed, if we use the basis defined by Equation (26) we get:

$$P(0|0) = \left|{}_a\langle\tilde{0}|\tilde{0}\rangle_b\right|^2 = \left|{}_a\langle\tilde{0}|\ \hat{R}\ |\tilde{0}\rangle_a\right|^2 = |\lambda_0|^2\ \left|{}_a\langle\tilde{0}|\tilde{0}\rangle_a\right|^2 = 1 \tag{32}$$

and similarly for $P(1|1)$ with eigenvalue λ_1, which renders $\epsilon = 0$ in this basis.

The physical interpretation of this fixed point is easy to understand. Indeed, the two Lorentz transformations involved are along the x and z axis. Under Lorentz transformations, physical variables such as length are contracted only across the axis parallel to the boost. Therefore, these Lorentz transformations affect the spin projection but only along the x and z axis leaving the y component invariant. It is easy to confirm that this is the case:

$$\hat{\sigma}_y\,|\tilde{0}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} = +\,|\tilde{0}\rangle \tag{33}$$

and also:

$$\hat{\sigma}_y\,|\tilde{1}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} i \\ 1 \end{pmatrix} = -\frac{1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \end{pmatrix} = -\,|\tilde{1}\rangle \tag{34}$$

And therefore, the Lorentz invariant basis corresponds to the $\hat{\sigma}_y$ basis.

4 The Teleportation Channel

As in the previous example, it is possible to steganographically teleport classical information by using an orthonormal basis to represent “0” and “1”, Equation (11), and access to an imperfectly entangled state:

$$|\Phi\rangle = \eta_{00}\,|0\rangle\,|0\rangle + \eta_{01}\,|0\rangle\,|1\rangle + \eta_{10}\,|1\rangle\,|0\rangle + \eta_{11}\,|1\rangle\,|1\rangle \tag{35}$$

It has been shown that, in this case, the probabilities of successful and failed teleportation of “0” and “1” form a binary symmetric channel and are given by [4]:

$$\begin{aligned} P &= |\eta_{00}|^2 + |\eta_{11}|^2 + 2\left(|\eta_{01}+\eta_{10}|^2 - |\eta_{00}-\eta_{11}|^2\right)|\alpha|^2|\beta|^2 \\ \epsilon &= 1 - |\eta_{00}|^2 - |\eta_{11}|^2 - 2\left(|\eta_{01}+\eta_{10}|^2 - |\eta_{00}-\eta_{11}|^2\right)|\alpha|^2|\beta|^2 \end{aligned} \tag{36}$$

The standard measure of entanglement for bipartite pure systems is given by the Shannon entropy of the Schmidt coefficients:

$$E\left(|\Phi\rangle\right) = -\sum_k |\lambda_k|^2 \log(|\lambda_k|^2) = -|\lambda_+|^2 \log(|\lambda_+|^2) - |\lambda_-|^2 \log(|\lambda_-|^2) \tag{37}$$

where:

$$\lambda_\pm^2 = \frac{1}{2}\left(1 \pm \sqrt{1 - 4|\eta_{01}\eta_{10} - \eta_{00}\eta_{11}|^2}\right) \tag{38}$$

A startling conclusion follows from these equations, the standard measure of entanglement for bipartite systems is not correlated with the amount of information than can be teleported using an entangled state [4]. That is, sometimes a perfectly entangled state does not correspond to maximum channel capacity (maximal probability of success). In what follows we will show that relativistic teleportation is an instance where this happens.

4.1 Relativistic Teleportation

Let us first consider the case where both qubits are in rest in Alice’s frame of reference. She has a perfectly entangled state given by:

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_a\,|0\rangle_a + |1\rangle_a\,|1\rangle_a\right) \tag{39}$$

Because Alice and Bob’s frames only differ from each other by a single boost, the rotation is $\Omega_p = 0$. In this situation, the Bell state remains invariant. In a similar manner, if Alice and Bob share entangled particles in their rest frames, their frames only differ by a single boost and $\Omega_p = 0$ once again.

As a more meaningful example, let us now consider the case where Alice produces an entangled pair in her frame of reference. She sends one particle to Bob in a similar manner as the one described in Figure 1. From the point of view of Alice, the entangled state will look like:

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle\,|0\rangle + |1\rangle\,|1\rangle\right) \tag{40}$$

with an amount of entanglement defined by the Schmidt coefficients $\lambda_{\pm}^2$:

$$\lambda_{\pm}^2 = \frac{1}{2}\left(1 \pm \sqrt{1 - 4|\eta_{01}\eta_{10} - \eta_{00}\eta_{11}|^2}\right) = \frac{1}{2}\left(1 \pm \sqrt{1 - 4\left|-\frac{1}{2}\right|^2}\right) = \frac{1}{2} \quad (41)$$

That is, Alice has a perfectly entangled state in the sense that $E(|\Phi\rangle) = 1$.

If the second qubit is sent to Bob, then he will perceive the entangled state to be:

$$\hat{R}|\Phi\rangle = \frac{1}{\sqrt{2}}\left(a|0\rangle|0\rangle + b|0\rangle|1\rangle - b|1\rangle|0\rangle + a|1\rangle|1\rangle\right) \quad (42)$$

So, for him, it looks like the entanglement is not perfect (in the sense that he does not look at a Bell state). However, it remains an entangled state with the exact same degree of entanglement as the original Bell state [2].

Indeed, the amount of entanglement measured by Bob is:

$$\lambda_{\pm}^2 = \frac{1}{2}\left(1 \pm \sqrt{1 - 4|\eta_{01}\eta_{10} - \eta_{00}\eta_{11}|^2}\right) = \frac{1}{2}\left(1 \pm \sqrt{1 - 4\left|\frac{a^2 + b^2}{2}\right|^2}\right) = \frac{1}{2} \quad (43)$$

These values also correspond to a maximally entangled state. Therefore, the net amount of entanglement remains the same after the Lorentz transformation:

$$E(|\Phi\rangle) = E\left(\hat{R}|\Phi\rangle\right) = 1 \quad (44)$$

that is, the amount of entanglement is a relativistic invariant. On the other hand, the probability of successful teleportation of classical information is not invariant:

$$\begin{aligned} P &= |\eta_{00}|^2 + |\eta_{11}|^2 + 2\left(|\eta_{01} + \eta_{10}|^2 - |\eta_{00} - \eta_{11}|^2\right)|\alpha|^2|\beta|^2 \quad (45) \\ &= \left|\frac{a}{\sqrt{2}}\right|^2 + \left|\frac{a}{\sqrt{2}}\right|^2 + 2\left(|\frac{b-b}{\sqrt{2}}|^2 - |\frac{a-a}{\sqrt{2}}|^2\right)|\alpha|^2|\beta|^2 \\ &= a^2 \end{aligned}$$

As expected, in the non-relativistic regime $v \ll c$ and as a consequence $\epsilon \approx 0$, the change to the capacity is negligible. On the other hand, in the ultra-relativistic regime $v \approx c$ and as a consequence $\epsilon \approx 1/2$, the teleportation channel capacity is reduced to zero.

It is also important to note that, in this case, the probability of success depends exclusively on the kinematic state (v), but not in the basis used to communicate the information (α and β). In other words, the kinematic noise does not depend on the basis used to represent the information.

[2] It is worth mentioning that if instead of using two momentum-spin eigenstates with entangled spin, we use states with a spread in momentum entangled in spin, the result of applying a boost is a state with some degree of spin entanglement and also momentum entanglement [2].

4.2 Absence of Relativistic Fixed Points

As noted in the previous example, the kinematic noise depends on the kinematic variable v, but it is independent of the basis (α, β) used to represent the information. Therefore, the notion of relativistic fixed points as described in Section 3.2 does not apply to the case of teleportation. However, we should explore as well the case where Alice and Bob use a different basis to represent the entangled state. Indeed, suppose we use the relativistic invariant base:

$$|\bar{0}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} \qquad |\bar{1}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix} \tag{46}$$

to generate the perfectly entangled Bell state (in Alice's frame):

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|\bar{0}\rangle|\bar{0}\rangle + |\bar{1}\rangle|\bar{1}\rangle\right) \tag{47}$$

Applying the Lorentz transformation leads to the following state perceived by Bob:

$$\hat{R}\,|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|\bar{0}\rangle\,\hat{R}\,|\bar{0}\rangle + |\bar{1}\rangle\,\hat{R}\,|\bar{1}\rangle\right) = \frac{1}{\sqrt{2}}\left(\lambda_0\,|\bar{0}\rangle|\bar{0}\rangle + \lambda_1\,|\bar{1}\rangle|\bar{1}\rangle\right) \tag{48}$$

and the probability of successful transmission is given by:

$$\begin{aligned} P &= |\eta_{00}|^2 + |\eta_{11}|^2 + 2\left(|\eta_{01} + \eta_{10}|^2 - |\eta_{00} - \eta_{11}|^2\right)|\alpha|^2|\beta|^2 \\ &= \frac{|\lambda_0|^2}{2} + \frac{|\lambda_1|^2}{2} - 2\,|\lambda_0 - \lambda_1|^2\,|\alpha|^2|\beta|^2 \\ &= 1 - 2b^2|\alpha|^2|\beta|^2 \end{aligned} \tag{49}$$

Therefore, even with the use of the relativistic invariant basis, the teleportation protocol feels the effects of kinematic noise. But now the kinematic error also depends on the basis (α, β) used to represent the information.

In the non-relativistic regime we have $v \ll c$ and as a consequence $\epsilon \approx 0$. However, in the ultra-relativistic regime:

$$v \approx c \implies \epsilon \approx |\alpha|^2|\beta|^2 \tag{50}$$

In such a case, the only way to reduce the kinematic noise is to move the basis (α, β) used to represent the information towards the standard measurement basis:

$$\left\{|\tilde{0}\rangle, |\tilde{1}\rangle\right\} \to \{|0\rangle, |1\rangle\} \implies (\alpha, \beta) \to (1, 0) \implies \epsilon \to 0 \tag{51}$$

Unfortunately, in the limiting case the use of quantum teleportation may become redundant.

We could also consider the case where Alice and Bob purposely use an imperfectly entangled state to try to overcome the effects of kinematic noise. The state they generate is:

$$|\Phi\rangle = \kappa_{00}\,|0\rangle|0\rangle + \kappa_{11}\,|1\rangle|1\rangle \tag{52}$$

For Bob, the entangled state looks like:

$$\begin{aligned} \hat{R}\,|\Phi\rangle &= \kappa_{00}\,|0\rangle \otimes (a\,|0\rangle + b\,|1\rangle) + \kappa_{11}\,|1\rangle \otimes (-b\,|0\rangle + a\,|1\rangle) \\ &= a\,\kappa_{00}\,|00\rangle + b\,\kappa_{00}\,|01\rangle - b\,\kappa_{11}\,|10\rangle + a\,\kappa_{11}\,|11\rangle \end{aligned} \tag{53}$$

The probability of successful transmission is given by:

$$\begin{aligned} P &= |\eta_{00}|^2 + |\eta_{11}|^2 + 2\left(|\eta_{01} + \eta_{10}|^2 - |\eta_{00} - \eta_{11}|^2\right)|\alpha|^2|\beta|^2 \\ &= a^2|\kappa_{00}|^2 + a^2|\kappa_{11}|^2 + 2\left(b^2|\kappa_{00} - \kappa_{11}|^2 - a^2|\kappa_{00} - \kappa_{11}|^2\right)|\alpha|^2|\beta|^2 \\ &= a^2 - 2\left(a^2 - b^2\right)|\kappa_{00} - \kappa_{11}|^2|\alpha|^2|\beta|^2 \end{aligned} \tag{54}$$

and because:

$$2\left(a^2 - b^2\right)|\kappa_{00} - \kappa_{11}|^2|\alpha|^2|\beta|^2 \geq 0 \tag{55}$$

we conclude that:

$$P \leq a^2 \implies \epsilon \geq b^2 \tag{56}$$

This is a bound that cannot be surpassed by any selection of the entanglement parameters κ_{00} and κ_{11}, nor for any selection of the encoding basis α and β.

Furthermore, in the ultra-relativistic regime $v \approx c$ and as a consequence $\epsilon \approx 1/2$, the channel capacity is reduced to zero without regards to the entanglement parameters κ_{00} and κ_{11}, and the encoding basis α and β.

5 Conclusions

In this paper we showed that motion can be considered as a form of noise. We analyzed the mathematical structure of kinematic noise in the context of quantum communications, and discussed the detrimental effects on the channel capacity for the send/receive and teleportation channels. Even though we limited our discussion to the case of massive spin-1/2 particles (i.e. electrons), similar conclusions follow for the case of massless spin-1 particles (i.e. photons). We are currently exploring the effects of non-inertial frames and gravitational fields on the capacity of quantum channels.

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