On quantum statistics in data analysis

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Outline

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Problem of latent semantics

E.g., the Netflix recommender system challenge:

Given a matrix of movie ratings:

	"Nemo"	"Solaris"	"Crash"	"Ikiru"
Abby	★★★★	★★★★★	★★	
Dusko	★★	★★★	★★	★★★★
Stef	★★	★	★★★★★	★
Temra	★		★★★	★★★★
Luka	★★★★★			★

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predict Luka's ratings for "Solaris" and "Crash".

Solution by classification

Analyze the matrix minors:

	"Nemo"	"Solaris"	"Crash"
Abby	1	2	–1
Dusko	–1	0	–1
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to classify the movie styles,

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. . . and the transposes:

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to classify viewers' tastes.

Relate Luka's taste to other viewer's tastes.
Extrapolate his future ratings.

Summary of the approach

Latent semantics

Extract the tastes and the styles from pattern matrices.

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Extract the tastes and the styles from pattern matrices.

Prediction from similarity.

Predict future ratings
from past ratings
of similar movies
by similar users.

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Pattern matrices

Latent semantics is mined from a map

$$\mathrm{J} \times \mathrm{U} \xrightarrow{A} \mathrm{R}$$

where

- J is a set of *objects*, or *items*,
- U is a set of *properties*, or *users*,
- R is a set of *values*, or *ratings*.

This map is conveniently presented as a *pattern matrix* $A = (A_{iu})_{\mathrm{J} \times \mathrm{U}}$.

Examples

domain	J	U	R	A_{iu}
text analysis	documents	terms	$\mathbb{N}$	occurrence
measurement	instances	quantities	$\mathbb{R}$	outcome
user preference	items	users	$\{0, \ldots, 5\}$	rating
topic search	authorities	hubs	$\mathbb{N}$	hyperlinks
concept analysys	objects	attributes	$\{0, 1\}$	satisfaction
elections	candidates	voters	$\{1, \ldots, n\}$	preference
market	producers	consumers	$\mathbb{Z}$	deliveries
digital images	images	pixels	$[0, 1]$	intensity

Pattern algebra

Matrix algebra over a rig

Pattern matrices induce linear operators

$$\cfrac{\cfrac{\mathsf{J}\times\mathsf{U}\xrightarrow{A}\mathsf{R}}{\mathsf{U}\xrightarrow{\overline{A}}\mathsf{R}^{\mathsf{J}}}}{\mathcal{U}=\mathsf{R}^{\mathsf{U}}\xrightarrow{B}\mathsf{R}^{\mathsf{J}}=\mathcal{J}}$$

which map tastes $x, y \ldots \in \mathsf{R}^{\mathsf{U}}$ to styles $a, b \ldots \in \mathsf{R}^{\mathsf{J}}$.

Rigs of ratings

Definition

A *rig*[1] is a structure

$$\mathsf{R} \quad = \quad (\mathsf{R}, +, \cdot, 0, 1)$$

where

- $(\mathsf{R}, +, 0)$ and $(\mathsf{R}, \cdot, 1)$ are commutative monoids, with
- $a(b + c) = ab + ac$ and $a0 = 0$.

[1]"ring without the negative elements"

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Examples

- natural numbers: $(\mathbb{N}, +, \cdot, 0, 1)$
- non-negative reals: $(\mathbb{R}_+, +, \cdot, 0, 1)$
- any distributive lattice: $(\mathbb{D}, \vee, \wedge, \bot, \top)$

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Conjugation

Definition

A *rig with conjugation* is rig R given with an automorphism (or antiisomorphism)

$$\overline{(-)} \quad : \quad \mathrm{R} \longrightarrow \mathrm{R}$$

such that

$$\overline{\overline{a}} \quad = \quad a$$

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Examples

- any complex cone: $\mathbb{C}_+$, with $\overline{a + ib} = a - ib$
- any boolean algebra: $\mathbb{B}$ with $\overline{a} = \neg a$,
- any rig R with $\overline{a} = a$.

Adjunction and inner product

Definition

Over a rig with conjugation R, the operation of *adjunction* maps the pattern matrices as follows:

$$\frac{J \times U \xrightarrow{A} R}{U \times J \xrightarrow{A^\ddagger} R}$$

where the entries of $A^\ddagger$ are

$$A^\ddagger_{ui} = \overline{A}_{iu}$$

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Inner product

The *inner product* of vectors $x, y \in \mathsf{R}^{\mathsf{U}}$ is defined

$$\langle x|y\rangle = y^{\ddagger} \circ x$$

Isometries and unitaries

Definition

An operator $U : \mathcal{U} \longrightarrow \mathcal{J}$ is an *isometry* if

$$\langle x|y\rangle \quad = \quad \langle Ux|Uy\rangle$$

holds for all $x, y \in \mathcal{U}$.

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A vector $u \in \mathcal{U}$ is a *unit* if $\langle u|u\rangle = 1$.

Upshot

Similarity is angle: the inner product of unit vectors.

- represent tastes by $x \in \mathbb{R}^U$, $\langle x|x\rangle = 1$
- the similarity $x, y \in \mathbb{R}^U$ is $\langle x|y\rangle$.

Upshot

Similarity is angle: the inner product of unit vectors.

- represent tastes by $x \in \mathbb{R}^U$, $\langle x|x\rangle = 1$
- the similarity $x, y \in \mathbb{R}^U$ is $\langle x|y\rangle$.

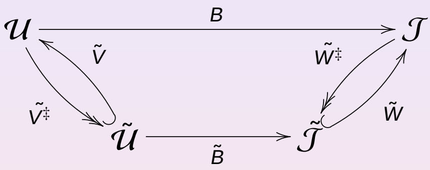
Concepts are geometric invariants: stalks of unit vectors

- preserved by isometries

Isometric decomposition of pattern operators

Definition

An *isometric decomposition* of an operator $B : \mathcal{U} \longrightarrow \mathcal{J}$ is in the form $B = \widetilde{W}\widetilde{B}\widetilde{V}^{\ddagger}$

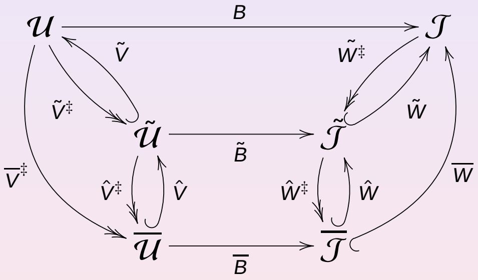


where $\widetilde{V}$ and $\widetilde{W}$ are isometries.

Spectral decomposition of pattern operators

Definition

The *spectral decomposition* $B = \overline{WBV}^{\ddagger}$ is minimal among B's isometric decompositions:



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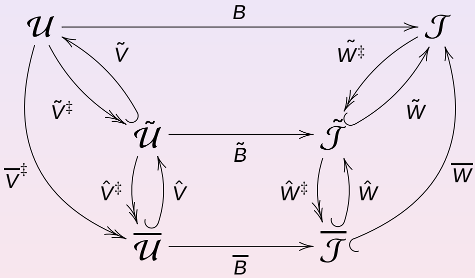

For every isometric decomposition $B = \widetilde{W}\widetilde{B}\widetilde{V}^{\ddagger}$
there is an isometric decomposition $\widetilde{B} = \widehat{W}\overline{B}\widehat{V}^{\ddagger}$,
such that $\overline{W} = \widetilde{W}\widehat{W}$ and $\overline{V} = \widetilde{V}\widehat{V}$.

Formal Concept Analysis

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where $2 = \{0, 1\}$

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The conjugation

$$\bar{\imath} = \neg i$$

is an automorphism $(2, \vee, \wedge, 0, 1) \longrightarrow (\widetilde{2}, \wedge, \vee, 1, 0)$.

Formal Concept Analysis

A pattern matrix is a binary relation $J \times U \xrightarrow{A} 2$.

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A pattern matrix is a binary relation $\mathsf{J} \times \mathsf{U} \xrightarrow{A} 2$.

The induced linear operators are antitone, so we write

$$\frac{\dfrac{\dfrac{\mathsf{J} \times \mathsf{U} \xrightarrow{A} \mathsf{R}}{\mathsf{U} \xrightarrow{\neg A} \mathsf{R}^{\mathsf{J}}}}{2^{\mathsf{U}} \xrightarrow{B} 2^{\mathsf{J}} \xrightarrow{\neg} \widetilde{2}^{\mathsf{J}}}}{\widetilde{2}^{\mathsf{J}} \xrightarrow{\widetilde{\neg}} 2^{\mathsf{J}} \xrightarrow{B^{\ddagger}} 2^{\mathsf{U}}}$$

where

$$\begin{aligned} B(X) &= \{i \in \mathsf{J} \mid \exists u \in X.\ \neg uAi\} \\ B^{\ddagger}(Y) &= \{u \in \mathsf{U} \mid \forall i \notin Y.\ uAi\} \end{aligned}$$

Formal Concept Analysis

The adjunction

$$B(X) \subseteq Y \quad \Longleftrightarrow \quad X \subseteq B^{\ddagger}(Y)$$

yields the Galois connection

$$Y \subseteq \neg B(X) \quad \Longleftrightarrow \quad X \subseteq B^{\ddagger}(\neg Y)$$

which induces the closure operators

$$M^{\mathsf{U}} = B^{\ddagger}\tilde{\neg}\tilde{\circ}\neg B(X) = \{u \in \mathsf{U} \mid \forall i \in \mathsf{J}.\ (\forall v \in X.\ iAv) \Rightarrow iAu\}$$

$$M^{\mathsf{J}} = \neg B \circ B^{\ddagger}(\tilde{\neg} Y) = \{i \in \mathsf{J} \mid \forall u \in \mathsf{U}.\ (\forall j \in Y.\ jAu) \Rightarrow iAu\}$$

where $\tilde{\circ}$ is the matrix multiplication over $\widetilde{2}$:

$$(P \tilde{\circ} Q)_{ik} = \bigwedge_j P_{ij} \vee Q_{kl}$$

Formal Concept Analysis

The lattices of closed sets

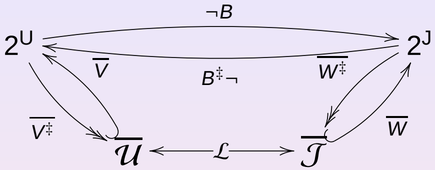
$$\overline{\mathcal{U}} = \{X \in 2^{\mathsf{U}} \mid M^{\mathsf{U}}(X) = X\}$$
$$\overline{\mathcal{J}} = \{Y \in 2^{\mathsf{J}} \mid M^{\mathsf{J}}(Y) = Y\}$$

are isomorphic, because they are both isomorphic with

$$\mathcal{L} = \Big\{\langle X, Y\rangle \in \wp\mathsf{U} \times \wp\mathsf{J} \mid \neg B(X) = Y \wedge B^{\ddagger}(\neg Y) = X\Big\}$$

Formal Concept Analysis

The spectral decomposition



is induced by this isomorphism of

- the strongest tastes in $\overline{\mathcal{U}}$, and
- the strongest styles in $\overline{\mathcal{J}}$.

Example

The pattern matrix

	Dusko	Stef	Temra	Luka
"Nemo"	−1	−1	−2	2
"Ikiru"	1	−2	1	−2

Example

. . . induces the relation

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...and the taste vectors in the form

$$x = M^{\mathrm{U}}x = B^{\ddagger} \,\tilde{\circ}\, B(x) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \tilde{\circ} \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \circ \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

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$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \quad \cdots \quad = \begin{pmatrix} x_0 \vee x_1 \vee x_2 \\ x_1 \\ x_0 \vee x_1 \vee x_2 \\ x_1 \vee x_3 \end{pmatrix} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

Latent Semantic Indexing

The rig of ratings $\mathsf{R} = \mathbb{R}$ is the field of real numbers, the conjugation $\overline{a} = a$ is trivial, adjunction $A^{\ddagger} = A^{T}$ is matrix transposition.

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Spectral decomposition of $A : \mathbb{R}^{\mathsf{U}} \longrightarrow \mathbb{R}^{\mathsf{J}}$ is the Singular Value Decomposition.

The styles are the eigenspaces of $AA^{\ddagger}$, while
the tastes are the eigenspaces of $A^{\ddagger}A$.

Concepts as geometric invariants

Summary so far

Isometric decomposition gives concepts (e.g. styles, or tastes) as invariant sets.

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Consequence

Concept lattices are not distributive.

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van Rijsbergen (2004)

The problems of Information Retrieval and Classification lead to quantum statistics:

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The problems of Information Retrieval and Classification lead to quantum statistics:

- pure concepts are rays,
  mixed concepts density operators,
- operations are unitary,
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- measurable spaces are orthomodular,
  not boolean,
- probability measures are à la Mackey-Gleason,
  not Kolmogorov.

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Problems of using quantum statistics in classification

- no simultaneous spectral decomposition of two or more pattern matrices
  - unless they commute with each other
- concepts may be indistinguishable by measurement (sampling)
  - unless they are orthogonal
- concepts may not be copied or deleted by semantical (isometric) operations
  - unless derived from a spectral decomposition

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Similarity

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Definition

The *similarity* of the tastes $x, y \in \mathrm{R}^{\mathrm{U}}$ is the angle of the styles to which they correspond by A

$$S(x,y) \quad = \quad \langle x|A^{\ddagger}A|y\rangle = \langle Ax|Ay\rangle$$

Deriving future agreement from past similarity

Data

- normalized vectors $x, y : \mathsf{J} \longrightarrow \mathsf{R}$
  - past preferences
- random variables $X, Y : \mathsf{J}' \longrightarrow \{0, 1\}$
  - future approvals

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- predict $\mathrm{P}(X = Y) \in [0, 1]$
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  - similarity
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  - probability of agreement

Try

- $\mathrm{P}(X = Y) \;=\; 2\mathrm{S}(x, y) - 1$

Lemma

Any three random variables $X, Y, Z : \mathsf{J}' \longrightarrow \{0, 1\}$ satisfy

$$\mathsf{P}(X \neq Z) \quad \leq \quad \mathsf{P}(X \neq Y) + \mathsf{P}(Y \neq Z) \tag{1}$$

Proof

Let $W_{XY} : \mathrm{U} \longrightarrow \{0,1\}$ be the random variable

$$W_{XY}(i) = \begin{cases} 1 & \text{if } X(i) \neq Y(i) \\ 0 & \text{if } X(i) = Y(i) \end{cases}$$

We claim that

$$W_{XZ} \leq W_{XY} + W_{YZ}$$

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Towards the contradiction, suppose that there is $j \in J$ with

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and thus

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which is impossible.

Proof (3)

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Hence

$$W_{XZ} \quad \leq \quad W_{XY} + W_{YZ}$$

But since $\mathsf{P}(X \neq Y) = \mathsf{E}(W_{XY})$, this gives

$$\mathsf{P}(X \neq Z) \quad \leq \quad \mathsf{P}(X \neq Y) + \mathsf{P}(Y \neq Z)$$

Proposition

Let the past preferences of the tastes x_0, x_1, y_0, y_1 be given as unit vectors $x_0, x_1, y_0, y_1 \in \mathbb{R}^U$.

If the probability of their future agreement is proportional to the similarity of their past preferences

$$P(X = Y) \quad = \quad 2S(x, y) - 1$$

then they must satisfy

$$S(x_0, y_1) + S(x_1, y_1) + S(x_1, y_0) - S(x_0, y_0) \quad \leq \quad 2$$

Proof

Since

$$P(X \neq Y) = 1 - P(X = Y) \text{ and}$$
$$P(X = Y) = \frac{1 + S(x, y)}{2}$$

it follows that

$$P(X \neq Y) = \frac{1 - S(x, y)}{2}$$

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From the Lemma it follows that

$$P(X_0 \neq Y_0) \leq \begin{cases} P(X_0 \neq Y_1) + \\ P(Y_1 \neq X_1) + \\ P(X_1 \neq Y_0) \end{cases}$$

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...which becomes

$$1 - S(x_0, y_0) \leq \begin{cases} 1 - S(x_0, y_1) + \\ 1 - S(x_1, y_1) + \\ 1 - S(x_1, y_0) \end{cases}$$

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. . . and finally

$$S(x_0, y_1) + S(x_1, y_1) + S(x_1, y_0) - S(x_0, y_0) \leq 2$$

Corollary

The probability of future agreement cannot be derived by rescaling past similarity.

Proof

The vectors

$$x_0 = (1,0) \qquad x_1 = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$y_0 = (-1,0) \qquad y_1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

provide a counterexample for the Proposition

$$\underbrace{S(x_0,y_1)}_{\frac{1}{2}} + \underbrace{S(x_1,y_1)}_{\frac{1}{2}} + \underbrace{S(x_1,y_0)}_{\frac{1}{2}} - \underbrace{S(x_0,y_0)}_{(-1)} \leq 2$$

Outline

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Try

- test spaces: probability beyond simplices

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- as soon as the past correspondences do not warrant joint sampling, or statistical mixing