Optimal Approximation of Queries Using Tractable Propositional Languages

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Motivation for approximation in databases

- Approximate query evaluation in probabilistic databases
  → Exact query evaluation is \#P-hard already for simple queries.

- Approximate explanations of query answers in provenance databases
  → Full explanations may have large size.

- Sampling-based approximation for query evaluation in relational databases
  → For aggregation queries in very large databases.
Given function $f$ and space of problem instances $C$. Assume complexity of $f$ on $C$ is too high.

How to approximate $f$ on $C$?
Approach 1: Modify $f$.
Find function $f'$ from nicer complexity class such that for all $\Phi \in C$

$$(1 - \epsilon) \cdot f(\Phi) \leq f'(\Phi) \leq (1 + \epsilon) \cdot f(\Phi)$$
Approach 1: Modify $f$.
Find function $f'$ from nicer complexity class such that for all $\Phi \in C$

$$(1 - \epsilon) \cdot f(\Phi) \leq f'(\Phi) \leq (1 + \epsilon) \cdot f(\Phi)$$

Approach 2: Modify $\Phi$.
Find $\Phi_{\text{Lower}}, \Phi_{\text{Upper}}$ from nicer problem class $C^{\text{easy}} \subset C$ such that

$$f(\Phi_{\text{Lower}}) \leq f(\Phi) \leq f(\Phi_{\text{Upper}})$$
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Find function $f'$ from nicer complexity class such that for all $\Phi \in \mathcal{C}$

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In this talk . . .

$C$: Unate Boolean propositional formulas in DNF

$f$: Probability computation or model counting

$C^{\text{easy}}$: Read-once formulas

- Probability computation for arbitrary formulas is $\#P$-hard
- Probability computation for read-once formulas is in $\text{PTIME}$
Annotated databases

- Tuples are annotated with event (“lineage”) expressions
- Here: Annotation with elements of the PosBool semiring

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- Queries map annotated databases to annotated databases. In particular, for every query, one can construct an expression $\Phi$ that is tightly connected to the query answer. (TJ Green et al., Provenance Semirings, PODS 2007)

\[
Q(A, B) \leftarrow R(A), S(A, B), T(B)
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Sandwich-bounds for event formulas

\[ Q \leftarrow R(A), S(A, B), T(B) \]
\[ \Phi = x_1 y_1 \lor x_1 y_2 \lor x_2 y_2 \]

- Find formulas \( \Phi_L, \Phi_U \) such that \( \Phi_L \models \Phi \models \Phi_U \)
- If \( \Phi_L, \Phi_U \) have “nicer” properties than \( \Phi \), then they provide convenient lower and upper bounds for \( \Phi \)
- For example, bound formulas in which every variable symbol occurs only once: \( \Phi_L = x_1 (y_1 \lor y_2) \), \( \Phi_U = (x_1 \lor x_2) (y_1 \lor y_2) \)
Application to provenance databases

\[ Q \leftarrow R(A), S(A, B), T(B) \]

\[ \Phi = x_1 y_1 \lor x_1 y_2 \lor x_2 y_2 \]

\[ x_1(y_1 \lor y_2) \models x_1 y_1 \lor x_1 y_2 \lor x_2 y_2 \models (x_1 \lor x_2)(y_1 \lor y_2) \]

- Lower bounds represent correct, yet not necessarily complete explanations
- Upper bounds represent complete, yet not necessarily correct explanations
- Idea: Choose bound formulas that admit small representation
Application to probabilistic databases

\[ Q \leftarrow R(A), S(A, B), T(B) \]

- Possible world semantics (database instances \( D \), interpretations \( I \)):

\[
P(Q) \overset{\text{def}}{=} \sum_{D \colon Q(D) \text{ is true}} P(D) = \sum_{I \colon I \models \Phi} P(I) \overset{\text{def}}{=} P(\Phi)
\]

- Probability computation for general propositional formulas is \#P-hard

- Model bounds imply probability bounds:

\[
\Phi_L \models \Phi \models \Phi_U \quad \Rightarrow \quad P(\Phi_L) \leq P(\Phi) \leq P(\Phi_U)
\]

- Idea: Choose bound formulas from a language that admits efficient probability computation
Key challenges for model-based query approximation

1. Which languages of propositional formulas are useful?
2. How to define optimality of bounds?
3. How to compute optimal bounds efficiently?
Key challenges for model-based query approximation

1. Which languages of propositional formulas are useful?
   - Read-once formulas or their DNF restrictions have size linear in the number of variables (and hence the size of the database) and admit linear time probability computation.
   - The event of every tractable conjunctive query without self-joins is equivalent to a read-once formula that can be computed in polynomial time.
   - More expressive languages? It is NP-hard to decide whether a formula has an equivalent read-2 formula. For read-3 formulas, probability computation is \#P-hard.

2. How to define optimality of bounds?

3. How to compute optimal bounds efficiently?
Key challenges for model-based query approximation

1. Which languages of propositional formulas are useful?
   ▶ Read-once formulas

2. How to define optimality of bounds?

3. How to compute optimal bounds efficiently?
Key challenges for model-based query approximation

1. Which languages of propositional formulas are useful?
   - Read-once formulas

2. How to define optimality of bounds?
   - Let $\mathcal{L}'$ and $\mathcal{L}$ be two languages of propositional formulas and $\Phi \in \mathcal{L}$. Formula $\Phi_L \in \mathcal{L}'$ is a \textit{lower bound for $\Phi$ with respect to $\mathcal{L}'$}, if
     
     $$\Phi_L \models \Phi \quad \text{(i.e. $\mathcal{M}(\Phi_L) \subseteq \mathcal{M}(\Phi)$)}.$$ 

     If in addition there is no formula $\Phi'_L \in \mathcal{L}'$ such that
     
     $$\mathcal{M}(\Phi_L) \subseteq \mathcal{M}(\Phi'_L) \subseteq \mathcal{M}(\Phi)$$

     then $\Phi_L$ is a \textit{greatest lower bound for $\Phi$ with respect to $\mathcal{L}'$}. Least upper bounds are defined analogously.

3. How to compute optimal bounds efficiently?
Key challenges for model-based query approximation

1. Which languages of propositional formulas are useful?
   ▶ Read-once formulas

2. How to define optimality of bounds?
   ▶ Greatest lower bounds and least upper bounds w.r.t. a language

3. How to compute optimal bounds efficiently?
Key challenges for model-based query approximation

1. Which languages of propositional formulas are useful?
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2. How to define optimality of bounds?
   ▶ Greatest lower bounds and least upper bounds w.r.t. a language

3. How to compute optimal bounds efficiently?
   ▶ Semantic definition is not very useful
   ▶ Seek equivalent syntactic definitions of optimal bounds
   ▶ Find algorithms to compute those bounds
Key challenges for model-based query approximation

1. Which languages of propositional formulas are useful?
   ▶ Read-once formulas

2. How to define optimality of bounds?
   ▶ Greatest lower bounds and least upper bounds w.r.t. a language

3. How to compute optimal bounds efficiently?
   ▶ Seek equivalent syntactic characterisation of optimal bounds
Syntactic characterisation of optimal iDNF lower bounds

- iDNF = class of read-once DNF formulas
- Consider monotone/unate input formulas, since non-trivial approximation of general formulas is NP-hard
- Starting point: Generic characterisation of lower bounds: \( \Phi_L \) is a lower bound of \( \Phi \) if and only if \( \Phi_L \) is obtainable by removing clauses from \( \Phi \) or adding literals to its clauses.
- Example: \( \Phi = x_1 y_1 \lor x_1 y_2 \lor x_2 y_2 \)
  
  Lower bounds: \( x_1 y_1, x_1 y_1 \lor x_2 y_2, x_1 y_1 y_2, \ldots \)

- Syntactic characterisation of optimal lower iDNF bounds:
  1. \((Lower \ bound)\) \( \Phi_L \) contains a subset of the clauses of \( \Phi \)
  2. \((Maximality)\) No further clause from \( \Phi \) can be added to \( \Phi_L \)
Syntactic characterisation of optimal iDNF lower bounds

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- Example: $\Phi = x_1 y_1 \lor x_1 y_2 \lor x_2 y_2$
  Lower bounds: $x_1 y_1, x_1 y_1 \lor x_2 y_2, x_1 y_1 y_2, \ldots$
  Optimal iDNF lower bounds: $x_1 y_2, x_1 y_1 \lor x_2 y_2$
  Non-iDNF lower bounds: $x_1 y_1 \lor x_1 y_2, \ldots$
  Non-optimal iDNF lower bounds: $x_1 y_1, x_2 y_2, \ldots$
- Syntactic characterisation of optimal lower iDNF bounds:
  1. (Lower bound) $\Phi_L$ contains a subset of the clauses of $\Phi$
  2. (Maximality) No further clause from $\Phi$ can be added to $\Phi_L$
Syntactic characterisation of optimal iDNF lower bounds

- Theorem: The semantic and syntactic characterisations of optimal iDNF lower bounds are equivalent.
- How many optimal lower bounds exist for a given formula? Exponentially many!

\[ \Phi = (x_1 y_1 \lor x_1 y_2) \lor \cdots \lor (x_n y_{2n-1} \lor x_n y_{2n}) \]

has 3\(n\) variables, 2\(n\) clauses and 2\(n\) iDNF greatest lower bounds.

- Polynomial enumeration of all optimal lower bounds is thus not possible. Next best thing: **Polynomial delay**
- Optimal lower bounds correspond to maximal independent sets in the clause dependency graph of the input formula
- There exist algorithms for polynomial-delay enumeration of maximal independent sets (e.g. Johnson&Yannakakis, 1988)
How good or bad can the optimal lower bound be?

- The bounds are *optimal* with respect to model inclusion and the iDNF class of formulas.
- However, they are also *incomparable* w.r.t. their models.
- But they *can* be compared w.r.t. probabilities.
- Is there a way to efficiently find an iDNF lower bound that is *good* in terms of its probability?
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- Is there a way to efficiently find an iDNF lower bound that is *good* in terms of its probability?

Let $\Phi$ be a $k$-partite unate DNF formula. There exists a polynomial time algorithm that constructs an iDNF greatest lower bound $\Phi_L$ for $\Phi$ such that $P(\Phi_L^{opt}) \leq k \cdot P(\Phi_L)$, where $\Phi_L^{opt}$ is the iDNF greatest lower bound for $\Phi$ with the highest probability amongst all of $\Phi$’s iDNF greatest lower bounds.
How good or bad can the optimal lower bound be?

- The bounds are *optimal* with respect to model inclusion and the iDNF class of formulas.
- However, they are also *incomparable* w.r.t. their models.
- But they *can* be compared w.r.t. probabilities.
- Is there a way to efficiently find an iDNF lower bound that is *good* in terms of its probability?

Idea: Sort clauses by descending probability and greedily pick in this order to construct an iDNF lower bound.
Syntactic characterisation of optimal iDNF upper bounds

- Starting point: Generic characterisation of upper bounds: $\Phi_U$ is an upper bound of $\Phi$ if and only if $\Phi_U$ is obtainable by adding clauses to $\Phi$ or removing literals from its clauses.

- Idea for syntactic and algorithmic treatment: Start with the most general upper bound $x_1 \lor \cdots \lor x_n$ and refine it until it gets optimal.
Syntactic characterisation of optimal iDNF upper bounds

Example: How to find upper bounds for $x_1 y_1 \lor x_1 y_2 \lor x_2 y_2$?

$\Phi = x_1 y_1 \lor x_1 y_2 \lor x_2 y_2$

$\Phi_U = x_1 \lor x_2 \lor y_1 \lor y_2$
Syntactic characterisation of optimal iDNF upper bounds

Example: How to find upper bounds for $x_1 y_1 \lor x_1 y_2 \lor x_2 y_2$?

$x_1 y_1$ implies both $x_1$ and $y_1$
which can be merged.

$\Phi = x_1 y_1 \lor x_1 y_2 \lor x_2 y_2$

$\Phi_U = x_1 \lor x_2 \lor y_1 \lor y_2$
Syntactic characterisation of optimal iDNF upper bounds

Example: How to find upper bounds for $x_1 y_1 \lor x_1 y_2 \lor x_2 y_2$?

$x_2$ is not necessary and can be removed.

$\Phi = x_1 y_1 \lor x_1 y_2 \lor x_2 y_2$

$\Phi_U = x_1 \lor x_2 \lor y_1 \lor y_2$
Syntactic characterisation of optimal iDNF upper bounds

Example: How to find upper bounds for $x_1 y_1 \lor x_1 y_2 \lor x_2 y_2$?

No non-necessary clauses.
No clause can be extended by $x_2$.

\[
\Phi = x_1 y_1 \lor x_1 y_2 \lor x_2 y_2
\]

\[
\Phi_U = x_1 \lor x_2 \lor y_1 \lor y_2
\]
Syntactic characterisation of optimal iDNF upper bounds

Example: How to find upper bounds for $x_1 y_1 \lor x_1 y_2 \lor x_2 y_2$?

\[ \Phi = \bigvee \begin{align*} x_1 y_1 & \lor x_1 y_2 & \lor x_2 y_2 \\ x_1 & \lor x_2 & \lor y_1 & \lor y_2 \end{align*} \]

\[ \Phi_U = \bigvee \begin{align*} x_1 y_1 & \lor x_1 y_2 & \lor x_2 y_2 \\ x_1 & \lor x_2 & \lor y_1 & \lor y_2 \end{align*} \]
Syntactic characterisation of optimal iDNF upper bounds

Example: How to find upper bounds for $x_1 y_1 \lor x_1 y_2 \lor x_2 y_2$?

\[
\Phi = \Phi_U = x_1 y_1 \lor x_1 y_2 \lor x_2 y_2
\]

\[
\Phi_U = x_1 \lor x_2 \lor y_1 \lor y_2
\]
Syntactic characterisation of optimal iDNF upper bounds

Ingredients to syntactic definition of optimal upper bounds:

- Every clause in $\Phi$ implies a clause in $\Phi_U$
- Every clause in $\Phi_U$ must be implied by one clause in $\Phi$ exclusively
- No unnecessary clauses in $\Phi_U$
- No clause in $\Phi_U$ can be extended by a variable from $\Phi$ while preserving the above conditions

$\Phi = x_1y_1 \lor x_1y_2 \lor x_2y_2$

$\Phi_U = x_1 \lor x_2 \lor y_1 \lor y_2$

$x_1y_1 \lor x_1y_2 \lor x_2y_2$

$x_1 \lor y_2$
Theorem: The semantic and syntactic characterisations of optimal iDNF upper bounds are equivalent.

How many optimal upper bounds exist for a given formula? Exponentially many!

\[\Phi = (x_1 y_1 \lor x_1 y_2) \lor \cdots \lor (x_n y_{2n-1} \lor x_n y_{2n})\]

has 3n variables, 2n clauses and 3^n iDNF greatest upper bounds.

Polynomial enumeration of all optimal upper bounds is thus not possible. Next best thing: **Polynomial delay**

We present two algorithms in the paper:

1. Enumeration of all optimal iDNF upper bounds.
2. Enumeration with polynomial delay of all optimal iDNF upper bounds that preserve the variables of the input formula.
Optimal bounds with respect to arbitrary read-once formulas

- So far: iDNF bounds
- Next best: Read-once bounds (that is, without the restriction to DNF formulas)
- We succeeded at finding optimal read-once k-partite bounds for k-partite formulas
- Those bounds are also optimal w.r.t. general read-once formulas.
- Conjunctive queries without self-joins have k-partite formulas as lineage
Optimal bounds with respect to arbitrary read-once formulas

- Query $Q:-R(A), S(A, B), T(B)$ with event formula

\[ \Phi = x_1 y_1 z_1 \lor x_1 y_2 z_2 \lor x_2 y_3 z_1 \lor x_2 y_4 z_2 \] is no read-once formula

- Find k-partite upper bounds by adding clauses to $\Phi$ such that it factorises. There may be several choices for this expansion:

\[ \Phi_{U,1} = (x_1 \lor x_2)[z_1(y_1 \lor y_3) \lor z_2(y_2 \lor y_4)] \]
\[ \Phi_{U,2} = [x_1(y_1 \lor y_2) \lor x_2(y_3 \lor y_4)](z_1 \lor z_2) \]

- Find k-partite lower bounds by removing clauses from $\Phi$ such that it factorises.

\[ \Phi_{L,1} = (x_1)[y_1 z_1 \lor y_2 z_2] \]
\[ \Phi_{L,2} = (x_2)[y_3 z_1 \lor y_4 z_2] \]
\[ \ldots \]
Characterising read-once formulas

A unate formula $\Phi$ is a read-once formula if and only if $\Phi$ is normal and $G(\Phi)$ is $P_4$-free. (Gurvich, 1991)

Examples:

- $xy + yz + xz$ is no read-once formula because its graph is not normal.
- $x_1y_1 \lor x_1y_2 \lor x_2y_1$ is no read-once formula because its graph contains a $P_4$.
- $x_1y_1 \lor x_1y_2 \lor x_2y_1 \lor x_2y_2$ is a read-once formula because its graph is normal and $P_4$-free.
Characterising \textbf{k-partite} read-once formulas

\textbf{Lemma.} In order to find optimal read-once bounds for a unate \textit{k}-partite formula \(\Phi\), it is sufficient to remove clauses from \(\Phi\) or add clauses to \(\Phi\).

(Note: This strategy will not find \textit{all} optimal read-once bounds.)
Characterising \textbf{k-partite} read-once formulas

**Lemma.** Let $B$ be the set of projection graphs of a unate $k$-partite formula. The set of connected components of the bipartite graphs in $B$ are complete and pairwise aligned if and only if the formula represented by $B$ is a read-once formula.

Example: $\Phi_1 = x_1 y_1 z_1 \lor x_1 y_2 z_2 \lor x_2 y_3 z_1 \lor x_2 y_4 z_2 \lor x_3 y_5 z_3 \lor x_3 y_6 z_4$
Optimal bounds with respect to arbitrary read-once formulas

- We give an algorithm to enumerate *some* optimal read-once upper bounds with polynomial delay. The problem of enumerating all optimal read-once upper bounds with polynomial delay is still open.

- We give an algorithm to compute all optimal read-once lower bounds. The problem of enumeration with polynomial delay is open.

- Excursion: “iDNF” is a *hereditary* property, but “read-once” is not. Does this observation help to determine the complexity of finding read-once lower bounds?
Approximation by queries

- Idea: Rewrite a given (hard) query $Q$ into bound queries $Q_L$ and $Q_U$ such that their event formulas are read-once bounds for the event of $Q$

- Catch 1: Expressing the query for upper bounds requires a query language that is able to express transitive closure

- Catch 2: Removing edges to get lower bounds requires non-deterministic choice, or a linear order on tuples

- There are different upper and lower bounds for a given formula. These choices correspond to different rewritings of $Q$. 
Approximation with arbitrary precision

- Model-based bounds do not provide precision guarantees
- But they can be obtained quickly
- Idea: Given a formula $\Phi$, construct partial decision diagram ("decomposition tree") for $\Phi$. Compute rough bounds for residual formulas and propagate them through the diagram to obtain overall probability bound.
- Can yield multiplicative and additive approximation guarantees
- See Olteanu, Huang, Koch, ICDE 2010.
Conclusion

- Framework for model-based characterisation of optimal bounds for propositional formulas
- Applications: Probabilistic databases, provenance databases
- Syntactic characterisations that are equivalent to model-based definitions yet much easier to turn into algorithms

Open questions

- The read-once results are so far only for k-partite formulas which is great for conjunctive queries without self-joins. What happens beyond k-partite approximations?
- Bounds for non-DNF input formulas?
- Complexity of obtaining read-once optimal lower bounds?
- Connection to recent work on *readability* of query answers? (Olteanu, Zavodny, ICDT 2012)
End.