Higher algebra in quantum information theory

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University of Oxford

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Based on joint work with Jamie Vicary:

Biunitary constructions in quantum information

Shaded tangles for the design and verification of quantum programs

Part 1
Shaded tensor networks & biunitaries

Quantum structures

Let's start with a very concrete problem.

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Hadamard matrices H

$$|H_{i,j}|^2 = 1 \qquad H^\dagger H = n\mathbb{1}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Quantum structures

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Hadamard matrices H	unitary error bases (UEB) $\{U_i\}_{1\leq i\leq n^2}$
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Important in quantum information ... but hard to construct.
Only a handful of known constructions, for example:

$$\text{Hadamard} + \text{Hadamard} + \text{Hadamard} \rightsquigarrow \text{UEB}$$

$$(U_{ab})_{c,d} = \frac{1}{\sqrt{n}} A_{a,d} B_{b,c} C_{c,d}$$

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An algebraic problem?

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Why do they work? Where do they come from? How can we find them?

A higher algebraic problem!

What is higher algebra?

- Ordinary algebra lets us compose along a line:

$$xy^2zyx^3$$

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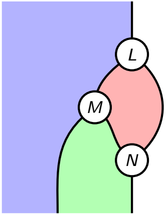
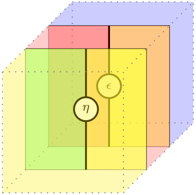
- *Higher algebra* lets us compose in higher dimensions:





Planar algebra = 2-category theory

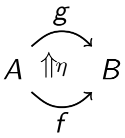
The language describing algebra in the plane is *2-category theory*:

A

objects

$A \xrightarrow{f} B$

1-morphism



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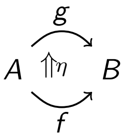
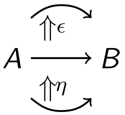
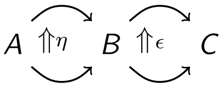


2-morphism

We can compose 2-morphisms like this:



vertical composition



horizontal composition

These are *pasting diagrams*.

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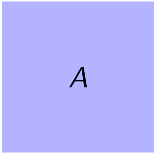
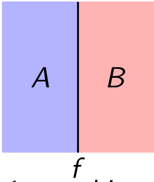


objects



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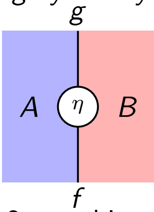

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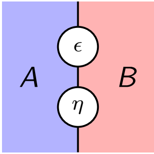

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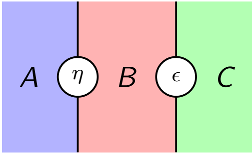

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The *dual* diagrams are the graphical calculus.

Monoidal dagger pivotal 2-categories

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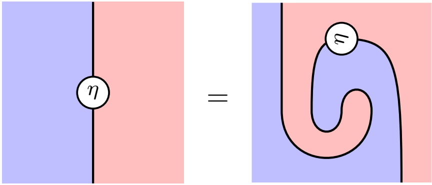

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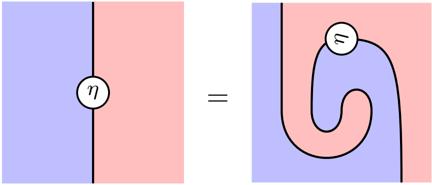

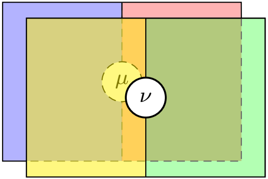

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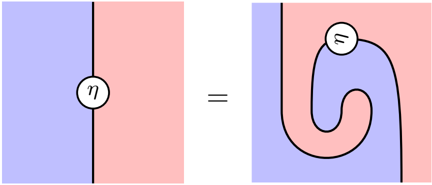

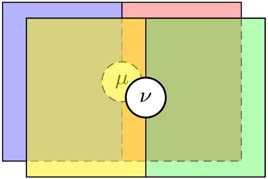

$\Rightarrow$ surfaces, wires and vertices in three-dimensional space

A model for quantum computation: **2Hilb**

We work in the 2-category **2Hilb**, a categorification of **Hilb**.

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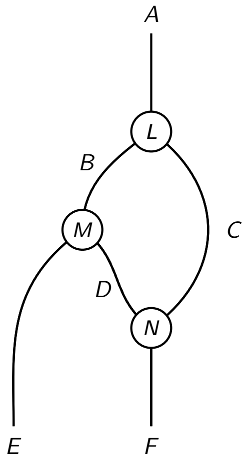
This well-studied structure plays a key role in *higher representation theory*.

A direct perspective: tensor networks

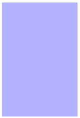
vector space
V

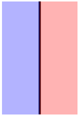
linear map
$F : V \to W$

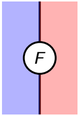


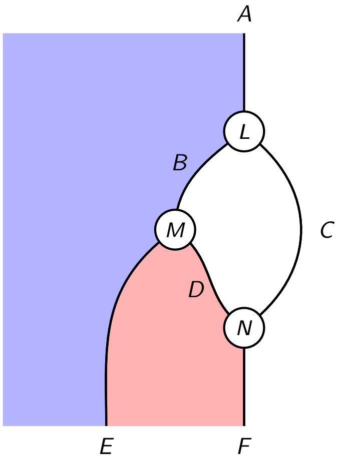
A (composed) linear map
$E \otimes F \to A$

A direct perspective: shaded tensor networks



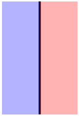


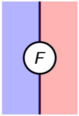


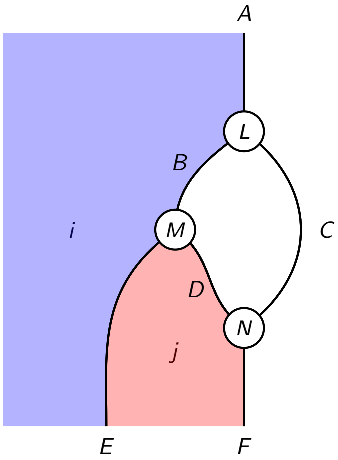


A direct perspective: shaded tensor networks

indexing set
$i \in S$







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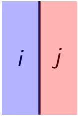
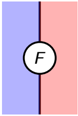
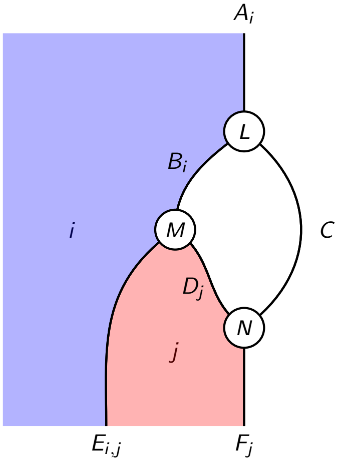


family of vector spaces $V_{i,j}$





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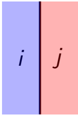

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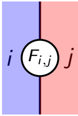
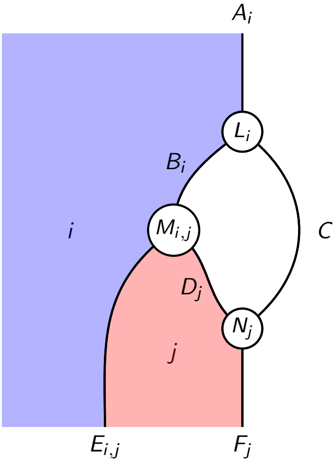


family of linear maps
$F_{i,j} : V_{i,j} \to W_{i,j}$



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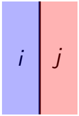

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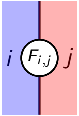

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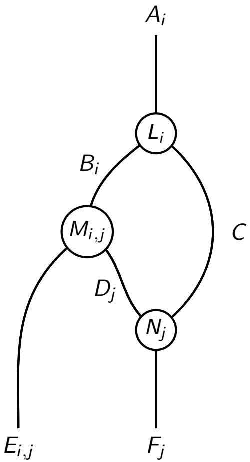

A family of linear maps, indexed by i and j
$E_{i,j} \otimes F_j \to A_i$

A direct perspective: shaded tensor networks

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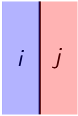

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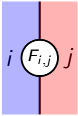

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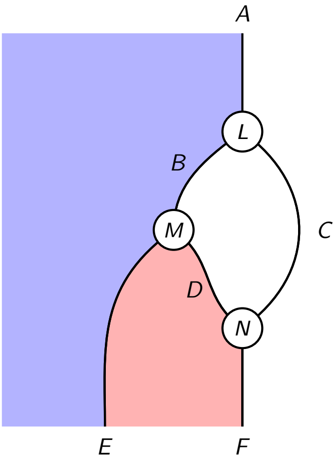

A family of linear maps, indexed by i

$$E_{i,j} \otimes F_j \to A_i$$

Biunitarity

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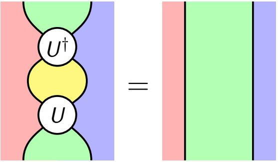
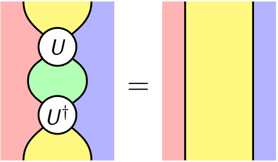
- *(vertically) unitary*:





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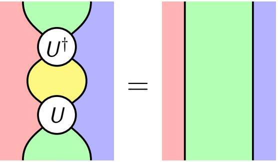

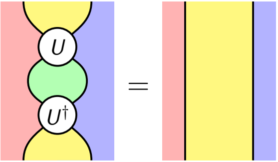

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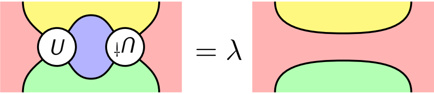

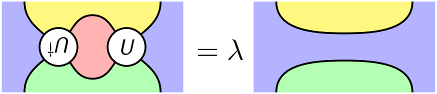

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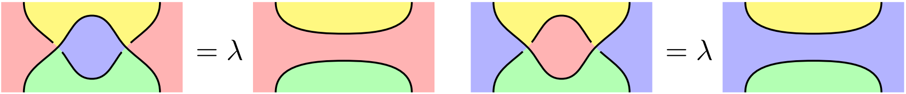
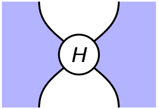


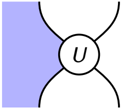
These look just like the *second Reidemeister move*.

Quantum structures are biunitaries in **2Hilb**

Result 1: Hadamards and UEBs are biunitaries of the following type:



Hadamard



UEB

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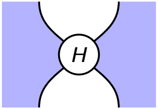
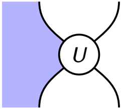
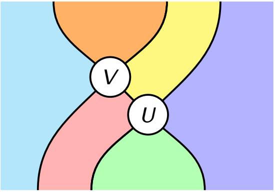


Hadamard



UEB

Result 2: We can compose biunitaries diagonally:



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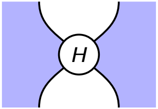

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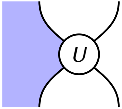

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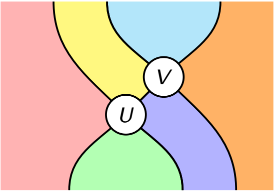
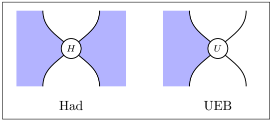
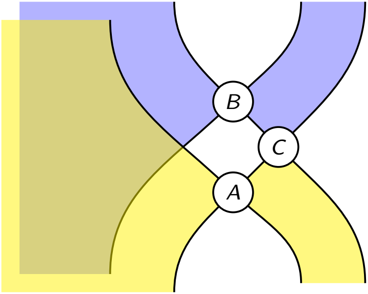


Composing quantum structures





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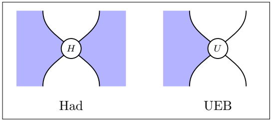

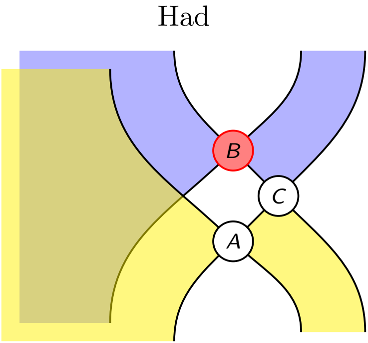

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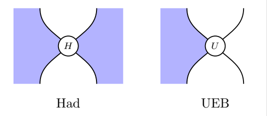
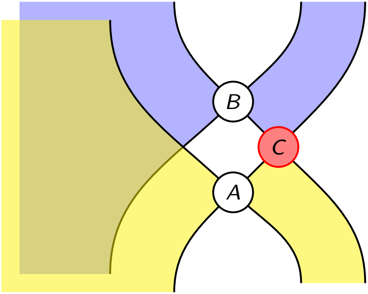


$$\text{Had} + \text{Had}$$



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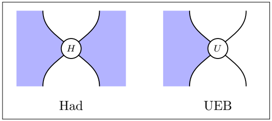

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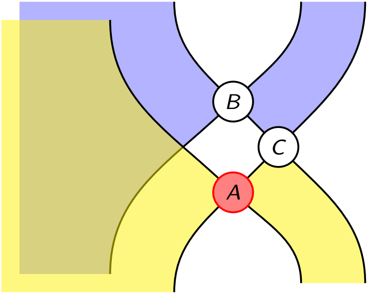

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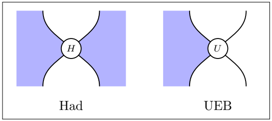
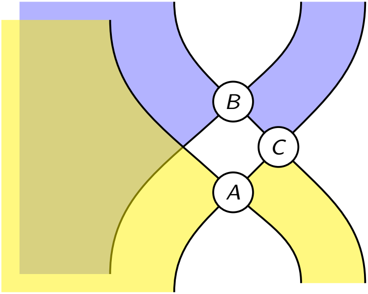


$$\text{Had} + \text{Had} + \text{Had} \rightsquigarrow \text{UEB}$$



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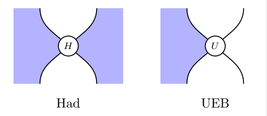

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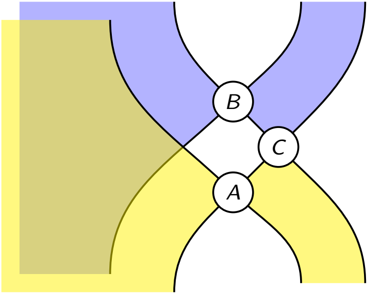

$$(U_{ab})_{c,d} = \frac{1}{\sqrt{n}} A_{a,d} B_{b,c} C_{c,d}$$

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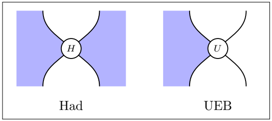

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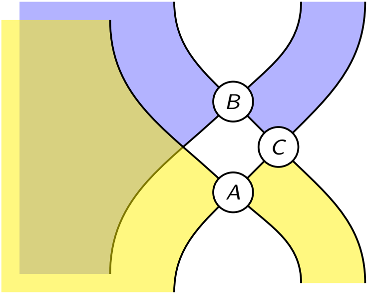

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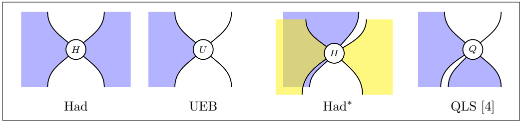
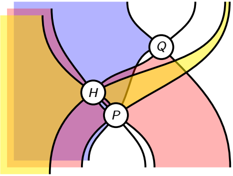
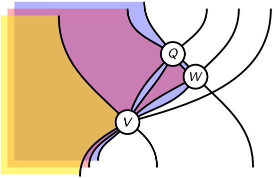




$U_{abc,de,fg} = H^{b,c}_{a,eg} P^{c,g}_{e,b,f} Q_{c,g,d}$



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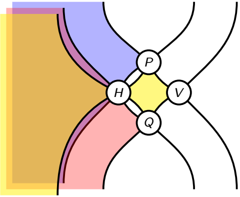

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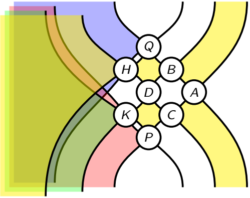

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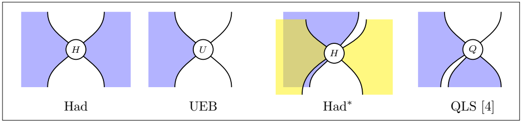

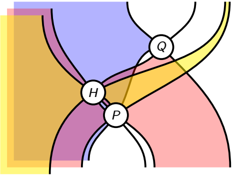

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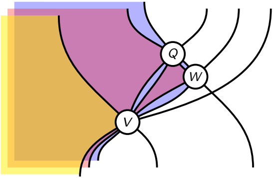

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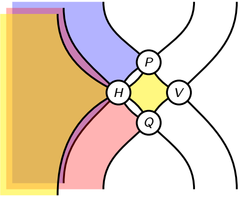

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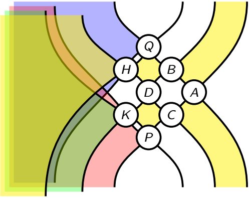

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  see structural properties hidden in conventional matrix notation

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Recall:

Hadamard matrix

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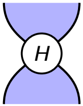
Hadamard matrix $\rightsquigarrow$ 

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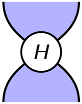
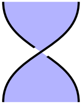
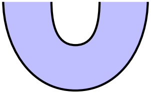
Hadamard matrix $\rightsquigarrow$ 

$\rightsquigarrow$ 

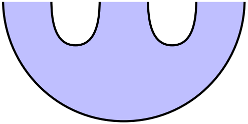
Part 2
Untangling quantum circuits

Basic states and gates

$$|+\rangle = |0\rangle + |1\rangle$$



$$|\mathrm{Bell}\rangle = |00\rangle + |11\rangle$$



$$|\mathrm{GHZ}\rangle = |000\rangle + |111\rangle$$

Basic states and gates

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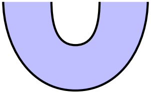

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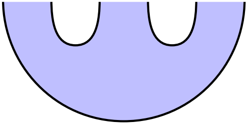

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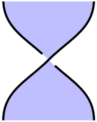
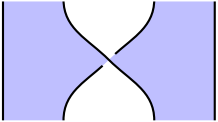


$|i\rangle \mapsto \sum_j H_{ij} |j\rangle$



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Basic states and gates

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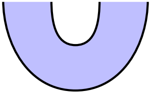

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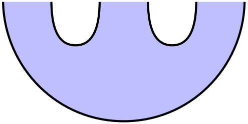

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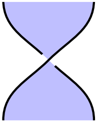
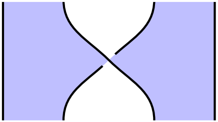


Hadamard gate



CZ gate

Creating GHZ states

How to create a GHZ state from $|+\rangle$ states?

Creating GHZ states

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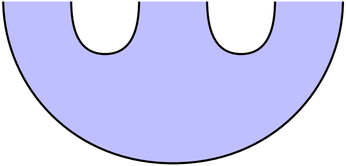

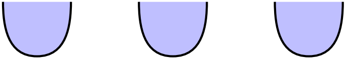

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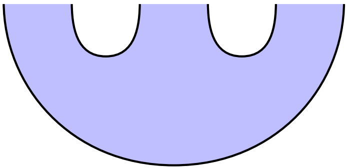

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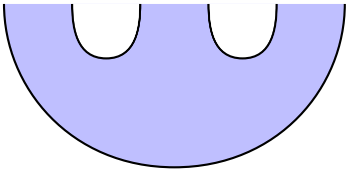

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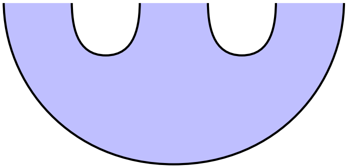

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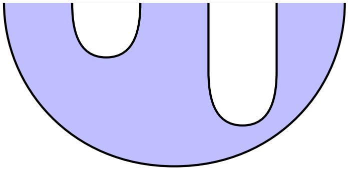

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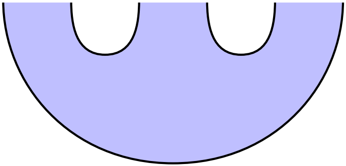

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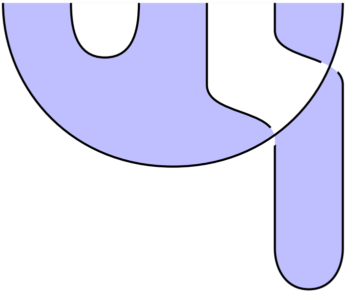

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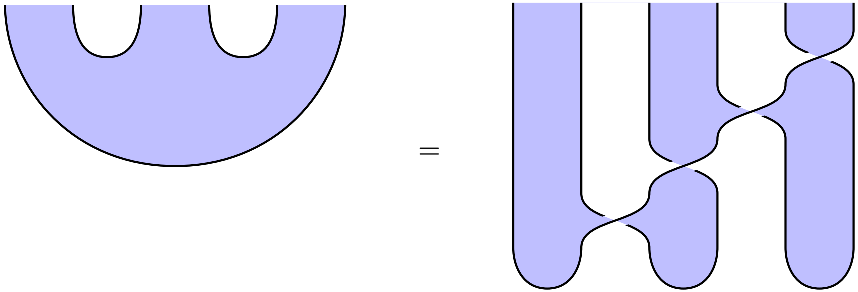
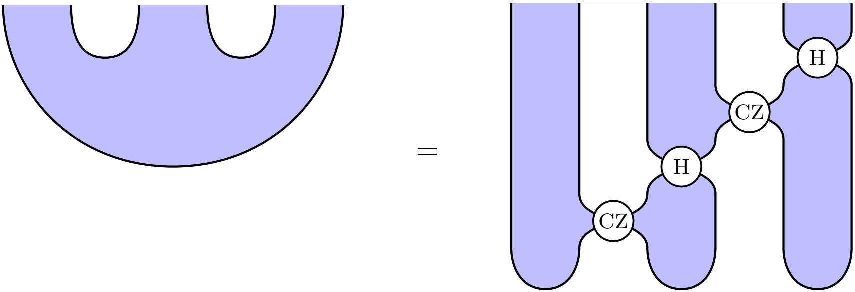


Creating GHZ states

How to create a GHZ state from $|+\rangle$ states?



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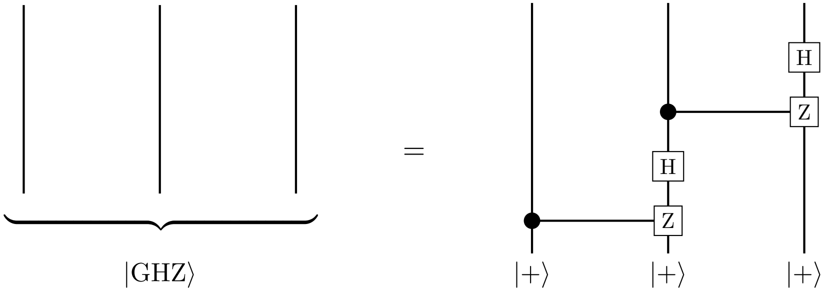

Quantum error correction

A k-local quantum code is an isometry $H \xrightarrow{\text{enc}} H^{\otimes n}$, s.t.

$$H \xrightarrow{\text{enc}} H^{\otimes n} \xrightarrow{E} H^{\otimes n} \xrightarrow{\text{enc}^\dagger} H$$

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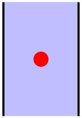

phase error

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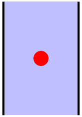
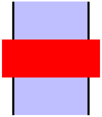


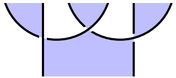
phase error



full error

The phase code

The following is a 2−local phase error code $H \to H^{\otimes 3}$:



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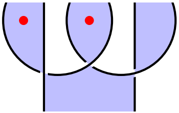

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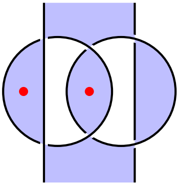

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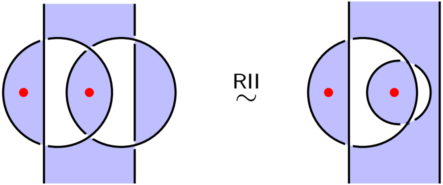

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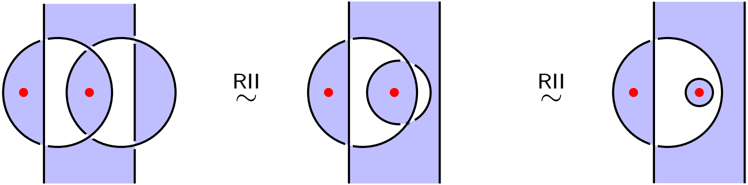

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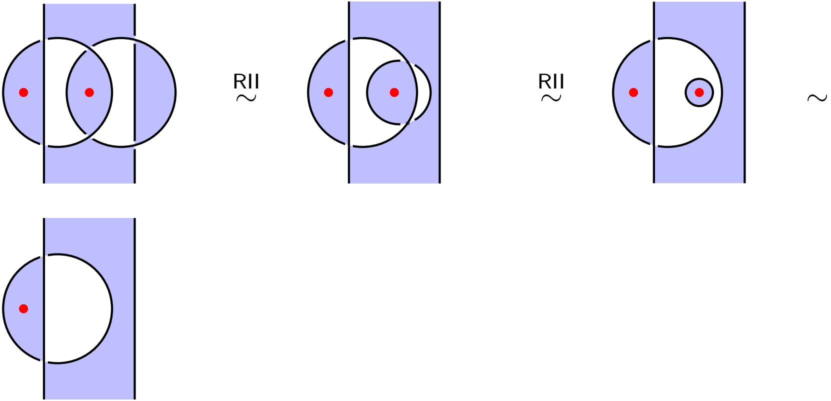

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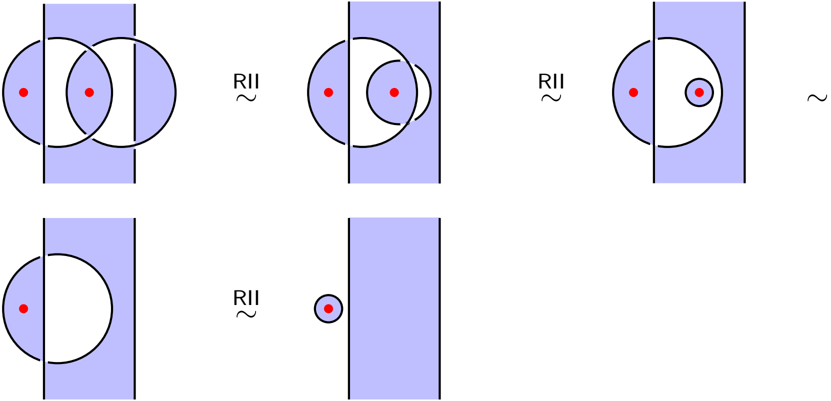

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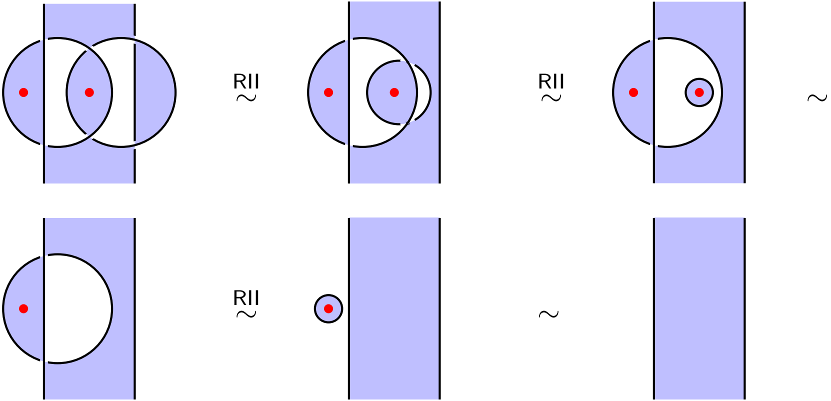

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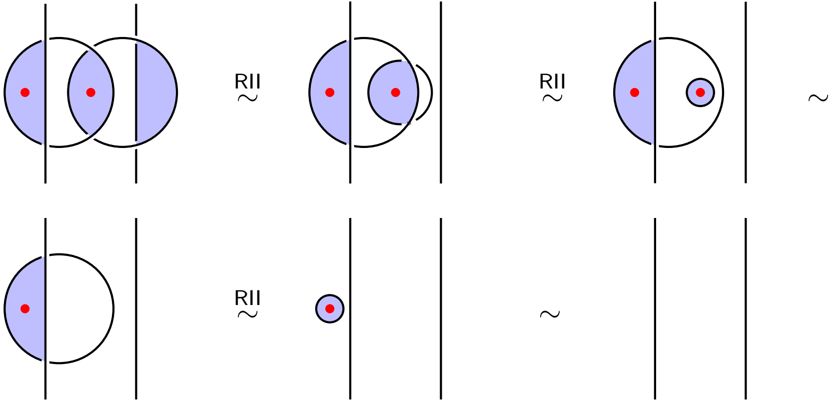
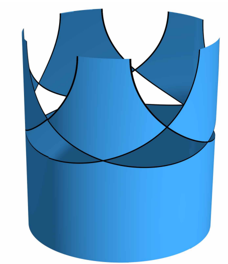
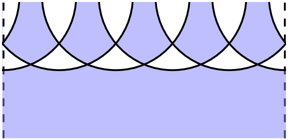


New construction of a phase code from unitary error bases.

Future work: The 5-qubit code

A 2−local full error correcting code $H \to H^{\otimes 5}$:





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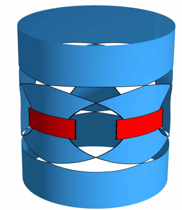

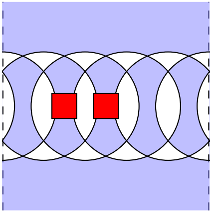

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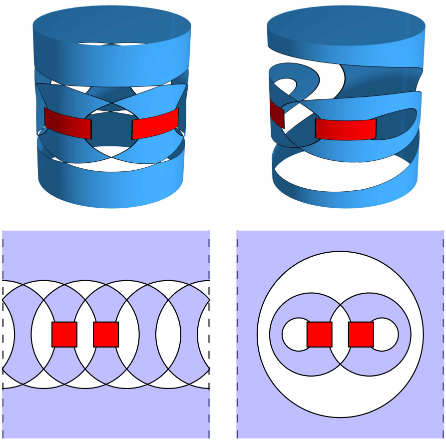

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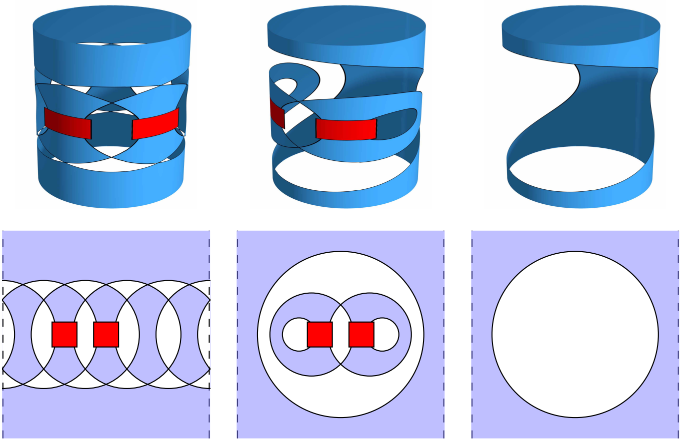

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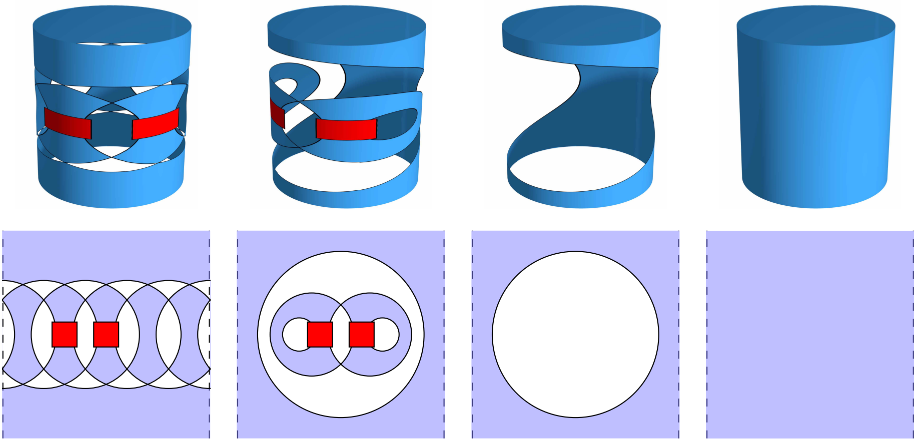

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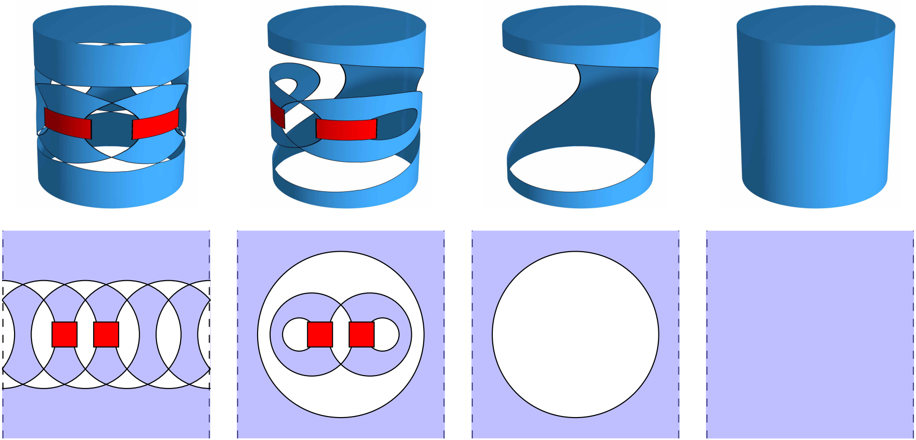

Caveat: We cannot yet handle two non-adjacent errors.

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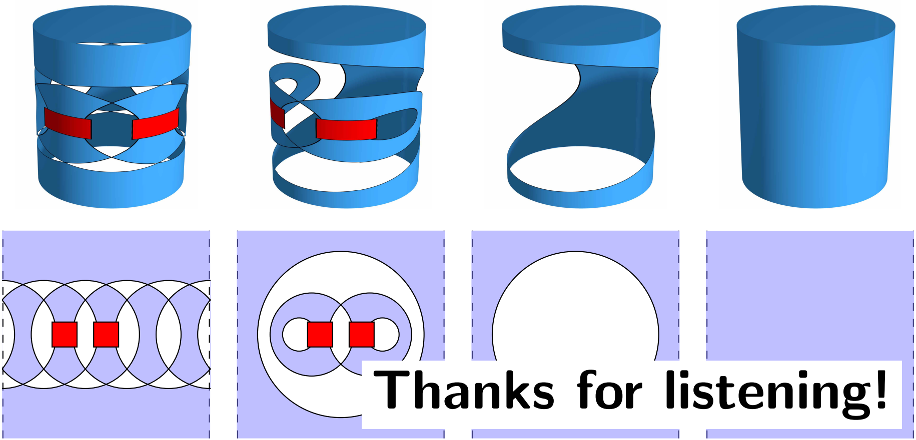

Thanks for listening!

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