

Local Reasoning for RCU

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Joint work with
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and Viktor Vafeiadis

Preparing Multicore Era

- Active research by verification communities.
- Various workshops/conferences (E.g. EC²).
- At least 9 CAV papers on concurrency.
- Concurrency is a hot topic inside separation logic.

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From “Shape-value abstraction for verifying linearizability” by Viktor Vafeiadis

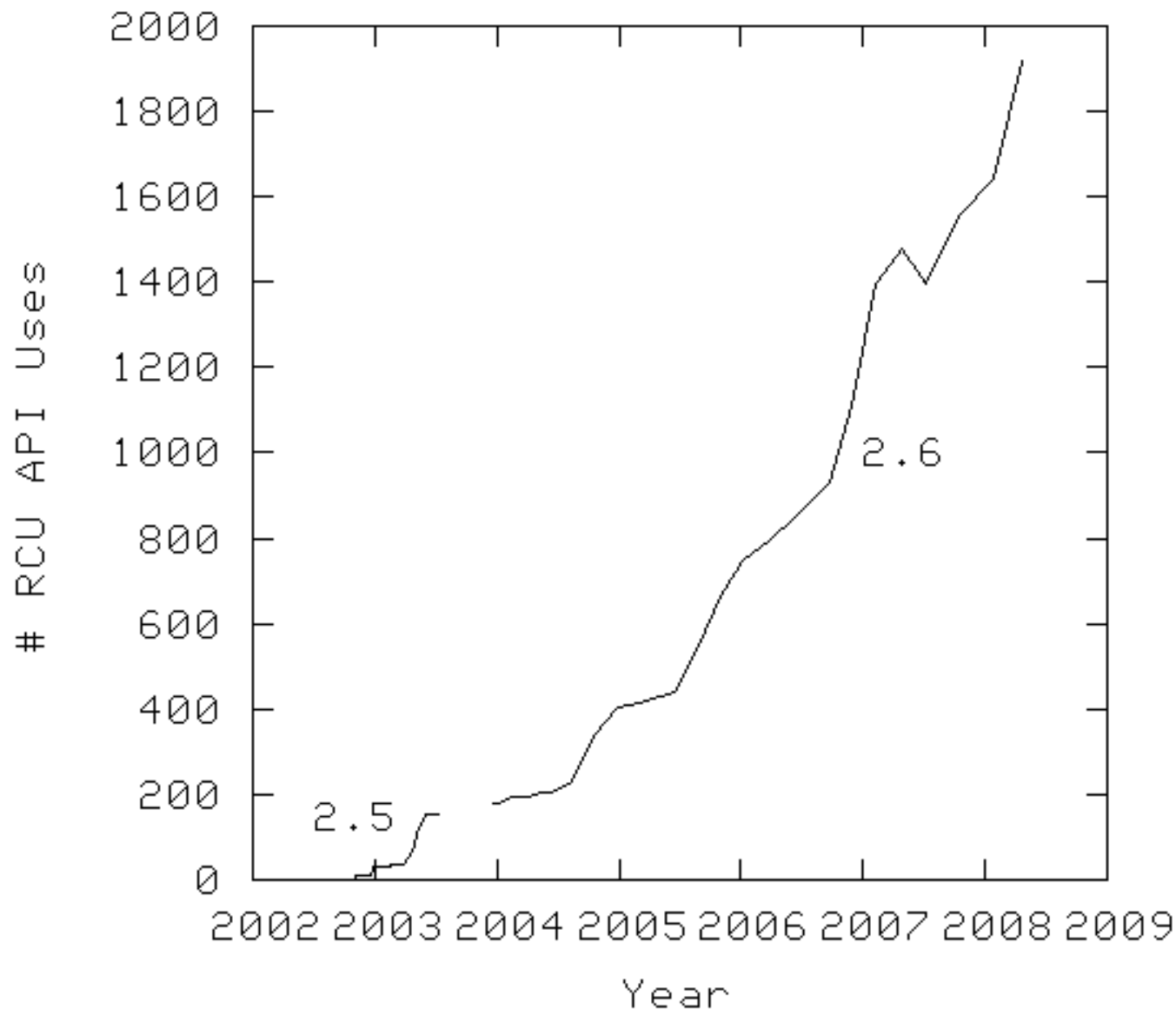
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RCU Usage in Linux



Data from
Paul McKenney's
RCU Web Page

Objectives

- Explain RCU and research problems related to RCU.
- Present a preliminary result on program logic for RCU.

RCU (Read-Copy-Update)

- Back to the future: old style locking for new style architecture.
- Very approximately, reader/writer locks.
- Allows the concurrent access by multiple readers and a single writer.
- Almost zero concurrency overhead for readers.
- Intended to work well with weak memory.


```
rcu_read_lock {  
    b' = b;  
    x = *b';  
    y = *(b'+1);  
}  
.. USE x,y ..
```

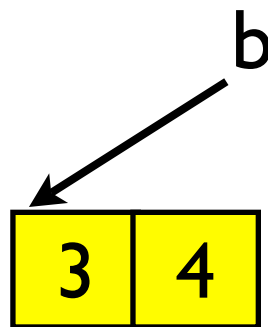
```
.. PRODUCE u,v ..  
nb = malloc(2);  
rcu_write_lock {  
    *nb = u; *(nb+1) = v;  
    barrier();  
    ob = b; b = nb;  
} sync {  
    free(ob); free(ob+1); }
```

RCU Reader/ Writer

reader0



reader1



writer0



writer1



```
rcu_read_lock {  
    b' = b;  
    x = *b';  
    y = *(b'+1);  
}  
.. USE x,y ..
```

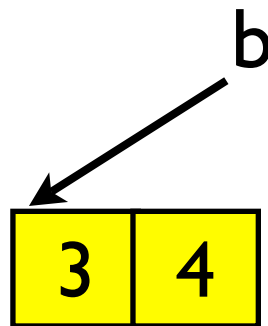
```
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    *nb = u; *(nb+1) = v;  
    barrier();  
    ob = b; b = nb;  
} sync {  
    free(ob); free(ob+1); }
```

RCU Reader/ Writer

reader0



reader1



writer0



writer1



RCU Reader/ Writer

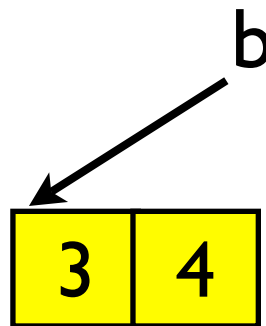
```
b' = b;  
x = *b';  
y = *(b'+1);  
  
.. USE x,y ..
```

```
.. PRODUCE u,v ..  
nb = malloc(2);  
  
*nb = u; *(nb+1) = v;  
  
ob = b; b = nb;  
  
free(ob); free(ob+1);
```

reader0



reader1



writer0



writer1



RCU Reader/ Writer

```
b' = b;  
x = *b';  
y = *(b'+1);  
  
.. USE x,y ..
```

```
.. PRODUCE u,v ..  
nb = malloc(2);  
  
*nb = u; *(nb+1) = v;  
  
ob = b; b = nb;  
  
free(ob); free(ob+1);
```

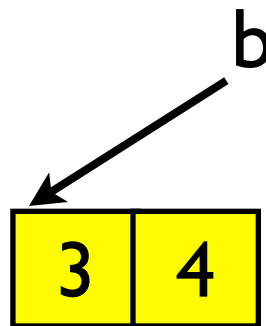
reader0

x: , y:
b'

reader1

writer0

writer1



RCU Reader/ Writer

b' = b;

x = *b';

y = *(b'+1);

.. USE x,y ..

.. PRODUCE u,v ..

nb = malloc(2);

*nb = u; *(nb+1) = v;

ob = b; b = nb;

free(ob); free(ob+1);

reader0

x: , y:

b'

b

writer0

writer1

reader1



RCU Reader/ Writer

```
b' = b;  
x = *b';  
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```

```
.. PRODUCE u,v ..  
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ob = b; b = nb;  
  
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```

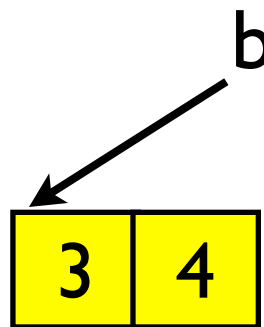
reader0

x:3 , y:
b'

reader1

writer0

writer1



RCU Reader/ Writer

```
b' = b;  
x = *b';  
y = *(b'+1);  
  
.. USE x,y ..
```

```
.. PRODUCE u,v ..  
nb = malloc(2);  
  
*nb = u; *(nb+1) = v;  
  
ob = b; b = nb;  
  
free(ob); free(ob+1);
```

reader0

x:3 , y:4

b'

b



writer0

writer1

RCU Reader/ Writer

```
b' = b;  
x = *b';  
y = *(b'+1);
```

.. USE x,y ..

```
.. PRODUCE u,v ..  
nb = malloc(2);  
  
*nb = u; *(nb+1) = v;  
  
ob = b; b = nb;  
  
free(ob); free(ob+1);
```

reader0

x:3 , y:4

b'

b



writer0

writer1

RCU Reader/ Writer

```
b' = b;  
x = *b';  
y = *(b'+1);  
  
.. USE x,y ..
```

.. PRODUCE u,v ..

```
nb = malloc(2);  
  
*nb = u; *(nb+1) = v;  
  
ob = b; b = nb;  
  
free(ob); free(ob+1);
```

writer0

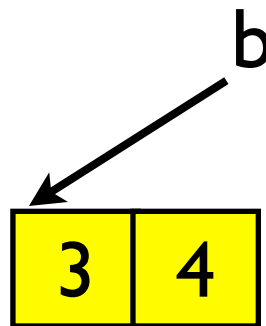
u:11, v:12
ob nb

writer1

reader0

x:3 , y:4
b'

reader1



RCU Reader/ Writer

```
b' = b;  
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.. PRODUCE u,v ..  
nb = malloc(2);  
  
*nb = u; *(nb+1) = v;  
  
ob = b; b = nb;  
  
free(ob); free(ob+1);
```

reader0

x:3 , y:4

b'

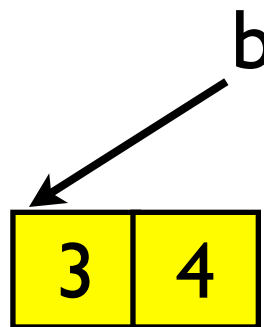
reader1

writer0

u:11, v:12

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RCU Reader/ Writer

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reader0

x:3 , y:4

b'

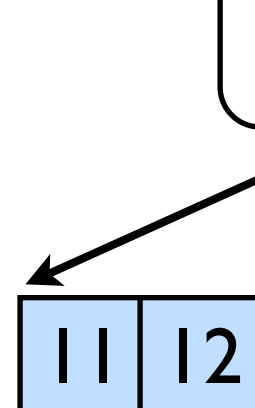
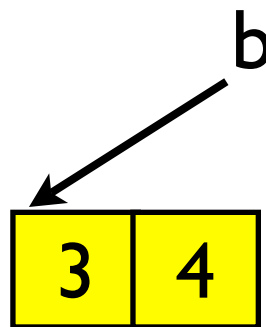
writer0

u:11, v:12

ob nb

reader1

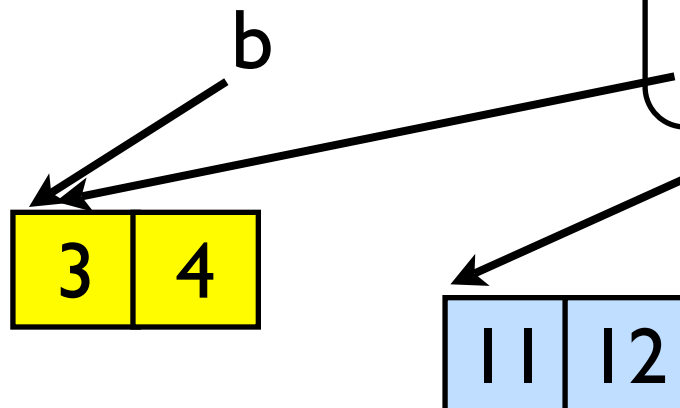
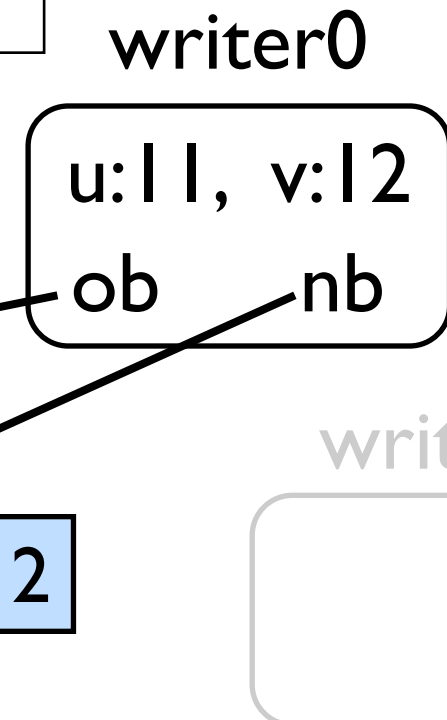
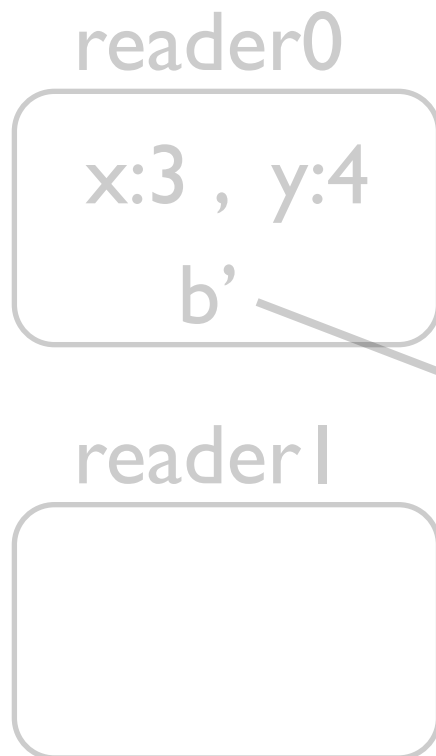
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RCU Reader/ Writer

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RCU Reader/ Writer

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reader0

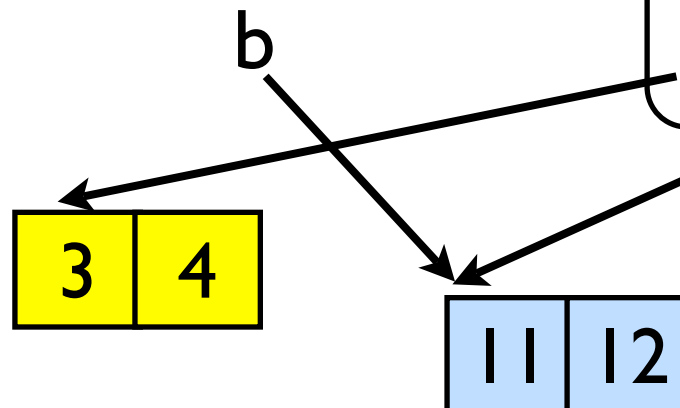
x:3 , y:4
b'

reader1

writer0

u:11, v:12
ob nb

writer1



RCU Reader/ Writer

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ob = b; b = nb;  
  
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```

reader0

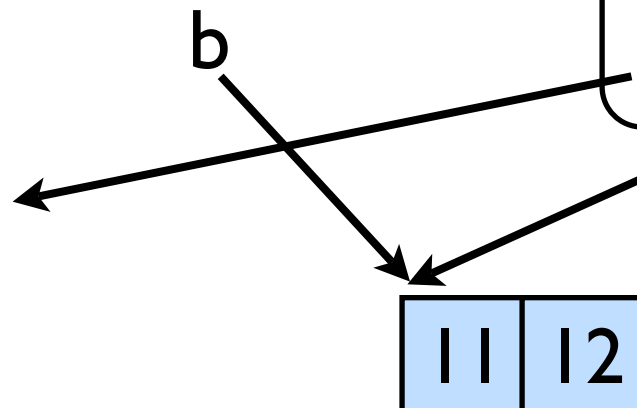
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reader1

writer0

u:11, v:12
ob nb

writer1



RCU Reader/ Writer

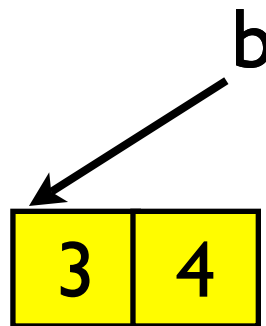
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reader0



reader1



writer0



writer1



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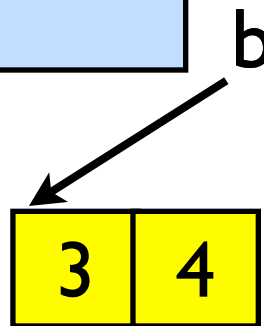
reader0

writer0

Pb I: Racy writers.

reader1

writer1



RCU Reader/ Writer

```
b' = b;  
x = *b';  
y = *(b'+1);  
  
.. USE x,y ..
```

```
.. PRODUCE u,v ..  
nb = malloc(2);  
rcu_write_lock {  
    *nb = u; *(nb+1) = v;  
  
    ob = b; b = nb;  
  
    free(ob); free(ob+1); }
```

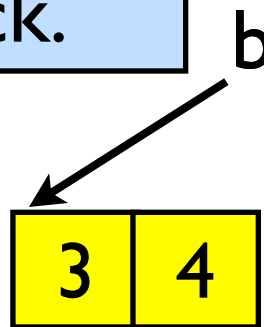
**Pb I: Racy writers.
Sol: Use a standard lock.**

reader0

writer0

reader1

writer1



RCU Reader/ Writer

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.. USE x,y ..
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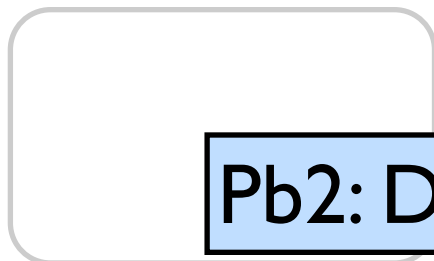
reader0



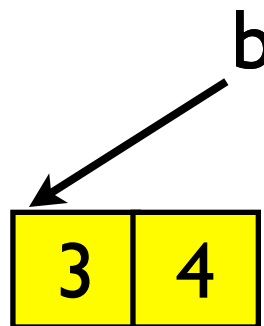
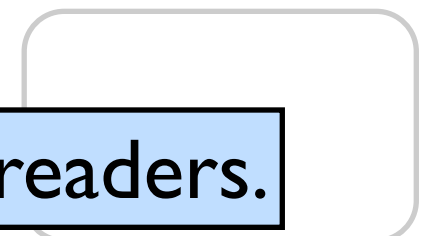
writer0



reader1



writer1



Pb2: Dangling pointer dereference by readers.

RCU Reader/ Writer

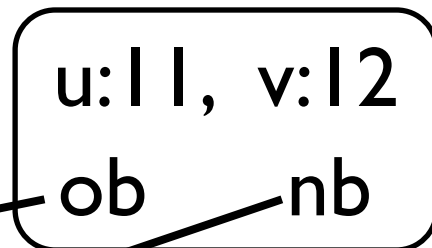
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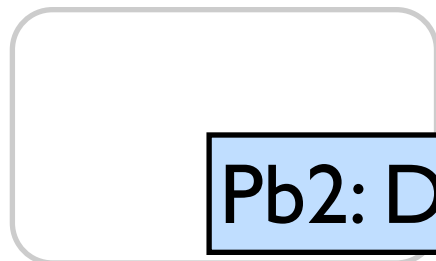
reader0



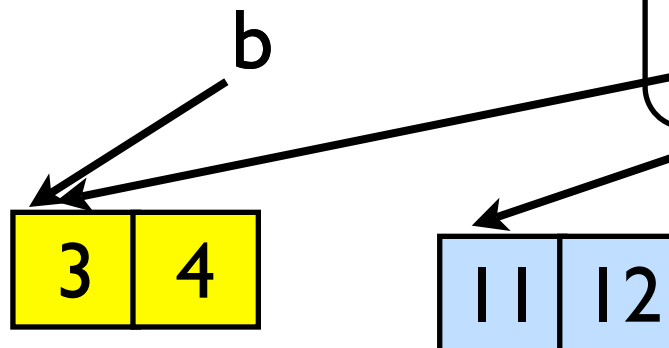
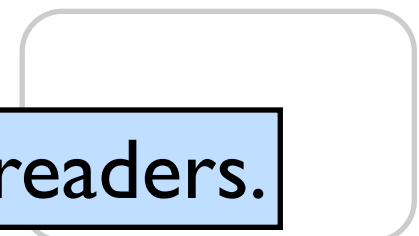
writer0



reader1



writer1



Pb2: Dangling pointer dereference by readers.

RCU Reader/ Writer

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b' = b;  
x = *b',  
y = *(b'+1);  
  
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.. PRODUCE u,v ..  
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    ob = b; b = nb;  
  
    free(ob); free(ob+1); }
```

reader0

x: , y:
b'

writer0

u:11, v:12
ob nb

reader1

writer1



Pb2: Dangling pointer dereference by readers.

RCU Reader/ Writer

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reader0

x: , y:
b'

writer0

u:11, v:12
ob nb

reader1

writer1

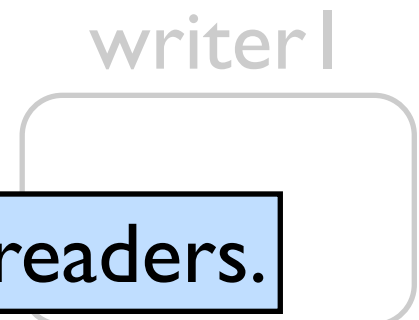
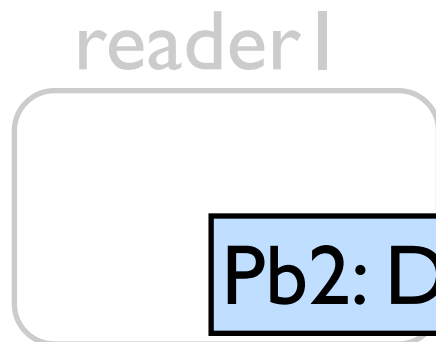
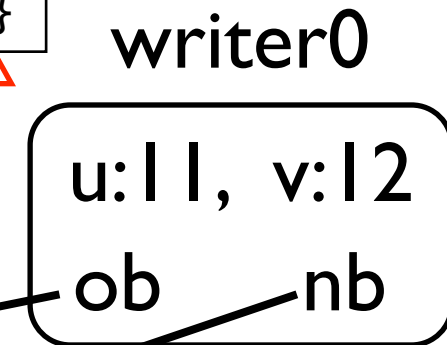
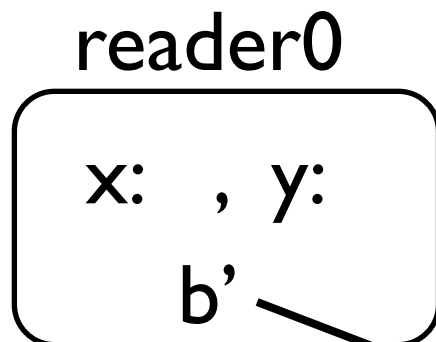


Pb2: Dangling pointer dereference by readers.

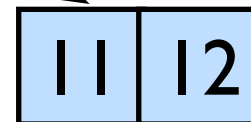
RCU Reader/ Writer

```
b' = b;  
x = *b',  
y = *(b'+1);  
  
.. USE x,y ..
```

```
.. PRODUCE u,v ..  
nb = malloc(2);  
rcu_write_lock {  
    *nb = u; *(nb+1) = v;  
  
    ob = b; b = nb;  
  
    free(ob); free(ob+1); }
```



b



Pb2: Dangling pointer dereference by readers.

RCU Reader/ Writer

```
b' = b;
```

```
x = *b';
```

```
y = *(b'+1);
```

```
.. USE x,y ..
```

```
.. PRODUCE u,v ..
```

```
nb = malloc(2);
```

```
{
```

```
  b+1) = v;
```

```
  ob = b; b = nb;
```

```
  free(ob); free(ob+1); }
```

**Dangling Pointer
Dereference**

reader0

x: , y:

b'

writer0

u:11, v:12

ob

nb

b

reader1

writer1

11 12

Pb2: Dangling pointer dereference by readers.

RCU Reader/ Writer

```
b' = b;  
x = *b';  
y = *(b'+1);  
  
.. USE x,y ..
```

```
.. PRODUCE u,v ..  
nb = malloc(2);  
rcu_write_lock {  
    *nb = u; *(nb+1) = v;  
  
    ob = b; b = nb;  
  
    free(ob); free(ob+1); }
```

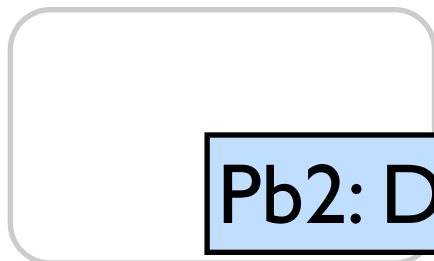
reader0



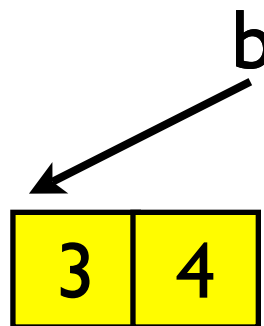
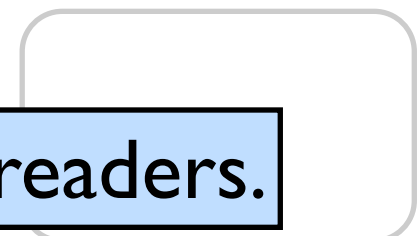
writer0



reader1



writer1



Pb2: Dangling pointer dereference by readers.

RCU

Reader/

Wait until no readers are accessing the old buffer.

```
b' = b;  
x = *b';  
y = *(b'+1);  
  
.. USE x,y ..
```

```
.. PRODUCE u,v ..  
nb = malloc(2);  
rcu_write_lock {  
    *nb = u; *(nb+1) = v;  
  
    ob = b; b = nb;  
  
    } sync {  
    free(ob); free(ob+1); }
```

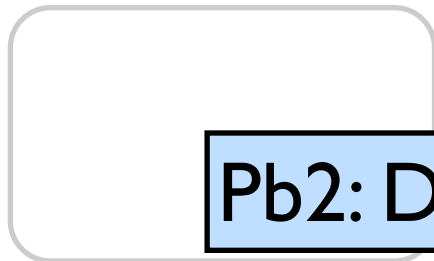
reader0



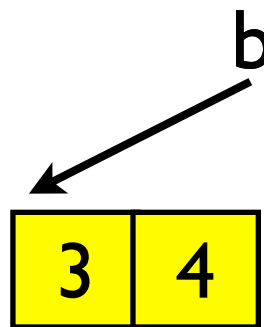
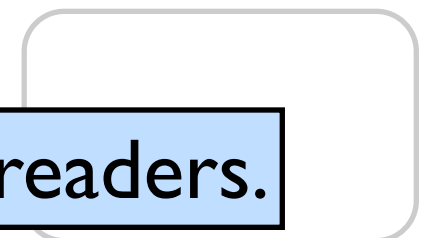
writer0



reader1



writer1



Pb2: Dangling pointer dereference by readers.

```
rcu_read_lock {
```

```
    b' = b;  
    x = *b';  
    y = *(b'+1);
```

```
}
```

Disable
context switch

RCU Reader/

Wait until no
readers are
accessing the
old buffer.

Enable
context switch

```
} sync {
```

```
    free(o, s, iter0)
```

Wait until all CPUs
context-switch.

reader I

writer I

3 4

Pb2: Dangling pointer dereference by readers.

```
rcu_read_lock {
```

```
    b' = b;
```

```
    x = *b';
```

```
    y = *(b'+1);
```

```
}
```

```
.. USE x,y ..
```

```
.. PRODUCE u,v ..
```

```
nb = malloc(2);
```

```
rcu_write_lock {
```

```
    *nb = u; *(nb+1) = v;
```

```
    ob = b; b = nb;
```

```
} sync {
```

```
    free(ob); free(ob+1); }
```

RCU Reader/

Wait until no
readers are
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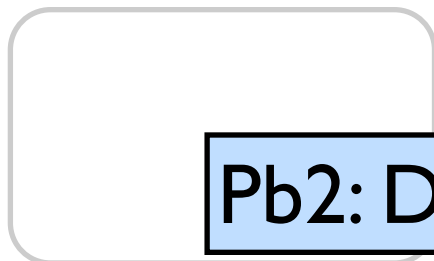
reader0



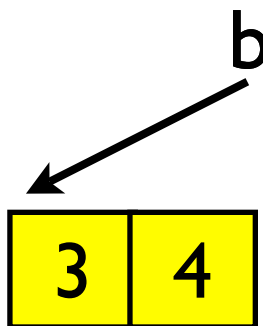
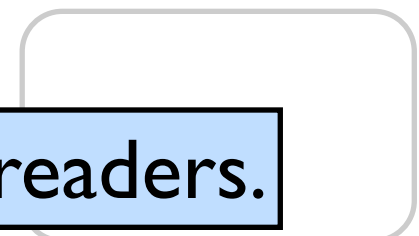
writer0



reader1



writer1



Pb2: Dangling pointer dereference by readers.

```
rcu_read_lock {
```

```
    b' = b;  
    x = *b',  
    y = *(b'+1);
```

```
}
```

```
.. USE x,y ..
```

```
.. PRODUCE u,v ..
```

```
nb = malloc(2);
```

```
rcu_write_lock {
```

```
    *nb = u; *(nb+1) = v;
```

```
    ob = b; b = nb;
```

```
} sync {
```

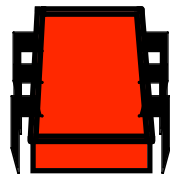
```
    free(ob); free(ob+1); }
```

RCU Reader/

Wait until no
readers are
accessing the
old buffer.

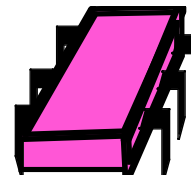
reader0

x: , y:
b'



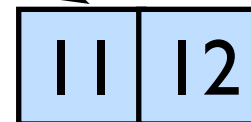
writer0

u:11, v:12
ob nb



reader1

writer1



Pb2: Dangling pointer dereference by readers.

```
rcu_read_lock {
```

```
    b' = b;
```

```
    x = *b';
```

```
    y = *(b'+1);
```

```
}
```

```
.. USE x,y ..
```

```
.. PRODUCE u,v ..
```

```
nb = malloc(2);
```

```
rcu_write_lock {
```

```
    *nb = u; *(nb+1) = v;
```

```
    ob = b; b = nb;
```

```
} sync {
```

```
    free(ob); free(ob+1); }
```

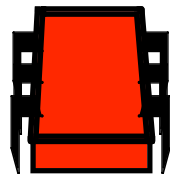
RCU Reader/

Wait until no
readers are
accessing the
old buffer.

reader0

x:3 , y:

b'

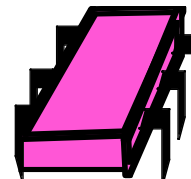


writer0

u:11, v:12

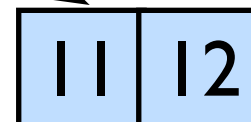
ob

nb



reader1

writer1



Pb2: Dangling pointer dereference by readers.

```
rcu_read_lock {
```

```
    b' = b;
```

```
    x = *b';
```

```
    y = *(b'+1);
```

```
}
```

```
.. USE x,y ..
```

```
.. PRODUCE u,v ..
```

```
nb = malloc(2);
```

```
rcu_write_lock {
```

```
    *nb = u; *(nb+1) = v;
```

```
    ob = b; b = nb;
```

```
} sync {
```

```
    free(ob); free(ob+1); }
```

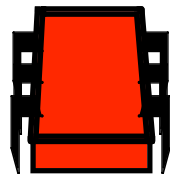
RCU Reader/

Wait until no
readers are
accessing the
old buffer.

reader0

x:3 , y:4

b'

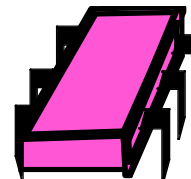


writer0

u:11, v:12

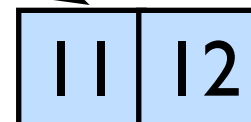
ob

nb



reader1

writer1



Pb2: Dangling pointer dereference by readers.

```
rcu_read_lock {
```

```
    b' = b;
```

```
    x = *b';
```

```
    y = *(b'+1);
```

```
}
```

.. USE x,y ..

```
.. PRODUCE u,v ..
```

```
nb = malloc(2);
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rcu_write_lock {
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```
    *nb = u; *(nb+1) = v;
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```
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    free(ob); free(ob+1); }
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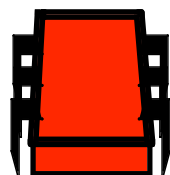
RCU Reader/

Wait until no
readers are
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old buffer.

reader0

x:3 , y:4

b'

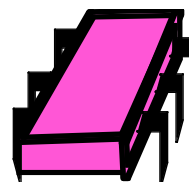


writer0

u:11, v:12

ob

nb



reader1

writer1

3 4

11 12

Pb2: Dangling pointer dereference by readers.

```
rcu_read_lock {
```

```
    b' = b;
```

```
    x = *b';
```

```
    y = *(b'+1);
```

```
}
```

```
.. USE x,y ..
```

```
.. PRODUCE u,v ..
```

```
nb = malloc(2);
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```
rcu_write_lock {
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```
    *nb = u; *(nb+1) = v;
```

```
    ob = b; b = nb;
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} sync {
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    free(ob); free(ob+1); }
```

RCU Reader/

Wait until no
readers are
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old buffer.

writer0

reader0

x:3 , y:4

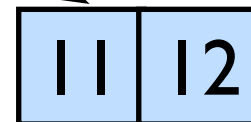
b'

u:11, v:12

ob

nb

b



writer1

Pb2: Dangling pointer dereference by readers.


```
rcu_read_lock {
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```
    b' = b;
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```

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RCU Reader/

Wait until no
readers are
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old buffer.

writer0

u:11, v:12

ob nb

writer1

reader0

x:3, y:4

b'

b

11 12

Pb2: Dangling pointer dereference by readers.

```
rcu_read_lock {  
    b' = b;  
    x = *b';  
    y = *(b'+1);  
}  
.. USE x,y ..
```

```
.. PRODUCE u,v ..  
nb = malloc(2);  
rcu_write_lock {  
    *nb = u; *(nb+1) = v;  
  
    ob = b; b = nb;  
} sync {  
    free(ob); free(ob+1); }
```

RCU Reader/ Writer

```
rcu_read_lock {  
    b' = b;  
    x = *b';  
    y = *(b'+1);  
}  
.. USE x,y ..
```

```
.. PRODUCE u,v ..  
nb = malloc(2);  
rcu_write_lock {  
    *nb = u; *(nb+1) = v;  
    barrier();  
    ob = b; b = nb;  
} sync {  
    free(ob); free(ob+1); }
```

RCU Reader/ Writer

- 1) Handles weak memory.
- 2) Ensures that a reader never reads values from a non-initialized buffer.

RCU Reader/ Writer

```
rcu_read_lock {  
    b' = b;  
    x = *b';  
    y = *(b'+1);  
}  
.. USE x,y ..
```

```
.. PRODUCE u,v ..  
nb = malloc(2);  
rcu_write_lock {  
    *nb = u; *(nb+1) = v;  
    barrier();  
    ob = b; b = nb;  
} sync {  
    free(ob); free(ob+1); }
```

- RCU constructs:

```
rcu_read_lock { .. },
```

```
rcu_write_lock { ... } sync { ... },    barrier
```

- Linux API also includes asynchronous sync and

Research Issues

- Full specification of RCU programs.

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- Program logic for RCU.

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 - Shape analysis, termination analysis.

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 - Prove correctness considering weak memory.

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- Full specification of RCU programs.
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 - Prove correctness considering weak memory.
- New algorithms based on RCU.

Research Issues

- Full specification of RCU programs.
- Program logic for RCU.
- Automatic program analysis.
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- Verify the implementations of RCU.
 - Implementations of RCU, SRCU.
 - Prove correctness considering weak memory.
- New algorithms based on RCU.

Overview of Logic

- Resource-invariant-based, thread-local reasoning.
- sync, barrier as ownership transfer operations.
- Reasoning about readers should use only those facts preserved by writers' operations.

Proof Sketch of RCU Readers/Writers

`rcu_read_lock {`

$b' = b;$

$x = *b';$

$y = *(b' + 1);$

`}`

`rcu_write_lock {`

$*nb = u; *(nb + 1) = v;$

`barrier $nb \mapsto \_, \_$ ;`

$ob = b; \quad b = nb;$

`} sync {`

`free( $ob$ ); free( $ob+1$ );`

`}`

Proof Sketch of RCU Readers/Writers

`rcu_read_lock {`

$b' = b;$

$x = *b';$

$y = *(b' + 1);$

`}`

`rcu_write_lock {`

$*nb = u; *(nb + 1) = v;$

`barrier $nb \mapsto \_, \_$ ;`

$ob = b; \quad b = nb;$

`} sync {`

`free(ob); free(ob+1);`

`}`

Invariant-based, thread-local reasoning.

Proof Sketch of RCU Readers/Writers

$\{\text{emp}\}$

`rcu_read_lock {`

$b' = b;$

$x = *b';$

$y = *(b' + 1);$

`}`
 $\{\text{emp}\}$

$\{nb \mapsto _, _ \}$

`rcu_write_lock {`

$*nb = u; *(nb + 1) = v;$

`barrier $nb \mapsto \_, \_$ ;`

$ob = b; b = nb;$

`} sync {`

`free(ob); free(ob+1);`

`}`
 $\{\text{emp}\}$

Invariant-based, thread-local reasoning.

Proof Sketch of RCU Readers/Writers

ResInv : $b \mapsto -, -$

$\{\text{emp}\}$

rcu_read_lock {

$\{nb \mapsto -, -\}$

rcu_write_lock {

$\{nb \mapsto -, - * \boxed{b \mapsto -, -}\}$

$*nb = u; *(nb + 1) = v;$

barrier _{$nb \mapsto -, -$} ;

$ob = b; b = nb;$

} sync {

free(ob); free($ob+1$);

}
 $\{\text{emp}\}$

}
 $\{\text{emp}\}$

**ResInv holds initially for
the shared heap.**

Invariant-based, thread-local reasoning.

Proof Sketch of RCU Readers/Writers

ResInv : $b \mapsto -, -$

$\{\text{emp}\}$

rcu_read_lock {

$b' = b;$

$x = *b';$

$y = *(nb + 1);$

$\{\text{emp}\}$

ResInv holds
right before releasing
the lock.

$\{nb \mapsto -, -\}$

rcu_write_lock {

$\{nb \mapsto -, - * \boxed{b \mapsto -, -}\}$

$*nb = u; *(nb + 1) = v;$

barrier $_{nb \mapsto -, -};$

$ob = b; b = nb;$

sync {

free(ob); free($ob+1$);

$\{\text{emp} * \boxed{b \mapsto -, -}\}$

}
 $\{\text{emp}\}$

Invariant-based, thread-local reasoning.

Proof Sketch of RCU Readers/Writers

ResInv : $b \mapsto -, -$

```

{emp}
rcu_read_lock {

     $b' = b;$ 

     $x = *b';$ 

     $y = *(b' + 1);$ 

}
{emp}
    
```

```

{ $nb \mapsto -, -$ }
rcu_write_lock {
    { $nb \mapsto -, - * \boxed{b \mapsto -, -}$ }
       $*nb = u; *(nb + 1) = v;$ 
    { $nb \mapsto -, - * \boxed{b \mapsto -, -}$ }
    barrier $_{nb \mapsto -, -};$ 

     $ob = b; b = nb;$ 

} sync {

     $\text{free}(ob); \text{free}(ob+1);$ 
    {emp *  $\boxed{b \mapsto -, -}$ }

}
{emp}
    
```

Proof Sketch of RCU Readers/Writers

ResInv : $b \mapsto -, -$

$\{\text{emp}\}$

rcu_read_lock {

$b' = b;$

$\{nb \mapsto -, -\}$

rcu_write_lock {

$\{nb \mapsto -, - * \boxed{b \mapsto -, -}\}$

$*nb = u; *(nb + 1) = v;$

$\{nb \mapsto -, - * \boxed{b \mapsto -, -}\}$

barrier _{$nb \mapsto -, -$} ;

$\{\text{emp} * \boxed{nb \mapsto -, - * b \mapsto -, -}\}$

$ob = b; b = nb;$

} sync {

free(ob); free($ob+1$);

$\{\text{emp} * \boxed{b \mapsto -, -}\}$

}

$\{\text{emp}\}$

barrier as ownership
transfer from private to shared

Proof Sketch of RCU Readers/Writers

ResInv : $b \mapsto _, _$

```

{emp}
rcu_read_lock {

     $b' = b;$ 

     $x = *b';$ 

     $y = *(b' + 1);$ 

}
{emp}
    
```

```

{nb  $\mapsto \_, \_$ }
rcu_write_lock {
    {nb  $\mapsto \_, \_ * b \mapsto \_, \_$ }
     $*nb = u; *(nb + 1) = v;$ 
    {nb  $\mapsto \_, \_ * b \mapsto \_, \_$ }
    barriernb  $\mapsto \_, \_$ ;
    {emp * nb  $\mapsto \_, \_ * b \mapsto \_, \_$ }
     $ob = b; b = nb;$ 
    {emp * b  $\mapsto \_, \_ * ob \mapsto \_, \_$ }
} sync {

     $\text{free}(ob); \text{free}(ob+1);$ 
    {emp * b  $\mapsto \_, \_$ }
}
{emp}
    
```

Proof Sketch of RCU Readers/Writers

ResInv : $b \mapsto -, -$

$\{\text{emp}\}$

rcu_read_lock {

$b' = b;$

$x = *b';$

$y = *(b' + 1);$

}

$\{nb \mapsto -, -\}$

rcu_write_lock {

$\{nb \mapsto -, - * \boxed{b \mapsto -, -}\}$

$*nb = u; *(nb + 1) = v;$

$\{nb \mapsto -, - * \boxed{b \mapsto -, -}\}$

barrier _{$nb \mapsto -, -$} ;

$\{\text{emp} * \boxed{nb \mapsto -, - * b \mapsto -, -}\}$

$ob = b; b = nb;$

$\{\text{emp} * \boxed{b \mapsto -, - * ob \mapsto -, -}\}$

} sync {

$\{ob \mapsto -, - * \boxed{b \mapsto -, -}\}$

free(ob); free(ob+1);

$\{\text{emp} * \boxed{b \mapsto -, -}\}$

}

$\{\text{emp}\}$

sync as ownership transfer
from shared to private

Proof Sketch of RCU Readers/Writers

ResInv : $b \mapsto -, -$

```
{emp}
rcu_read_lock {

     $b' = b;$ 

     $x = *b';$ 

     $y = *(b' + 1);$ 

}
{emp}
```

```
{ $nb \mapsto -, -$ }
rcu_write_lock {
    { $nb \mapsto -, - * b \mapsto -, -$ }
     $*nb = u; *(nb + 1) = v;$ 
    { $nb \mapsto -, - * b \mapsto -, -$ }
    barrier $nb \mapsto -, -$ ;
    {emp *  $nb \mapsto -, - * b \mapsto -, -$ }
     $ob = b; b = nb;$ 
    {emp *  $b \mapsto -, - * ob \mapsto -, -$ }
} sync {
    { $ob \mapsto -, - * b \mapsto -, -$ }
     $\text{free}(ob); \text{free}(ob+1);$ 
    {emp *  $b \mapsto -, -$ }
}
{emp}
```

Proof Sketch of RCU Readers/Writers

$\{\text{emp}\}$

rcu_read_lock {

$b' = b;$

$x = *b';$

$y = *(b' + 1);$

}
 $\{\text{emp}\}$

$\{nb \mapsto _, _ \}$

rcu_write_lock {

$\{nb \mapsto _, _ * \boxed{b \mapsto _, _} \}$

$*nb = u; *(nb + 1) = v;$

$\{nb \mapsto _, _ * \boxed{b \mapsto _, _} \}$

barrier _{$nb \mapsto _, _;$}

$\{\text{emp} * \boxed{nb \mapsto _, _ * b \mapsto _, _} \}$

$ob = b; b = nb;$

$\{\text{emp} * \boxed{b \mapsto _, _ * ob \mapsto _, _} \}$

} sync {

$\{ob \mapsto _, _ * \boxed{b \mapsto _, _} \}$

free(ob); free(ob+1);

$\{\text{emp} * \boxed{b \mapsto _, _} \}$

}
 $\{\text{emp}\}$

ResInv : $b \mapsto _, _$

AlwaysResInv : $b \mapsto _, _ * \text{true}$

Proof Sketch of RCU Readers/Writers

$\{\text{emp}\}$

rcu_read_lock {

$b' = b;$

$x = *b';$

$y = *(b' + 1);$

}
 $\{\text{emp}\}$

$\{nb \mapsto _, _ \}$

rcu_write_lock {

$\{nb \mapsto _, _ * \boxed{b \mapsto _, _} \}$

$*nb = u; *(nb + 1) = v;$

$\{nb \mapsto _, _ * \boxed{b \mapsto _, _} \}$

barrier _{$nb \mapsto _, _;$}

$\{\text{emp} * \boxed{nb \mapsto _, _ * b \mapsto _, _} \}$

$ob = b; b = nb;$

$\{\text{emp} * \boxed{b \mapsto _, _ * ob \mapsto _, _} \}$

} sync {

$\{ob \mapsto _, _ * \boxed{b \mapsto _, _} \}$

$\text{free}(ob); \text{free}(ob+1);$

$\{\text{emp} * \boxed{b \mapsto _, _} \}$

}
 $\{\text{emp}\}$

ResInv : $b \mapsto _, _$

AlwaysResInv : $b \mapsto _, _ * \text{true}$

Pres : $\forall a. \left(\boxed{a \mapsto _, _ * \text{true}} \rightsquigarrow \boxed{a \mapsto _, _ * \text{true}} \right)$

Proof Sketch of RCU Readers/Writers

<pre> {emp} rcu_read_lock { {emp * $b \mapsto _, _ * \text{true}$} b' = b; x = *b'; y = *(b' + 1); } {emp} </pre>	<pre> {nb $\mapsto _, _$} rcu_write_lock { {nb $\mapsto _, _ * b \mapsto _, _$} *nb = u; *(nb + 1) = v; * $b \mapsto _, _$ free nb $\mapsto _, _$; {nb $\mapsto _, _ * b \mapsto _, _$} ob = b; b = nb; {emp * $b \mapsto _, _ * ob \mapsto _, _$} } sync { {ob $\mapsto _, _ * b \mapsto _, _$} free(ob); free(ob+1); {emp * $b \mapsto _, _$} } {emp} </pre>
--	--

Start with
AlwaysResInv.

ResInv : $b \mapsto _, _$

AlwaysResInv : $b \mapsto _, _ * \text{true}$

Pres : $\forall a. (a \mapsto _, _ * \text{true} \rightsquigarrow a \mapsto _, _ * \text{true})$

Proof Sketch of RCU Readers/Writers

```

{emp}
rcu_read_lock {
  {emp *  $b \mapsto \_, \_ * \text{true}$ }
  b' = b;

  x = *b';

  y = *(b' + 1);
  {emp *  $b' \mapsto \_, \_ * \text{true}$ }
}
{emp}
  
```

```

{nb  $\mapsto \_, \_$ }
rcu_write_lock {
  {nb  $\mapsto \_, \_ * b \mapsto \_, \_$ }
  *nb = u; *(nb + 1) = v;
  {nb  $\mapsto \_, \_ * b \mapsto \_, \_$ }
  barriernb  $\mapsto \_, \_$ ;
  {emp *  $nb \mapsto \_, \_ * b \mapsto \_, \_$ }
  ob = b; b = nb;
  {emp *  $b \mapsto \_, \_ * ob \mapsto \_, \_$ }
}
sync {
  {nb  $\mapsto \_, \_ * b \mapsto \_, \_$ }
  free(ob); free(ob+1);
  {nb  $\mapsto \_, \_$ }
}
{emp}
  
```

ResInv : $b \mapsto _, _$

Can end with
any assertion.

AlwaysResInv : $b \mapsto _, _ * \text{true}$

Pres : $\forall a. \left(a \mapsto _, _ * \text{true} \rightsquigarrow a \mapsto _, _ * \text{true} \right)$

Proof Sketch of RCU Readers/Writers

$\{emp\}$

`rcu_read_lock {`

$\{emp * \boxed{b \mapsto _, _ * true}\}$

`b' = b;`

$\{emp * \boxed{b' \mapsto _, _ * true}\}$

`x = *b';`

`y = *(b' + 1);`

$\{emp * \boxed{b' \mapsto _, _ * true}\}$

`}`
 $\{emp\}$

$\{nb \mapsto _, _ \}$

`rcu_write_lock {`

$\{nb \mapsto _, _ * \boxed{b \mapsto _, _}\}$

`*nb = u; *(nb + 1) = v;`

$\{nb \mapsto _, _ * \boxed{b \mapsto _, _}\}$

`barriernb;`

$\{emp * \boxed{nb \mapsto _, _ * b \mapsto _, _}\}$

`ob = b; b = nb;`

$\{emp * \boxed{b \mapsto _, _ * ob \mapsto _, _}\}$

`} sync {`

$\{ob \mapsto _, _ * \boxed{b \mapsto _, _}\}$

`free(ob); free(ob+1);`

$\{emp * \boxed{b \mapsto _, _}\}$

`}`
 $\{emp\}$

ResInv : $b \mapsto _, _$

AlwaysResInv : $b \mapsto _, _ * true$

Pres : $\forall a. (\boxed{a \mapsto _, _ * true} \rightsquigarrow \boxed{a \mapsto _, _ * true})$

Proof Sketch of RCU Writers

```

{emp}
rcu_read_lock {
  {emp *  $b \mapsto \_, \_ * \text{true}$ }
   $b' = b$ ;
  {emp *  $b' \mapsto \_, \_ * \text{true}$ }
   $x = *b'$ ;

   $y = *(b' + 1)$ ;
  {emp *  $b' \mapsto \_, \_ * \text{true}$ }
}
{emp}
    
```

Not modify the
shared heap.

```

{emp *  $b \mapsto \_, \_ * \text{true}$ }
 $*nb = u; *(nb + 1) = v$ ;
{ $nb \mapsto \_, \_ * b \mapsto \_, \_$ }
barrier $nb \mapsto \_, \_$ ;
{emp *  $nb \mapsto \_, \_ * b \mapsto \_, \_$ }
 $ob = b; b = nb$ ;
{emp *  $b \mapsto \_, \_ * ob \mapsto \_, \_$ }
} sync {
  { $ob \mapsto \_, \_ * b \mapsto \_, \_$ }
  free( $ob$ ); free( $ob + 1$ );
  {emp *  $b \mapsto \_, \_$ }
}
{emp}
    
```

AlwaysResInv : $b \mapsto _, _ * \text{true}$

Pres : $\forall a. \left(a \mapsto _, _ * \text{true} \rightsquigarrow a \mapsto _, _ * \text{true} \right)$

Proof Sketch of RCU Writers

```

{emp}
rcu_read_lock {
  {emp *  $b \mapsto \_, \_ * \text{true}$ }
   $b' = b$ ;
  {emp *  $b' \mapsto \_, \_ * \text{true}$ }
   $x = *b'$ ;

   $y = *(b' + 1)$ ;
  {emp *  $b' \mapsto \_, \_ * \text{true}$ }
}
{emp}

```

Not modify the shared heap.

Should be preserved by writers.

AlwaysResInv : $b \mapsto _, _ * \text{true}$

Pres : $\forall a. (a \mapsto _, _ * \text{true} \rightsquigarrow a \mapsto _, _ * \text{true})$

Proof Sketch of RCU Readers/Writers

$\{\text{emp}\}$

rcu_read_lock {

$\{\text{emp} * \boxed{b \mapsto _, _ * \text{true}}\}$

$b' = b;$

$\{\text{emp} * \boxed{b' \mapsto _, _ * \text{true}}\}$

$x = *b';$

$\{\text{emp} * \boxed{b' \mapsto _, _ * \text{true}}\}$

$y = *(b' + 1);$

$\{\text{emp} * \boxed{b' \mapsto _, _ * \text{true}}\}$

}

$\{\text{emp}\}$

$\{nb \mapsto _, _ \}$

rcu_write_lock {

$\{nb \mapsto _, _ * \boxed{b \mapsto _, _}\}$

$*nb = u; *(nb + 1) = v;$

$\{nb \mapsto _, _ * \boxed{b \mapsto _, _}\}$

barrier _{$nb \mapsto _, _;$}

$\{\text{emp} * \boxed{nb \mapsto _, _ * b \mapsto _, _}\}$

$ob = b; b = nb;$

$\{\text{emp} * \boxed{b \mapsto _, _ * ob \mapsto _, _}\}$

} sync {

$\{ob \mapsto _, _ * \boxed{b \mapsto _, _}\}$

free(ob); free(ob+1);

$\{\text{emp} * \boxed{b \mapsto _, _}\}$

}

$\{\text{emp}\}$

ResInv : $b \mapsto _, _$

AlwaysResInv : $b \mapsto _, _ * \text{true}$

Pres : $\forall a. (\boxed{a \mapsto _, _ * \text{true}} \rightsquigarrow \boxed{a \mapsto _, _ * \text{true}})$

Proof Sketch of RCU Readers/Writers

$\{\text{emp}\}$

```
rcu_read_lock {
  {emp *  $b \mapsto \_, \_ * \text{true}$ }
   $b' = b;$ 
  {emp *  $b' \mapsto \_, \_ * \text{true}$ }
   $x = *b';$ 
```

```
  {emp *  $b' \mapsto \_, \_ * \text{true}$ }
   $y = *(b' + 1);$ 
  {emp *  $b' \mapsto \_, \_ * \text{true}$ }
```

}

$\{\text{emp}\}$

$\{nb \mapsto _, _ \}$

```
rcu_write_lock {
  { $nb \mapsto \_, \_ * b \mapsto \_, \_$ }
   $*nb = u; *(nb + 1) = v;$ 
  { $nb \mapsto \_, \_ * b \mapsto \_, \_$ }
  barrier $nb \mapsto \_, \_;$ 
```

```
  {emp *  $nb \mapsto \_, \_ * b \mapsto \_, \_$ }
   $ob = b; b = nb;$ 
  {emp *  $b \mapsto \_, \_ * ob \mapsto \_, \_$ }
```

```
} sync {
  { $ob \mapsto \_, \_ * b \mapsto \_, \_$ }
  free( $ob$ ); free( $ob + 1$ );
  {emp *  $b \mapsto \_, \_$ }
```

}

$\{\text{emp}\}$

ResInv : $b \mapsto _, _$

AlwaysResInv : $b \mapsto _, _ * \text{true}$

Pres : $\forall a. (a \mapsto _, _ * \text{true} \rightsquigarrow a \mapsto _, _ * \text{true})$

Proof Sketch of RCU Readers/Writers

$\{emp\}$

```
rcu_read_lock {
  {emp *  $b \mapsto \_, \_ * true$ }
   $b' = b$ ;
  {emp *  $b' \mapsto \_, \_ * true$ }
   $x = *b'$ ;
  {emp *  $b' \mapsto \_, \_ * true$ }
   $y = *(b' + 1)$ ;
  {emp *  $b' \mapsto \_, \_ * true$ }
}
```

$\{emp\}$

$\{nb \mapsto _, _ \}$

```
rcu_write_lock {
  { $nb \mapsto \_, \_ * b \mapsto \_, \_$ }
   $*nb = u$ ;  $*(nb + 1) = v$ ;
  { $nb \mapsto \_, \_ * b \mapsto \_, \_$ }
  barrier $nb \mapsto \_, \_$ ;
  {emp *  $nb \mapsto \_, \_ * b \mapsto \_, \_$ }
   $ob = b$ ;  $b = nb$ ;
  {emp *  $b \mapsto \_, \_ * ob \mapsto \_, \_$ }
} sync {
  { $ob \mapsto \_, \_ * b \mapsto \_, \_$ }
  free( $ob$ ); free( $ob + 1$ );
  {emp *  $b \mapsto \_, \_$ }
}
```

$\{emp\}$

ResInv : $b \mapsto _, _$

AlwaysResInv : $b \mapsto _, _ * true$

Pres : $\forall a. (a \mapsto _, _ * true \rightsquigarrow a \mapsto _, _ * true)$

Proof Sketch of RCU Readers/Writers

$\{\text{emp}\}$

`rcu_read_lock {`

$\{\text{emp} * \boxed{b \mapsto _, _ * \text{true}}\}$

`b' = b;`

$\{\text{emp} * \boxed{b' \mapsto _, _ * \text{true}}\}$

$\{nb \mapsto _, _ \}$

`rcu_write_lock {`

$\{nb \mapsto _, _ * \boxed{b \mapsto _, _}\}$

`*nb = u; *(nb + 1) = v;`

$\{nb \mapsto _, _ * \boxed{b \mapsto _, _}\}$

ResInv : $b \mapsto _, _$

- 1) Invariant-based, thread-local reasoning.
- 2) barrier, sync as ownership transfer operations.
- 3) Use only preserved facts, when verifying readers.

$\}$
 $\{\text{emp}\}$

`} sync {`

$\{ob \mapsto _, _ * \boxed{b \mapsto _, _}\}$

`free(ob); free(ob+1);`

$\{\text{emp} * \boxed{b \mapsto _, _}\}$

$\}$
 $\{\text{emp}\}$

AlwaysResInv : $b \mapsto _, _ * \text{true}$

Pres : $\forall a. (\boxed{a \mapsto _, _ * \text{true}} \rightsquigarrow \boxed{a \mapsto _, _ * \text{true}})$

Assertion Language

$$P ::= \text{true} \mid E \mapsto F \mid \text{emp} \mid P * Q \mid \dots$$
$$K ::= P \mid \boxed{P} \mid K * L \mid \dots$$

- P describes the heap local to the thread.
- $\boxed{P}$ describes the shared heap.
- K describes both.

Judgments

$$(R, I(a)) \vdash^{\text{rd/wr/no}} \{K\}C\{L\}$$

Judgments

Precise
res. invariant.
(E.g. $b \mapsto _, _$)

$$(\boxed{R}, I(a)) \vdash^{\text{rd/wr/no}} \{K\}C\{L\}$$

Judgments

Precise
res. invariant.
(E.g. $b \mapsto _, _$)

$(R, I(a)) \vdash^{\text{rd/wr/no}} \{K\}C\{L\}$

Facts
preserved by writers.
(E.g. $a \mapsto _, _ * \text{true}$)

Judgments

Precise
res. invariant.
(E.g. $b \mapsto _, _$)

Indicates
“lock” held.

$(R, I(a)) \vdash \boxed{\text{rd/wr/no}} \{K\}C\{L\}$

Facts
preserved by writers.
(E.g. $a \mapsto _, _ * \text{true}$)

Judgments

Precise
res. invariant.
(E.g. $b \mapsto _, _$)

Indicates
“lock” held.

$(R, I(a)) \vdash^{\text{rd/wr/no}} \{K\}C\{L\}$

Facts
preserved by writers.
(E.g. $a \mapsto _, _ * \text{true}$)

Proof Rule for Lock/Sync

$$\frac{\begin{array}{l} (R, I(a)) \vdash^{\text{wr}} \{P * \boxed{R}\} C \{Q' * \boxed{R' * R}\} \\ (R, I(a)) \vdash^{\text{wr}} \{Q' * R' * \boxed{R}\} D \{Q * \boxed{R}\} \end{array}}{(R, I(a)) \vdash^{\text{no}} \{P\} \text{rcu_write_lock } C \text{ sync } D \{Q\}}$$

Proof Rule for Lock/Sync

Allowed to
access the shared heap
described by R .

$$\frac{\begin{array}{l} (R, I(a)) \vdash^{wr} \{P * \boxed{R}\} C \{Q' * \boxed{R' * R}\} \\ (R, I(a)) \vdash^{wr} \{Q' * R' * \boxed{R}\} D \{Q * \boxed{R}\} \end{array}}{(R, I(a)) \vdash^{no} \{P\} \text{rcu_write_lock } C \text{ sync } D \{Q\}}$$

Proof Rule for Lock/Sync

$$\frac{\begin{array}{l} (R, I(a)) \vdash^{wr} \{P * \boxed{R}\} C \boxed{\{Q' * \boxed{R' * R}\}} \\ (R, I(a)) \vdash^{wr} \boxed{\{Q' * R' * \boxed{R}\}} D \{Q * \boxed{R}\} \end{array}}{(R, I(a)) \vdash^{no} \{P\} \text{rcu_write_lock } C \boxed{\text{sync}} D \{Q\}}$$

sync transfers R' from
shared to private.

Proof Rule for Lock/Sync

Every op in C and D should preserve $R * \text{true}$ and $I(a)$.

$$\frac{\begin{array}{l} (R, I(a)) \vdash^{\text{wr}} \{P * \boxed{R}\} \boxed{C} \{Q' * \boxed{R' * R}\} \\ (R, I(a)) \vdash^{\text{wr}} \{Q' * R' * \boxed{R}\} \boxed{D} \{Q * \boxed{R}\} \end{array}}{(R, I(a)) \vdash^{\text{no}} \{P\} \text{rcu_write_lock } C \text{ sync } D \{Q\}}$$

Proof Rule for Readers

$$\frac{(R, I(a)) \vdash^{\text{rd}} \{P * \boxed{R * \text{true}}\} C \{Q * \boxed{R'}\}}{(R, I(a)) \vdash^{\text{no}} \{P\} \text{rcu_read_lock } C \{Q\}}$$

Proof Rule for Readers

Start with
 $\text{ResInv} * \text{true}$

Anything

$$\frac{(R, I(a)) \vdash^{\text{rd}} \{P * \boxed{R * \text{true}}\} C \{Q * \boxed{R'}\}}{(R, I(a)) \vdash^{\text{no}} \{P\} \text{rcu_read_lock } C \{Q\}}$$

Proof Rule for Readers

Every op in
C can only read
the shared.

$$\frac{(R, I(a)) \vdash^{\text{rd}} \{P * \boxed{R * \text{true}}\} \boxed{C} \{Q * \boxed{R'}\}}{(R, I(a)) \vdash^{\text{no}} \{P\} \text{rcu_read_lock } C\{Q\}}$$

Proof Rule

$$(R, I(a)) \vdash^{\text{wr}} \{P * Q * \boxed{R}\} \text{barrier}_{\boxed{Q}} \{P * \boxed{Q * R}\}$$

Transfer Q from
private to shared.

Q has to
be precise.

Soundness

- No weak memory yet.
- Proved by the compilation into RGSep by Vafeiadis and Parkinson.

Messages

- Sync transfers a heaplet from shared to private.
- Barrier transfers a heaplet from private to shared.

RCU Reader/Writer

```
while (TRUE) {  
    rcu_read_lock {  
        b' = b;  
        x0 = *b';  
        x1 = *(b'+1);  
    }  
    .. USE x0,x1 ..  
}
```

```
while (TRUE) {  
    .. PRODUCE y0,y1 ..  
    nb = malloc(2);  
    rcu_write_lock {  
        *nb = y0; *(nb+1) = y1;  
        barrier();  
        ob = b; b = nb;  
    } sync {  
        free(ob); free(ob+1);  
    }  
}
```

RCU Reader/Writer **Optimized**

```
nb = malloc(2);
```

```
while (TRUE) {  
    rcu_read_lock {  
        b' = b;  
        x0 = *b';  
        x1 = *(b'+1);  
    }  
    .. USE x0,x1 ..  
}
```

```
while (TRUE) {  
    .. PRODUCE y0,y1 ..  
    nb = malloc(2);  
    rcu_write_lock {  
        *nb = y0; *(nb+1) = y1;  
        barrier();  
        ob = b; b = nb;  
    } sync {  
        nb = ob;  
        free(b);  
    }  
}}
```