Optimal Approximation of Queries Using Tractable Propositional Languages

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Motivation for approximation in databases

- Approximate query evaluation in probabilistic databases
  $\rightarrow$ Exact query evaluation is #P-hard already for simple queries.

- Approximate explanations of query answers in provenance databases
  $\rightarrow$ Full explanations may have large size.

- Sampling-based approximation for query evaluation in relational databases
  $\rightarrow$ For aggregation queries in very large databases.

Given function f and space of problem instances $\mathcal{C}$. Assume complexity of f on $\mathcal{C}$ is *too* high.

How to approximate f on $\mathcal{C}$?

Approach 1: Modify f.

Find function f' from nicer complexity class such that for all $\Phi \in \mathcal{C}$

$$(1-\epsilon) \cdot f(\Phi) \leq f'(\Phi) \leq (1+\epsilon) \cdot f(\Phi)$$

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Approach 2: Modify Φ.

Find $\Phi_{\text{Lower}}, \Phi_{\text{Upper}}$ from nicer problem class $\mathcal{C}^{\text{easy}} \subset \mathcal{C}$ such that

$$f(\Phi_{\text{Lower}}) \;\leq\; f(\Phi) \;\leq\; f(\Phi_{\text{Upper}})$$

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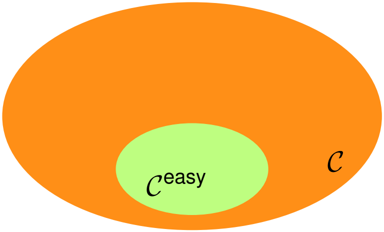

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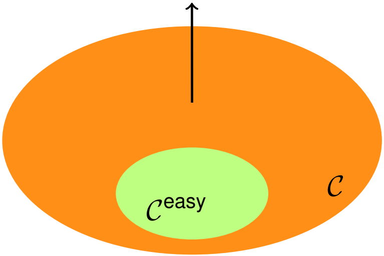

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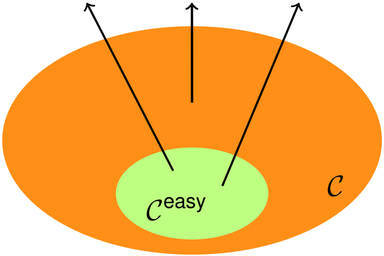

In this talk . . .

$\mathcal{C}$: Unate Boolean propositional formulas in DNF

f : Probability computation or model counting

$\mathcal{C}^{\text{easy}}$: Read-once formulas

- Probability computation for arbitrary formulas is #P-hard
- Probability computation for read-once formulas is in PTIME

Annotated databases

- Tuples are annotated with event ("lineage") expressions
- Here: Annotation with elements of the *PosBool* semiring

R

A	E
1	x_1
2	x_2

S

A	B	E
1	1	$\top$
1	2	$\top$
2	2	$\top$

T

B	E
1	y_1
2	y_2

- Queries map annotated databases to annotated databases. In particular, for every query, one can construct an expression Φ that is tightly connected to the query answer.
(TJ Green et al., Provenance Semirings, PODS 2007)

$Q(A, B) \leftarrow R(A), S(A, B), T(B)$

A	B	E
1	1	$x_1 y_1$
1	2	$x_1 y_2$
2	2	$x_2 y_2$

$Q \leftarrow R(A), S(A, B), T(B)$

	E
()	$x_1 y_1 \lor x_1 y_2 \lor x_2 y_2$

Sandwich-bounds for event formulas

R

A	E
1	x_1
2	x_2

S

A	B	E
1	1	$\top$
1	2	$\top$
2	2	$\top$

T

B	E
1	y_1
2	y_2

$$Q \leftarrow R(A), S(A, B), T(B)$$

$$\Phi = x_1 y_1 \vee x_1 y_2 \vee x_2 y_2$$

- Find formulas Φ_L, Φ_U such that $\Phi_L \models \Phi \models \Phi_U$
- If Φ_L, Φ_U have „nicer“ properties than Φ, then they provide convenient lower and upper bounds for Φ
- For example, bound formulas in which every variable symbol occurs only once: $\Phi_L = x_1(y_1 \vee y_2)$, $\Phi_U = (x_1 \vee x_2)(y_1 \vee y_2)$

Application to provenance databases

R

A	E
1	x_1
2	x_2

S

A	B	E
1	1	$\top$
1	2	$\top$
2	2	$\top$

T

B	E
1	y_1
2	y_2

$$Q \leftarrow R(A), S(A, B), T(B)$$

$$\Phi = x_1 y_1 \vee x_1 y_2 \vee x_2 y_2$$

$$x_1(y_1 \vee y_2) \models x_1 y_1 \vee x_1 y_2 \vee x_2 y_2 \models (x_1 \vee x_2)(y_1 \vee y_2)$$

- Lower bounds represent correct, yet not necessarily complete explanations
- Upper bounds represent complete, yet not necessarily correct explanations
- Idea: Choose bound formulas that admit small representation

Application to probabilistic databases

R

A	E
1	x_1
2	x_2

S

A	B	E
1	1	$\top$
1	2	$\top$
2	2	$\top$

T

B	E
1	y_1
2	y_2

$$Q \leftarrow R(A), S(A, B), T(B)$$

- Possible world semantics (database instances D, interpretations I):

$$P(Q) \stackrel{def}{=} \sum_{D:Q(D) \text{ is true}} P(D) = \sum_{I:I \models \Phi} P(I) \stackrel{def}{=} P(\Phi)$$

- Probability computation for general propositional formulas is #P-hard
- Model bounds imply probability bounds:

$$\Phi_L \models \Phi \models \Phi_U \qquad \Rightarrow \qquad P(\Phi_L) \leq P(\Phi) \leq P(\Phi_U)$$

- Idea: Choose bound formulas from a language that admits efficient probability computation

Key challenges for model-based query approximation

1. Which languages of propositional formulas are useful?
2. How to define optimality of bounds?
3. How to compute optimal bounds efficiently?

Key challenges for model-based query approximation

1. Which languages of propositional formulas are useful?
   - Read-once formulas or their DNF restrictions have size linear in the number of variables (and hence the size of the database) and admit linear time probability computation.
   - The event of every tractable conjunctive query without self-joins is equivalent to a read-once formula that can be computed in polynomial time.
   - More expressive languages? It is NP-hard to decide whether a formula has an equivalent read-2 formula. For read-3 formulas, probability computation is #P-hard.
2. How to define optimality of bounds?
3. How to compute optimal bounds efficiently?

Key challenges for model-based query approximation

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Key challenges for model-based query approximation

1. Which languages of propositional formulas are useful?
   - Read-once formulas
2. How to define optimality of bounds?
   - Let $\mathcal{L}'$ and $\mathcal{L}$ be two languages of propositional formulas and $\Phi \in \mathcal{L}$. Formula $\Phi_L \in \mathcal{L}'$ is a *lower bound for* Φ *with respect to* $\mathcal{L}'$, if

     $$\Phi_L \models \Phi \qquad \text{(i.e. } \mathcal{M}(\Phi_L) \subseteq \mathcal{M}(\Phi)\text{).}$$

     If in addition there is no formula $\Phi'_L \in \mathcal{L}'$ such that

     $$\mathcal{M}(\Phi_L) \subset \mathcal{M}(\Phi'_L) \subseteq \mathcal{M}(\Phi)$$

     then Φ_L is a *greatest lower bound for* Φ *with respect to* $\mathcal{L}'$. Least upper bounds are defined analogously.
3. How to compute optimal bounds efficiently?

Key challenges for model-based query approximation

1. Which languages of propositional formulas are useful?
   - Read-once formulas
2. How to define optimality of bounds?
   - Greatest lower bounds and least upper bounds w.r.t. a language
3. How to compute optimal bounds efficiently?

Key challenges for model-based query approximation

1. Which languages of propositional formulas are useful?
   - Read-once formulas
2. How to define optimality of bounds?
   - Greatest lower bounds and least upper bounds w.r.t. a language
3. How to compute optimal bounds efficiently?
   - Semantic definition is not very useful
   - Seek equivalent syntactic definitions of optimal bounds
   - Find algorithms to compute those bounds

Key challenges for model-based query approximation

1. Which languages of propositional formulas are useful?
   - Read-once formulas
2. How to define optimality of bounds?
   - Greatest lower bounds and least upper bounds w.r.t. a language
3. How to compute optimal bounds efficiently?
   - Seek equivalent syntactic characterisation of optimal bounds

Syntactic characterisation of optimal iDNF lower bounds

- iDNF = class of read-once DNF formulas
- Consider monotone/unate input formulas, since non-trivial approximation of general formulas is NP-hard
- Starting point: Generic characterisation of lower bounds: Φ_L is a lower bound of Φ if and only if Φ_L is obtainable by removing clauses from Φ or adding literals to its clauses.
- Example: $\Phi = x_1 y_1 \vee x_1 y_2 \vee x_2 y_2$
  Lower bounds: $x_1 y_1$, $x_1 y_1 \vee x_2 y_2$, $x_1 y_1 y_2$, ...

- Syntactic characterisation of optimal lower iDNF bounds:
  1. (*Lower bound*) Φ_L contains a subset of the clauses of Φ
  2. (*Maximality*) No further clause from Φ can be added to Φ_L

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  Lower bounds: $x_1 y_1$, $x_1 y_1 \vee x_2 y_2$, $x_1 y_1 y_2$, ...
  Optimal iDNF lower bounds: $x_1 y_2$, $x_1 y_1 \vee x_2 y_2$
  Non-iDNF lower bounds: $x_1 y_1 \vee x_1 y_2$, ...
  Non-optimal iDNF lower bounds: $x_1 y_1$, $x_2 y_2$, ...
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  1. (*Lower bound*) Φ_L contains a subset of the clauses of Φ
  2. (*Maximality*) No further clause from Φ can be added to Φ_L

Syntactic characterisation of optimal iDNF lower bounds

- Theorem: The semantic and syntactic characterisations of optimal iDNF lower bounds are equivalent.
- How many optimal lower bounds exist for a given formula? Exponentially many!

$$\Phi = (x_1 y_1 \vee x_1 y_2) \vee \cdots \vee (x_n y_{2n-1} \vee x_n y_{2n})$$

  has $3n$ variables, $2n$ clauses and 2^n iDNF greatest lower bounds.
- Polynomial enumeration of all optimal lower bounds is thus not possible. Next best thing: **Polynomial delay**
- Optimal lower bounds correspond to maximal independent sets in the clause dependency graph of the input formula
- There exist algorithms for polynomial-delay enumeration of maximal independet sets (e.g. Johnson&Yannakakis, 1988)

How good or bad can the optimal lower bound be?

- The bounds are *optimal* with respect to model inclusion and the iDNF class of formulas.
- However, they are also *incomparable* w.r.t. their models
- But they *can* be compared w.r.t. probabilities.
- Is there a way to efficiently find an iDNF lower bound that is *good* in terms of its probability?

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Let Φ be a k-partite unate DNF formula. There exists a polynomial time algorithm that constructs an iDNF greatest lower bound Φ_L for Φ such that $P(\Phi_L^{\text{opt}}) \leq k \cdot P(\Phi_L)$, where Φ_L^{opt} is the iDNF greatest lower bound for Φ with the highest probability amongst all of Φ's iDNF greatest lower bounds.

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- But they *can* be compared w.r.t. probabilities.
- Is there a way to efficiently find an iDNF lower bound that is *good* in terms of its probability?

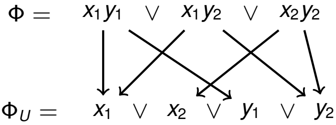
Idea: Sort clauses be descending probability and greedily pick in this order to construct an iDNF lower bound.

Syntactic characterisation of optimal iDNF upper bounds

- Starting point: Generic characterisation of upper bounds: Φ_U is an upper bound of Φ if and only if Φ_U is obtainable by adding clauses to Φ or removing literals from its clauses.
- Idea for syntactic and algorithmic treatment: Start with the most general upper bound $x_1 \vee \cdots \vee x_n$ and refine it until it gets optimal.

Syntactic characterisation of optimal iDNF upper bounds

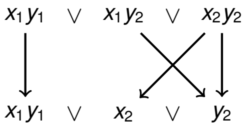
Example: How to find upper bounds for $x_1 y_1 \vee x_1 y_2 \vee x_2 y_2$?

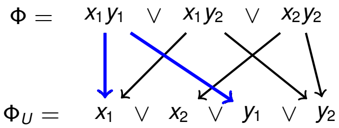


Syntactic characterisation of optimal iDNF upper bounds

Example: How to find upper bounds for $x_1 y_1 \vee x_1 y_2 \vee x_2 y_2$?

$x_1 y_1$ implies both x_1 and y_1 which can be merged.

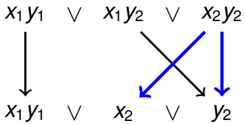


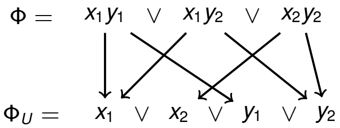


Syntactic characterisation of optimal iDNF upper bounds

Example: How to find upper bounds for $x_1 y_1 \vee x_1 y_2 \vee x_2 y_2$?

x_2 is not necessary and can be removed.

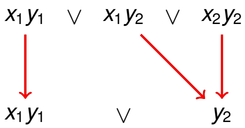


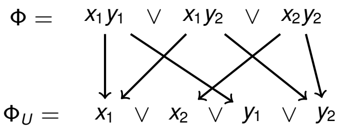


Syntactic characterisation of optimal iDNF upper bounds

Example: How to find upper bounds for $x_1y_1 \vee x_1y_2 \vee x_2y_2$?

No non-necessary clauses.
No clause can be extended by x_2.





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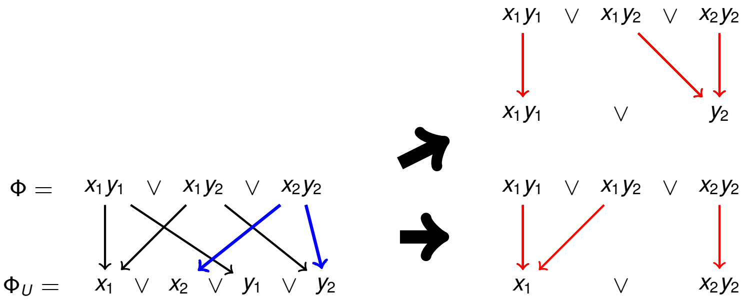

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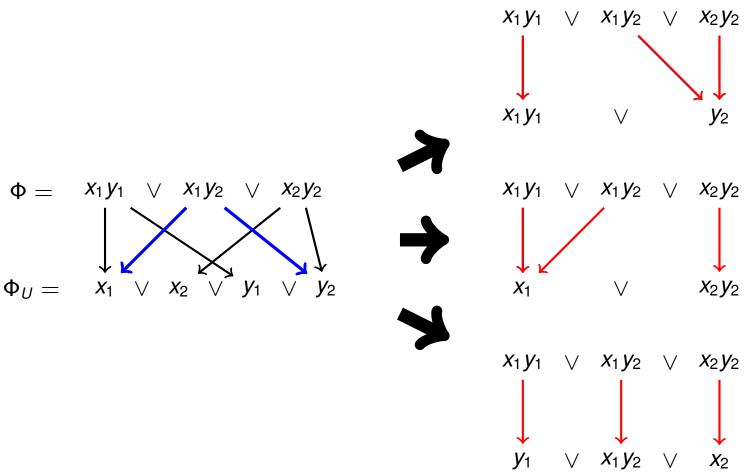
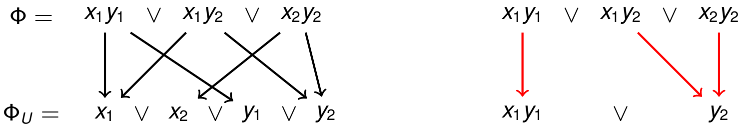


Syntactic characterisation of optimal iDNF upper bounds

Ingredients to syntactic definition of optimal upper bounds:

- Every clause in Φ implies a clause in Φ_U
- Every clause in Φ_U must be implied by one clause in Φ *exclusively*
- No unnecessary clauses in Φ_U
- No clause in Φ_U can be extended by a variable from Φ while preserving the above conditions



Syntactic characterisation of optimal iDNF upper bounds

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- How many optimal upper bounds exist for a given formula? Exponentially many!

$$\Phi = (x_1 y_1 \vee x_1 y_2) \vee \cdots \vee (x_n y_{2n-1} \vee x_n y_{2n})$$

  has $3n$ variables, $2n$ clauses and 3^n iDNF greatest upper bounds.
- Polynomial enumeration of all optimal upper bounds is thus not possible. Next best thing: **Polynomial delay**
- We present two algorithms in the paper:
  1. Enumeration of all optimal iDNF upper bounds.
  2. Enumeration with polynomial delay of all optimal iDNF upper bounds that preserve the variables of the input formula.

Optimal bounds with respect to arbitrary read-once formulas

- So far: iDNF bounds
- Next best: Read-once bounds (that is, without the restriction to DNF formulas)
- We succeeded at finding optimal read-once k-partite bounds for k-partite formulas
- Those bounds are also optimal w.r.t. general read-once formulas.
- Conjunctive queries without self-joins have k-partite formulas as lineage

Optimal bounds with respect to arbitrary read-once formulas

- Query Q:-$R(A), S(A, B), T(B)$ with event formula

  $\Phi = x_1 y_1 z_1 \vee x_1 y_2 z_2 \vee x_2 y_3 z_1 \vee x_2 y_4 z_2$ is no read-once formula

- Find k-partite upper bounds by adding clauses to Φ such that it factorises. There may be several choices for this expansion:

$$\Phi_{U,1} = (x_1 \vee x_2)[z_1(y_1 \vee y_3) \vee z_2(y_2 \vee y_4)]$$
$$\Phi_{U,2} = [x_1(y_1 \vee y_2) \vee x_2(y_3 \vee y_4)](z_1 \vee z_2)$$

- Find k-partite lower bounds by removing clauses from Φ such that it factorises.

$$\Phi_{L,1} = (x_1)[y_1 z_1 \vee y_2 z_2)]$$
$$\Phi_{L,2} = (x_2)[y_3 z_1 \vee y_4 z_2)]$$
$$\ldots$$

Characterising read-once formulas

A unate formula Φ is a read-once formula if and only if Φ is *normal* and $G(\Phi)$ is P_4*-free*. (Gurvich, 1991)

Examples:

- $xy + yz + xz$ is no read-once formula because its graph is not normal
- $x_1y_1 \vee x_1y_2 \vee x_2y_1$ is no read-once formula because its graph contains a P_4.
- $x_1y_1 \vee x_1y_2 \vee x_2y_1 \vee x_2y_2$ is a read-once formula because its graph is normal and P_4-free

Characterising **k-partite** read-once formulas

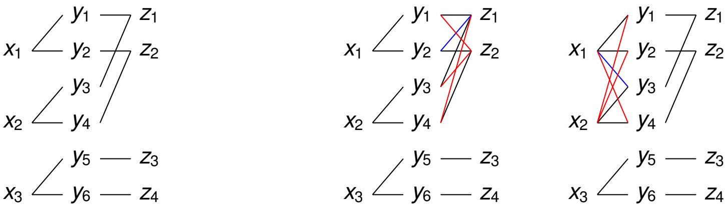
Lemma. In order to find optimal read-once bounds for a unate k-partite formula Φ, it is sufficient to remove clauses from Φ or add clauses to Φ.

(Note: This strategy will not find *all* optimal read-once bounds.)

Characterising **k-partite** read-once formulas

Lemma. Let $\mathcal{B}$ be the set of projection graphs of a unate k-partite formula. The set of connected components of the bipartite graphs in $\mathcal{B}$ are complete and pairwise aligned if and only if the formula represented by $\mathcal{B}$ is a read-once formula.

Example: $\Phi_1 = x_1y_1z_1 \vee x_1y_2z_2 \vee x_2y_3z_1 \vee x_2y_4z_2 \vee x_3y_5z_3 \vee x_3y_6z_4$



Optimal bounds with respect to arbitrary read-once formulas

- We give an algorithm to enumerate *some* optimal read-once upper bounds with polynomial delay. The problem of enumerating all optimal read-once upper bounds with polynomial delay is still open.
- We give an algorithm to compute all optimal read-once lower bounds. The problem of enumeration with polynomial delay is open.
- Excursion: “iDNF” is a *hereditary* property, but “read-once” is not. Does this observation help to determine the complexity of finding read-once lower bounds?

Approximation by queries

- Idea: Rewrite a given (hard) query Q into bound queries Q_L and Q_U such that their event formulas are read-once bounds for the event of Q
- Catch 1: Expressing the query for upper bounds requires a query language that is able to express transitive closure
- Catch 2: Removing edges to get lower bounds requires non-deterministic choice, or a linear order on tuples
- There are different upper and lower bounds for a given formula. These choices correspond to different rewritings of Q.

Approximation with arbitrary precision

- Model-based bounds do not provide precision guarantees
- But they can be obtained quickly
- Idea: Given a formula Φ, construct partial decision diagram ("decomposition tree") for Φ. Compute rough bounds for residual formulas and propagate them through the diagram to obtain overall probability bound.
- Can yield multiplicative and additive approximation guarantees
- See Olteanu, Huang, Koch, ICDE 2010.

Conclusion

- Framework for model-based characterisation of optimal bounds for propositional formulas
- Applications: Probabilistic databases, provenance databases
- Syntactic characterisations that are equivalent to model-based definitions yet much easier to turn into algorithms

Open questions

- The read-once results are so far only for k-partite formulas which is great for conjunctive queries without self-joins. What happens beyond k-partite approximations?
- Bounds for non-DNF input formulas?
- Complexity of obtaining read-once optimal lower bounds?
- Connection to recent work on *readability* of query answers? (Olteanu, Zavodny, ICDT 2012)

End. ?