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A Toolbox of Query Evaluation Techniques for Probabilistic Databases

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Uncertain and Probabilistic Data

Uncertain and probabilistic data is commonplace:

- (Web) information extraction
- Processing manually entered data (such as census forms)
- Data integration, data cleaning
- Risk management: Decision support queries, hypothetical queries
- Social network analysis
- Managing scientific data; sensor data
- Crime fighting, surveillance, plagiarism detection, predicting terrorist actions

Relational DBMSs are not flexible enough to accommodate such data

- one tries to find a "good" yet partial fit of the data in the relational format

Recent years have seen advances in developing

- representation models for uncertain/probabilistic data,
- uncertainty-aware query languages, and
- scalable query evaluation techniques for such data.

Probabilistic Databases Today

Many active projects, for instance

- Mystiq, Lahar (Washington U.)
- Trio (Stanford)
- MCDB (IBM Almaden & Florida)
- BayesStore (Berkeley)
- Orion (Purdue)
- PrDB (Maryland)
- Also at UMass, Waterloo, Hong Kong, Florida State, Wisconsin, MIT, ...

Projects I am involved in

- MayBMS (Cornell+Oxford)
  - Uncertainty-aware query languages and data representation models
- SPROUT = Scalable Query PROcessing on Uncertain Tables (Oxford)
  - Query engine that extends PostgreSQL backend

MayBMS and SPROUT are available at `maybms.sourceforge.net`.

Outline of this talk

- Two (discrete) probabilistic data models
  - U-relational databases: a complete but "hard" model
  - Tuple-independent databases: a restricted but "easier" model

- Query evaluation in probabilistic databases
  - The non-probabilistic case
  - Data complexity in the probabilistic case

- Exact query evaluation techniques
  - Evaluation of tractable conjunctive queries
    - using OBDDs (ordered binary decision diagrams)
    - using relational plans
  - Detection of large tractable query & data sub-instances

- Approximate query evaluation techniques with error guarantees
  - Monte Carlo techniques (FPRAS)
  - Incremental evaluation techniques based on lineage factorization

U-relational Probabilistic Databases

Syntax.
Probabilistic databases are relational databases where

- There is a finite set of independent random variables $\mathbf{X} = \{x_1, \ldots, x_n\}$ with finite domains $\mathrm{Dom}_{x_1}, \ldots, \mathrm{Dom}_{x_n}$.
- Tuples are associated with *lineage*, i.e., conjunctions of atomic events of the form $x_i = a$ or $x_i \neq a$ where $x_i \in \mathbf{X}$ and $a \in \mathrm{Dom}_{x_i}$.
- There is a probability distribution over the assignments of each variable.

Semantics.

- *Possible worlds* defined by total assignments θ over $\mathbf{X}$.
- The world defined by assignment θ
  - consists of all tuples with condition ϕ such that $\theta(\phi) = \mathit{true}$.
  - has probability defined by the product of probabilities of each assignment in θ.

This formalism can represent any discrete probability distribution over relational databases.

Example: Probabilistic Databases

Consider a simplified TPC-H scenario with customers (Cust) and orders (Ord):

Cust

ckey	cname	V_1	P_1	V_2	P_2
1	Joe	x_1	0.1	x_3	0.1
2	Dan	$\overline{x_1}$	0.9	x_4	0.5
3	Li	x_2	0.3	$\overline{x_4}$	0.5
4	Mo	$\overline{x_2}$	0.7	$\overline{x_5}$	0.2

Ord

okey	ckey	odate	V_1	P_1	V_2	P_2
1	1	1995-01-10	y_1	0.1	$\overline{x_5}$	0.2
2	1	1996-01-09	y_2	0.2	$\overline{x_4}$	0.5
3	2	1994-11-11	y_3	0.3	x_3	0.1

- Variables are Boolean (wlog); write x instead of $x = 1$, $\overline{x}$ instead of $x = 0$.
- A pair (V_i, P_i) states that the variable assignment given by V_i has the probability given by P_i.
- Lineage can represent arbitrary correlations between tuples, eg,
  - (1,Joe) and (3,Li) are independent: They use disjoint sets of variables.
  - (1,Joe) and (2,Dan) are mutually exclusive: x_1 is either true or false.

Example: Probabilistic Databases

Consider the world $\mathcal{A}$ defined by a total assignment θ:

- x_1, x_2, y_1, y_2 are true, and all other variables are false.

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2	**1**	**1996-01-09**	y_2	**0.2**	$\overline{x_4}$	**0.5**
3	2	1994-11-11	y_3	0.3	x_3	0.1

The world $\mathcal{A}$ is as follows:

Cust

ckey	cname
3	Li

Ord

okey	ckey	odate
1	1	1995-01-10
2	1	1996-01-09

Probability of $\mathcal{A}$ = product of probabilities of the assignments in θ:

$$Pr(\mathcal{A}) = Pr(\theta) = Pr(x_1) \cdot Pr(x_2) \cdot Pr(y_1) \cdot Pr(y_2) \cdot Pr(\overline{x_3}) \cdot Pr(\overline{x_4}) \cdot Pr(\overline{x_5}) \cdot Pr(\overline{y_3}).$$

Tuple-independent Probabilistic Databases

Tuple-independent: Tuples have independent lineage, or equivalently

- Each tuple t is associated with a Boolean random variable x_t.
- Tuple t is in the world defined by θ if $x_t = true$ holds in θ.

Cust

ckey	cname	V	P
1	Joe	x_1	0.1
2	Dan	x_2	0.2
3	Li	x_3	0.3
4	Mo	x_4	0.4

Ord

okey	ckey	odate	V	P
1	1	1995-01-10	y_1	0.1
2	1	1996-01-09	y_2	0.2
3	2	1994-11-11	y_3	0.3
4	2	1993-01-08	y_4	0.4
5	3	1995-08-15	y_5	0.5
6	3	1996-12-25	y_6	0.6

Item

okey	disc	ckey	V	P
1	0.1	1	z_1	0.1
1	0.2	1	z_2	0.2
3	0.4	2	z_3	0.3
3	0.1	2	z_4	0.4
4	0.4	2	z_5	0.5
5	0.1	3	z_6	0.6

Query Evaluation in Probabilistic Databases

Subsumed by general probabilistic inference, which was investigated in AI for many years. Is there something left to do for the database community?

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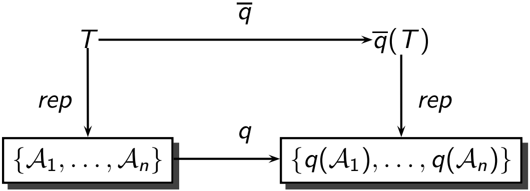
The database approach is based on two fundamental observations:

- the separation of (very large) data and (small and fixed) query, and
- the use of mature relational query engines to achieve scalability.

Query Evaluation in Probabilistic Databases

The MayBMS/SPROUT approach:

- Given probabilistic database $T = \{\mathcal{A}_1, \ldots, \mathcal{A}_n\}$ and query q.
- Under *possible world semantics*: q is evaluated in each possible world of T.
- Is there $\overline{q}$ such that $\overline{q}(T) = \{q(\mathcal{A}_1), \ldots, q(\mathcal{A}_n)\}$? [Imielinski&Lipski84]



- Compute the probability of each *distinct* tuple in $\overline{q}(T)$.

How hard is query evaluation in probabilistic databases?

- $\overline{q}(T)$ can be computed in PTIME (wrt data complexity) for relational algebra queries and U-relational databases [AJK&**O**.08]
- Probability computation is in general *very* hard, but there are tractable cases

Example: Query Evaluation

Query asking for the dates of discounted orders shipped to customer 'Joe':

$$Q(odate) \text{ :- } \mathrm{Cust}(ckey, 'Joe'), \mathrm{Ord}(okey, ckey, odate), \mathrm{Item}(okey, disc, ckey), disc > 0$$

odate	V_c	P_c	V_o	P_o	V_i	P_i	tuple probability
1995-01-10	x_1	0.1	y_1	0.1	z_1	0.1	$0.1 \cdot 0.1 \cdot 0.1$
1995-01-10	x_1	0.1	y_1	0.1	z_2	0.2	$0.1 \cdot 0.1 \cdot 0.2$

- Query $\overline{Q}$ is Q changed so that the lineage of input tuples is copied in the answer tuples.
- Probability of distinct answer tuple (1995-01-10) is the probability of the associated lineage $x_1y_1z_1 + x_1y_1z_2$.

Difficulty:

- The sets of satisfying assigments of any two clauses may overlap.
- It may require to iterate over its (exponentially many) satisfying assignments.

Dichotomy Property

Discussed here [Dalvi&Suciu07]:

- Conjunctive queries without self-joins (CQ^1) on
- Tuple-independent databases.

> The data complexity of any CQ^1 query is either FP or #P-hard.

- #P = class of functions $f(x)$ for which there exists a PTIME non-deterministic Turing machine M such that $f(x)$ = number of accepting computations of M on input x.
- FP = class of functions that can be solved by a deterministic Turing machine in PTIME. These functions can have *any* output, not only true/false.

Further tractability results not discussed here:

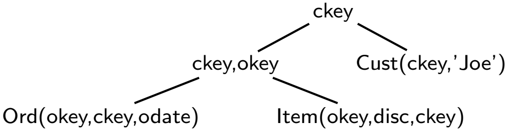
- Dichotomy for conjunctive queries with self-joins [Dalvi&Suciu07b]
- Tractable queries with inequalities ($<, \leq, \neq$) [**O.**&Huang08,**O.**&Huang09]
- Extension to the block-independent disjoint model follow easily [DRS07]

All tractable CQ[1] queries are hierarchical

A query is *hierarchical* if for any two non-head variables, either their sets of subgoals are disjoint, or one set is contained in the other.

$Q(odate)$:- $\text{Cust}(ckey, 'Joe'), \text{Ord}(okey, ckey, odate), \text{Item}(okey, disc, ckey), disc > 0.$
is hierarchical; also without odate as head variable.

subgoals(disc)=$\{\text{Item}\}$, subgoals(okey)=$\{\text{Ord}, \text{Item}\}$, subgoals(ckey)=$\{\text{Cust}, \text{Ord}, \text{Item}\}$.
It holds that subgoals(disc)$\subseteq$ subgoals(okey)$\subseteq$ subgoals(ckey).



Exact Query Evaluation

Exact Query Evaluation using SPROUT

Cast the query evaluation problem as a decision diagram construction problem.

- Given a query q and a probabilistic database D, each distinct tuple $t \in q(D)$ is associated with a DNF expression ϕ_t.
- Probability of t is probability of ϕ_t.
- Compile ϕ_t into an equivalent **binary decision diagram (BDD)**.
- Probability of ϕ_t is then the probability of its BDD.
- SPROUT employs secondary-storage techniques for BDD construction and probability computation.

BDDs

- Commonly used to represent compactly large Boolean expressions.
- Idea: Decompose Boolean expressions using variable elimination and avoid redundancy in the representation.
  Variable elimination by Shannon's expansion: $\phi = x \cdot \phi\mid_x + \bar{x} \cdot \phi\mid_{\bar{x}}$.
- Supports linear-time probability computation.

$$
\begin{aligned}
Pr(\phi) &= Pr(x \cdot \phi\mid_x + \bar{x} \cdot \phi\mid_{\bar{x}}) \\
&= Pr(x \cdot \phi\mid_x) + Pr(\bar{x} \cdot \phi\mid_{\bar{x}}) \\
&= Pr(x) \cdot Pr(\phi\mid_x) + Pr(\bar{x}) \cdot Pr(\phi\mid_{\bar{x}})
\end{aligned}
$$

Ordered BDDs (OBDDs):

- Variable order π = order of variable eliminations;
  the same variable order on all root-to-leaf paths $\Rightarrow$ ordered BDDs (OBDDs)
- An OBDD for ϕ is uniquely identified by the pair (ϕ, π).

Compilation example

R	A	B	V_r
	a_1	b_1	x_1
	a_2	b_1	x_2
	a_2	b_2	x_3
	a_3	b_3	x_4

S	A	C	V_s
	a_1	c_1	y_1
	a_1	c_2	y_2
	a_2	c_1	y_3
	a_4	c_2	y_4

q :- $R(A, B), S(A, C)$

	V_r	V_s
	x_1	y_1
	x_1	y_2
	x_2	y_3
	x_3	y_3

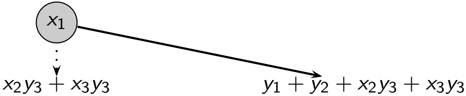
Query q has lineage $\phi = x_1 y_1 + x_1 y_2 + x_2 y_3 + x_3 y_3$.
Assume variable order: $\pi = x_1 y_1 y_2 x_2 x_3 y_3$.
Task: Construct the OBDD (ϕ, π).

Compilation example

Task: Construct OBDD (ϕ, π), where

- $\phi = x_1 y_1 + x_1 y_2 + x_2 y_3 + x_3 y_3$ and $\pi = x_1 y_1 y_2 x_2 x_3 y_3$.

Step 1: Eliminate variable x_1 in ϕ.

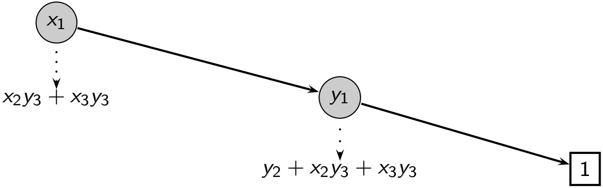


Compilation example

Task: Construct OBDD (ϕ, π), where

- $\phi = x_1 y_1 + x_1 y_2 + x_2 y_3 + x_3 y_3$ and $\pi = x_1 y_1 y_2 x_2 x_3 y_3$.

Step 2: Eliminate variable y_1.

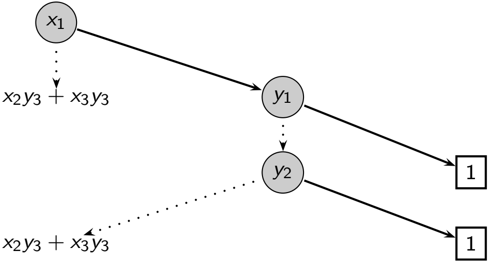


Compilation example

Task: Construct OBDD (ϕ, π), where

- $\phi = x_1y_1 + x_1y_2 + x_2y_3 + x_3y_3$ and $\pi = x_1y_1y_2x_2x_3y_3$.

Step 3: Eliminate variable y_2.



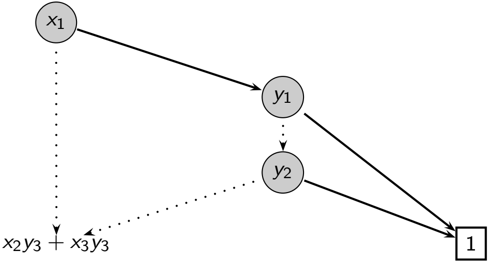
Some leaves have the same expressions $\Rightarrow$ Represent them only once!

Compilation example

Task: Construct OBDD (ϕ, π), where

- $\phi = x_1 y_1 + x_1 y_2 + x_2 y_3 + x_3 y_3$ and $\pi = x_1 y_1 y_2 x_2 x_3 y_3$.

Step 4: Merge leaves with the same expressions.

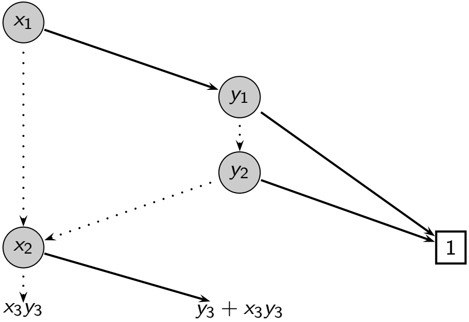


Compilation example

Task: Construct OBDD (ϕ, π), where

- $\phi = x_1 y_1 + x_1 y_2 + x_2 y_3 + x_3 y_3$ and $\pi = x_1 y_1 y_2 x_2 x_3 y_3$.

Step 5: Eliminate variable x_2.

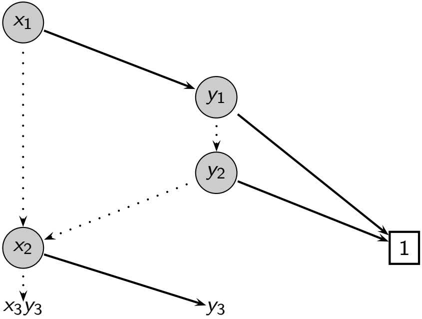


Compilation example

Task: Construct OBDD (ϕ, π), where

- $\phi = x_1 y_1 + x_1 y_2 + x_2 y_3 + x_3 y_3$ and $\pi = x_1 y_1 y_2 x_2 x_3 y_3$.

Step 6: Replace $y_3 + x_3 y_3$ by y_3.

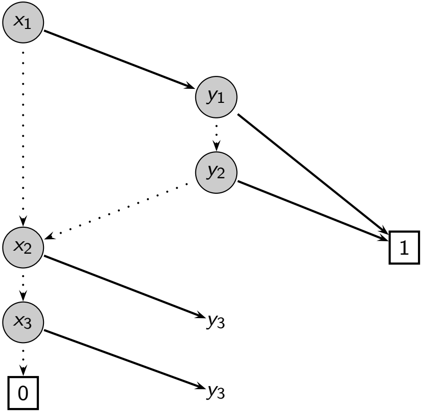


Compilation example

Task: Construct OBDD (ϕ, π), where

- $\phi = x_1 y_1 + x_1 y_2 + x_2 y_3 + x_3 y_3$ and $\pi = x_1 y_1 y_2 x_2 x_3 y_3$.

Step 7: Eliminate variable x_3.

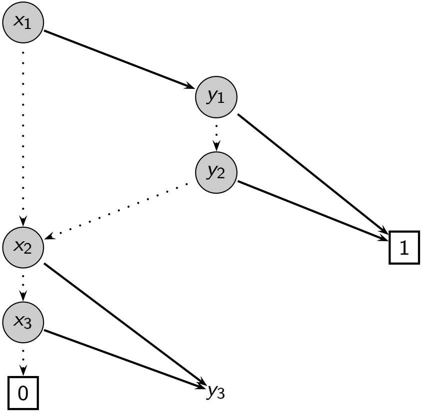


Compilation example

Task: Construct OBDD (ϕ, π), where

- $\phi = x_1y_1 + x_1y_2 + x_2y_3 + x_3y_3$ and $\pi = x_1y_1y_2x_2x_3y_3$.

Step 8: Merge leaves with the same expression y_3.

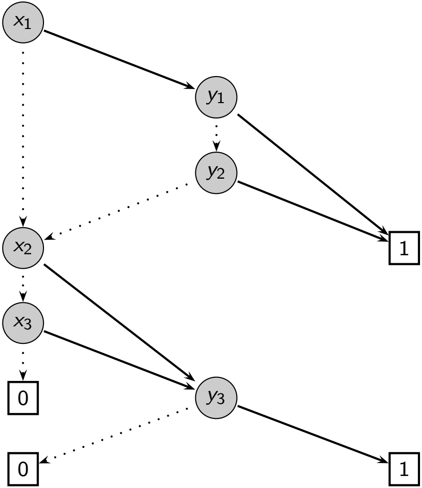


Compilation example

Task: Construct OBDD (ϕ, π), where

- $\phi = x_1 y_1 + x_1 y_2 + x_2 y_3 + x_3 y_3$ and $\pi = x_1 y_1 y_2 x_2 x_3 y_3$.

Step 9: Eliminate variable y_3.

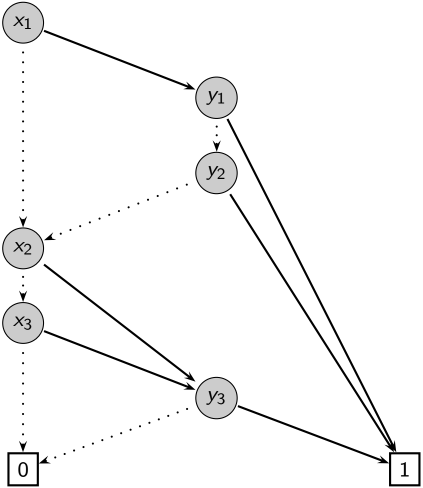


Compilation example

Task: Construct OBDD (ϕ, π), where

- $\phi = x_1y_1 + x_1y_2 + x_2y_3 + x_3y_3$ and $\pi = x_1y_1y_2x_2x_3y_3$.

Step 10 (final): Merge leaves with the same expression (0 or 1).



Compilation example: Summing Up

OBDD (ϕ, π) has size bounded in the number of literals in ϕ.
(exactly one node per variable in ϕ in our example)

Questions

1. Is this property shared by the BDDs of many queries?
2. Can we efficiently construct such succinct BDDs?

Compilation example: Summing Up

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(exactly one node per variable in ϕ in our example)

Questions

1. Is this property shared by the BDDs of many queries?
2. Can we efficiently construct such succinct BDDs?

The answer is in the affirmative for both questions!

Tractable Queries and Succinct BDDs

For any hierarchical query q and database D, $\forall t \in q(D)$, and lineage ϕ_t,

- There is a variable order π computable in time $O(|\phi_t| \cdot \log^2 |\phi_t|)$ such that
- The OBDD (ϕ_t, π) has size $O(|Vars(\phi_t))$ and can be computed in time $O(|\phi_t| \cdot \log |\phi_t|)$.
- ϕ_t can be factorized into read-once functions! [**O.**&Huang08]

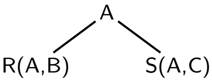
BDD construction in polynomial time for a class of tractable conjunctive queries with inequalities. [**O.**&Huang09]

Good variable orders can be *statically* derived from the query structure!

Static Query Analysis: Query Signatures

Query signatures for TQ queries capture

- the structures of queries and
- the one/many-to-one/many relationships between the query tables;
- variable orders for succinct BDDs representing compiled lineage!



Query q :- $R(A,B), S(A,C)$ has signature $(R^*S^*)^*$.

- There may be several R-tuples with the same A-value, hence R^*
- There may be several S-tuples with the same A-value, hence S^*
- R and S join on A, hence R^*S^*
- There may be several A-values in R and S, hence $(R^*S^*)^*$

Variable orders captured by $(R^*S^*)^*$ (x_i's are from R, y_j's are from S):
$\{[x_1(y_1y_2)][(x_2x_3)y_3]\}$, $\{[(x_2x_3)y_3][x_1(y_1y_2)]\}$, $\{[y_3(x_3x_2)][x_1(y_2y_1)]\}$, etc.

Query Rewriting under Functional Dependencies (FDs)

FDs on tuple-independent databases can help deriving better query signatures.

Given a set of FDs Σ and a conjunctive query of the form

$$Q = \pi_{\overline{A_0}}(\sigma_\phi(R_1(\overline{A_1}) \bowtie \ldots \bowtie R_n(\overline{A_n}))$$

where ϕ is a conjunction of unary predicates. Let $\Sigma_0 = CLOSURE_\Sigma(\overline{A_0})$.
Then, the Boolean query

$$\pi_\emptyset(\sigma_\phi(R_1(CLOSURE_\Sigma(\overline{A_1}) - \Sigma_0) \bowtie \ldots \bowtie R_n(CLOSURE_\Sigma(\overline{A_n}) - \Sigma_0)))$$

is called the **FD-reduct** of Q under Σ. [**O.**&HK09]

If there is a sequence of chase steps under Σ that turns Q into a hierarchical query, then the fixpoint of the chase (the FD-reduct) is hierarchical.

Importance of FD-reducts

The signature of Q's FD-reduct captures the structure of Q's lineage.

Two relevant cases

1. Intractable queries may admit tractable FD-reducts.

   Under $X \to Y$, the hard query Q :- $R(X), S(X, Y), T(Y)$ admits the hierarchical FD-reduct Q' :- $R(X, Y), S(X, Y), T(Y)$ with signature $((RS)^* T)^*$.

2. FD-reducts have more precise query signatures.

   In the presence of keys ckey and okey, the query $Q(\mathit{odate})$:- $\text{Cust}(\mathit{ckey}, \mathit{cname}), \text{Ord}(\mathit{okey}, \mathit{ckey}, \mathit{odate}), \text{Item}(\mathit{okey}, \mathit{disc}, \mathit{ckey})$ with signature $(\text{Cust}^*(\text{Ord}^*\text{Item}^*)^*)^*$ rewrites into

   Q' :- $\text{Cust}(\mathit{ckey}, \mathit{cname}), \text{Ord}(\mathit{okey}, \mathit{ckey}, \mathit{cname}), \text{Item}(\mathit{okey}, \mathit{disc}, \mathit{ckey}, \mathit{cname})$ with signature $(\text{Cust}(\text{Ord Item}^*)^*)^*$.

Case Study: TPC-H Queries

Considered the conjunctive part of each of the 22 TPC-H queries

- Boolean versions (B)
- with original selection attributes, but without aggregates (O)

Hierarchical in the absence of key constraints

- 8 queries (B)
- 13 queries (O)

Hierarchical in the presence of key constraints

- 8+4 queries (B)
- 13+4 queries (O)

In-depth study at

`http://www.comlab.ox.ac.uk/people/dan.olteanu/papers/icde09queries.html`

Secondary-storage Query Evaluation

Query evaluation approached in two logically-independent steps

1. Compute query answer using a *relational* query plan of your choice.
2. Compute probabilities of each distinct answer (or temporary) tuple.

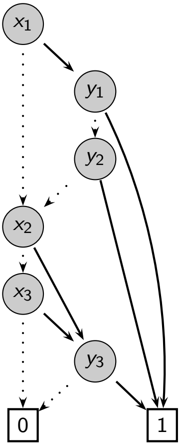
Probability computation supported by a new aggregation operator that can

- blend itself in any relational query plan,
- be placed on top of the query plan, or *partially* pushed down past joins.
- compute in parallel different fragments of the BDD for the lineage *without* materializing the BDD.

Our aggregation operator is a sequence of

- **aggregation** steps. Effect on query signature: $\alpha^* \to \alpha$
- **propagation** steps. Effect on query signature: $\alpha\beta \to \alpha$

Example of Probability Computation



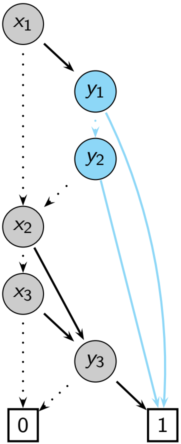
q :- $R(A, B), S(A, C)$

	V_r	V_s
	x_1	y_1
	x_1	y_2
	x_2	y_3
	x_3	y_3

How to proceed?

1. **Sort query answer by** (V_r, V_s).
   Initial signature: $(R^* S^*)^*$
2. Apply aggregation step $S^* \rightarrow S$.
   New signature: $(R^* S)^*$
3. Apply aggregation step $R^* \rightarrow R$.
   New signature: $(RS)^*$
4. Apply propagation step $RS \rightarrow R$.
   New signature: R^*
5. Apply aggregation step $R^* \rightarrow R$.
   New signature: R

Example of Probability Computation



q :- $R(A, B), S(A, C)$

	V_r	V_s
	x_1	$y_1 + y_2$
	x_2	y_3
	x_3	y_3

How to proceed?

1. Sort query answer by (V_r, V_s).
   Initial signature: $(R^* S^*)^*$
2. **Apply aggregation step $S^* \rightarrow S$.**
   New signature: $(R^* S)^*$
3. Apply aggregation step $R^* \rightarrow R$.
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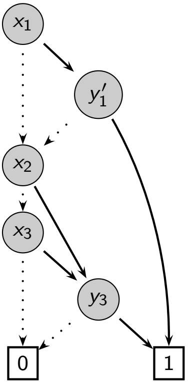
Example of Probability Computation



q :- $R(A,B), S(A,C)$

	V_r	V_s
	x_1	y'_1
	x_2	y_3
	x_3	y_3

How to proceed?

1. Sort query answer by (V_r, V_s).
   Initial signature: $(R^* S^*)^*$
2. **Apply aggregation step $S^* \rightarrow S$.**
   New signature: $(R^* S)^*$
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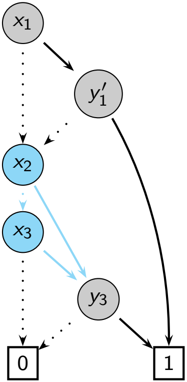
Example of Probability Computation



q :- $R(A,B), S(A,C)$

	V_r	V_s
	x_1	y'_1
	$x_2 + x_3$	y_3

How to proceed?

1. Sort query answer by (V_r, V_s).
   Initial signature: $(R^* S^*)^*$
2. Apply aggregation step $S^* \to S$.
   New signature: $(R^* S)^*$
3. **Apply aggregation step $R^* \to R$.**
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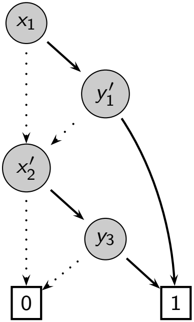
Example of Probability Computation



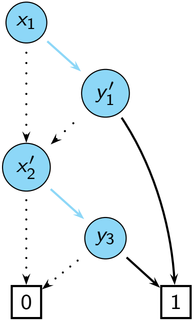
q :- $R(A, B), S(A, C)$

	V_r	V_s
	x_1	y_1'
	x_2'	y_3

How to proceed?

1. Sort query answer by (V_r, V_s).
   Initial signature: $(R^* S^*)^*$
2. Apply aggregation step $S^* \to S$.
   New signature: $(R^* S)^*$
3. **Apply aggregation step $R^* \to R$.**
   New signature: $(RS)^*$
4. Apply propagation step $RS \to R$.
   New signature: R^*
5. Apply aggregation step $R^* \to R$.
   New signature: R

Example of Probability Computation



q :- $R(A, B), S(A, C)$

	V_r
	$x_1 y_1'$
	$x_2' y_3$

How to proceed?

1. Sort query answer by (V_r, V_s).
   Initial signature: $(R^* S^*)^*$
2. Apply aggregation step $S^* \to S$.
   New signature: $(R^* S)^*$
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   New signature: R^*
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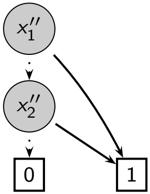
Example of Probability Computation



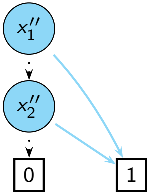
q :- $R(A, B), S(A, C)$

	V_r
	x_1''
	x_2''

How to proceed?

1. Sort query answer by (V_r, V_s).
   Initial signature: $(R^* S^*)^*$
2. Apply aggregation step $S^* \to S$.
   New signature: $(R^* S)^*$
3. Apply aggregation step $R^* \to R$.
   New signature: $(RS)^*$
4. **Apply propagation step** $RS \to R$**.**
   New signature: R^*
5. Apply aggregation step $R^* \to R$.
   New signature: R

Example of Probability Computation



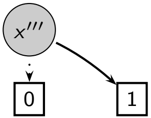
q :- $R(A, B), S(A, C)$

	V_r
	$x_1'' + x_2''$

How to proceed?

1. Sort query answer by (V_r, V_s).
   Initial signature: $(R^* S^*)^*$
2. Apply aggregation step $S^* \to S$.
   New signature: $(R^* S)^*$
3. Apply aggregation step $R^* \to R$.
   New signature: $(RS)^*$
4. Apply propagation step $RS \to R$.
   New signature: R^*
5. **Apply aggregation step $R^* \to R$.**
   New signature: R

Example of Probability Computation



q :- $R(A,B), S(A,C)$

	V_r
	x'''

Return the probability of x'''.

How to proceed?

1. Sort query answer by (V_r, V_s).
   Initial signature: $(R^* S^*)^*$
2. Apply aggregation step $S^* \to S$.
   New signature: $(R^* S)^*$
3. Apply aggregation step $R^* \to R$.
   New signature: $(RS)^*$
4. Apply propagation step $RS \to R$.
   New signature: R^*
5. **Apply aggregation step $R^* \to R$.**
   New signature: R

Can we leverage existing results on BDD construction?

- Generic AI compilation techniques construct BDDs whose sizes are exponential in the treewidth of the lineage [Huang&Darwiche01]

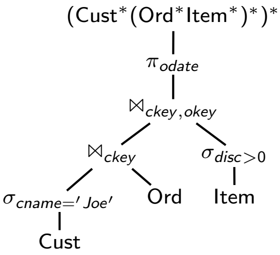
- Conjunctive queries **do** generate lineage of unbounded treewidth.
  - The product query Q :- $R(X), S(Y)$ generates lineage that has a clause for each pair of random variables of R and S $\Rightarrow$ unbounded treewidth.

- Reconciling the two techniques [Jha,**O.**&Suciu10]:
  - Partition input query+data into a tractable subinstance and a (usually much smaller) hard subinstance.
  - Tractable subinstance: largest sub-relation satisfying functional dependency
  - Apply scalable database-specific techniques to the tractable part and generic AI compilation techniques to the hard part.

Query Optimization: Types of Query Plans

Our previous examples considered *lazy* plans, where

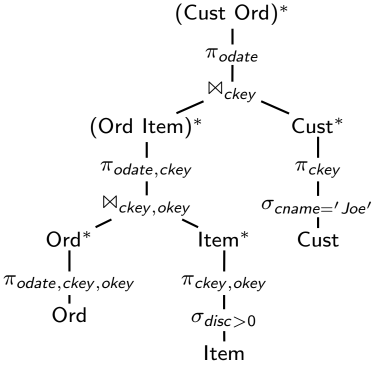
- probability computation done *after* the computation of answer tuples
- unrestricted search space for good query plans
- especially desirable when join conditions are selective (eg, TPC-H)!



BUT, we can push down probability computation!

Query Optimization: Types of Query Plans

Eager plans discard duplicates and compute probabilities on each temporary table.



MystiQ's safe plans are special cases of eager plans!

- mirror the hierarchical structure of the query signature
- probability computation restricts join ordering!
- suboptimal join ordering, which is more costly than probability computation

Approximate Query Evaluation

Approximate Query Evaluation using Monte Carlo

Using Karp-Luby FPRAS [Karp&Luby83], [Graedel&Gurevitch&Hirsch98]

Input: Boolean formula in DNF $\phi = C_1 + \cdots + C_m$ with variables $V(\phi)$

$Cnt \leftarrow 0; S \leftarrow Pr(C_1) + \cdots + Pr(C_m)$

repeat N times

  randomly choose $1 \leq i \leq m$ with probability $Pr(C_i)/S$

  randomly choose a total valuation λ over $V(\phi)$ such that $\lambda(C_i) =$ true

  if $\forall 1 \leq j < i : \lambda(C_j) =$ false then $Cnt = Cnt + 1$

$P = Cnt/N \times S/2^{|V(\phi)|}$

return P/* $\approx Pr(\phi)$*/

If $N \geq (1/m) \times (4 \ln(2/\delta)/\epsilon^2)$ then $Pr[| P/Pr(\phi) - 1 |> \epsilon] < \delta$.

- Slightly modified algorithms are used in MayBMS, MystiQ, and MCDB.
- Veeeery sloooow in practice: SPROUT query plans are about two orders of magnitude faster that the optimized Monte Carlo.

Approximate Evaluation with SPROUT

- Monte Carlo simulations are very powerful and generic
  - only require sampling the formula, no knowledge of its structure

- Why not exploit the structure of the input formula? [**O.**&HK10]
  - incrementally compile it into an equivalent decomposed form that allows for efficient probability computation
  - in practice, good approximations are obtained after a few decomposition steps
  - Topic of my ICDE10 talk tomorrow :-)

Thanks!

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