Reasoning about consistency choices in distributed systems

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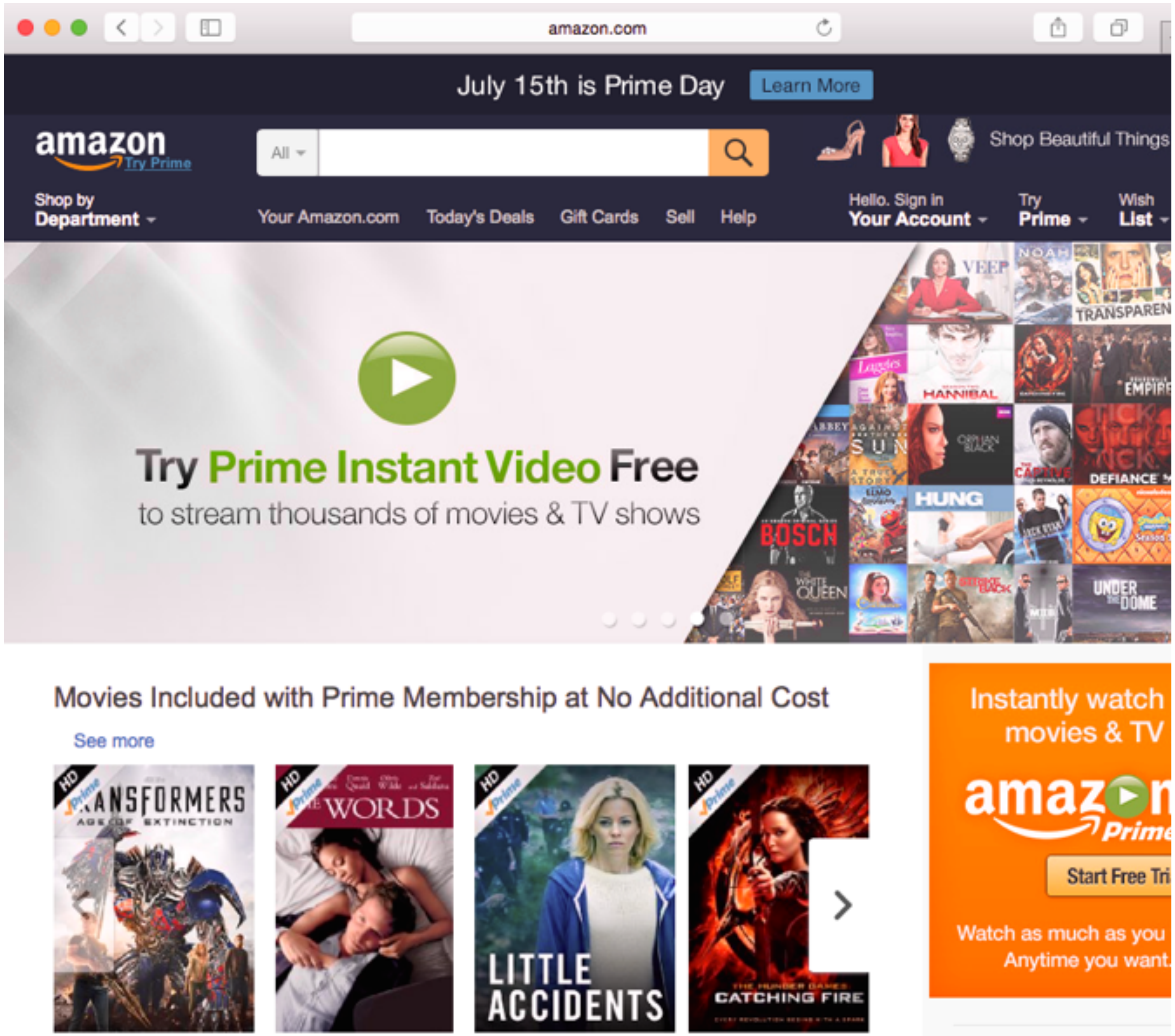
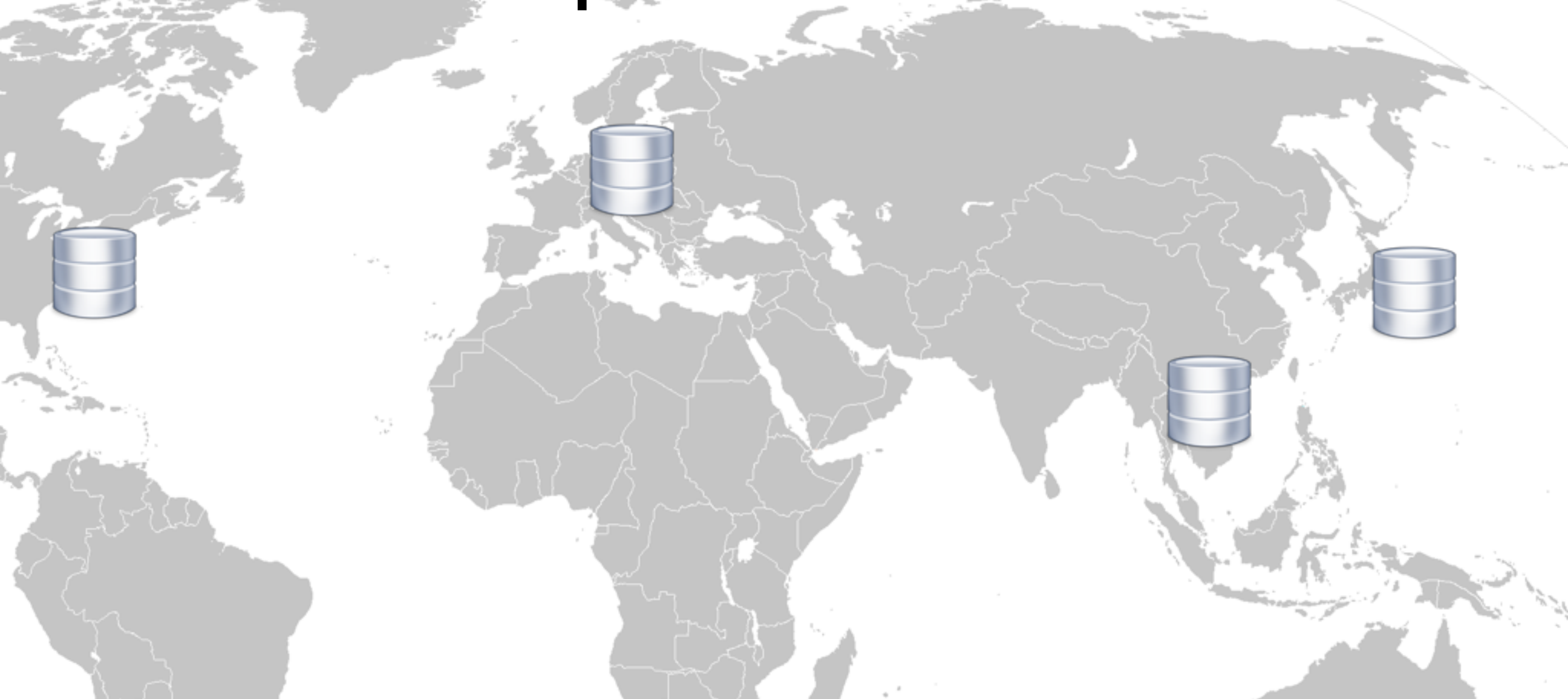
Joint work with Alexey Gotsman (IMDEA, Spain), Carla Ferreira (U Nova Lisboa), Mahsa Najafzadeh, Marc Shapiro (INRIA)

Global-scale Internet service



Geo-replicated databases



- Every data centre stores a complete replica of data
- Purpose: Minimising latency. Fault tolerance.

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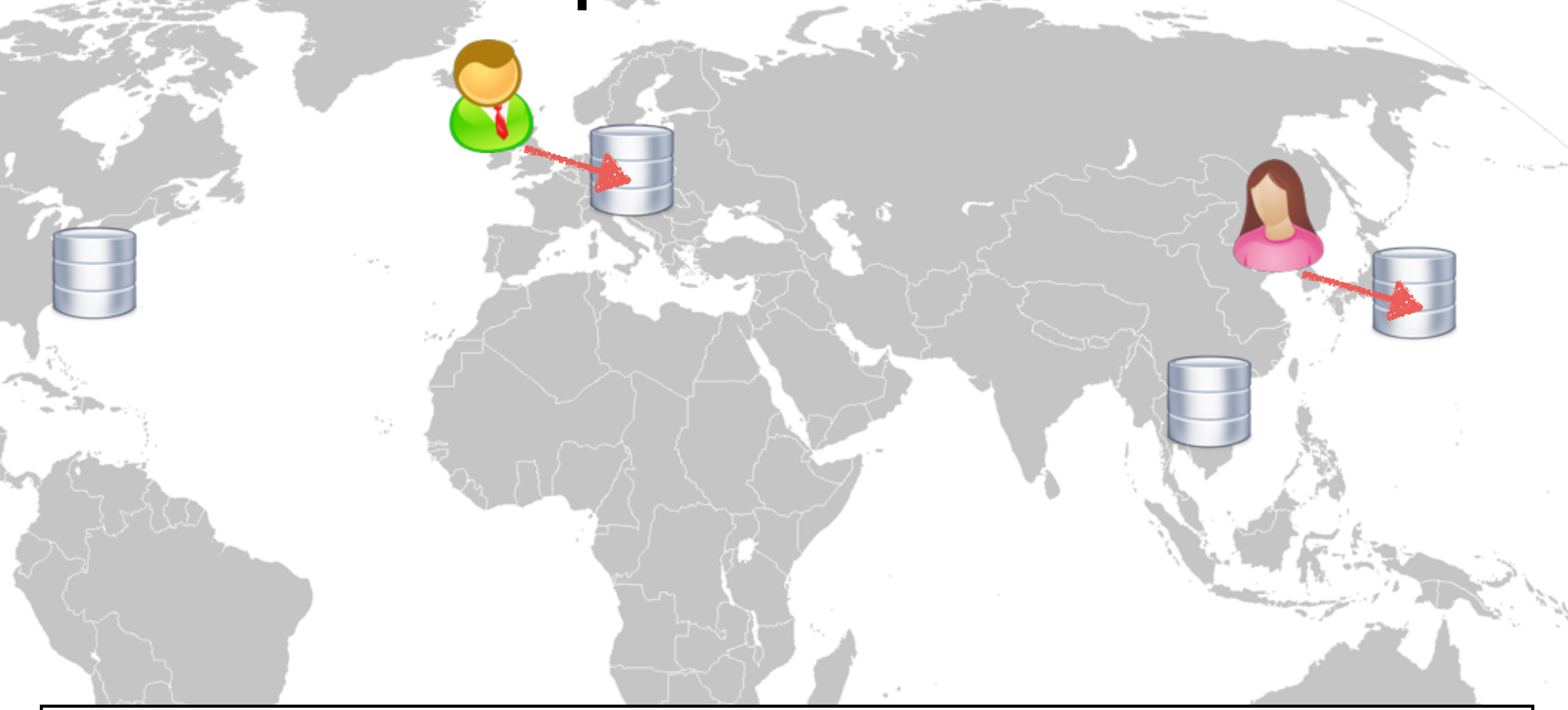

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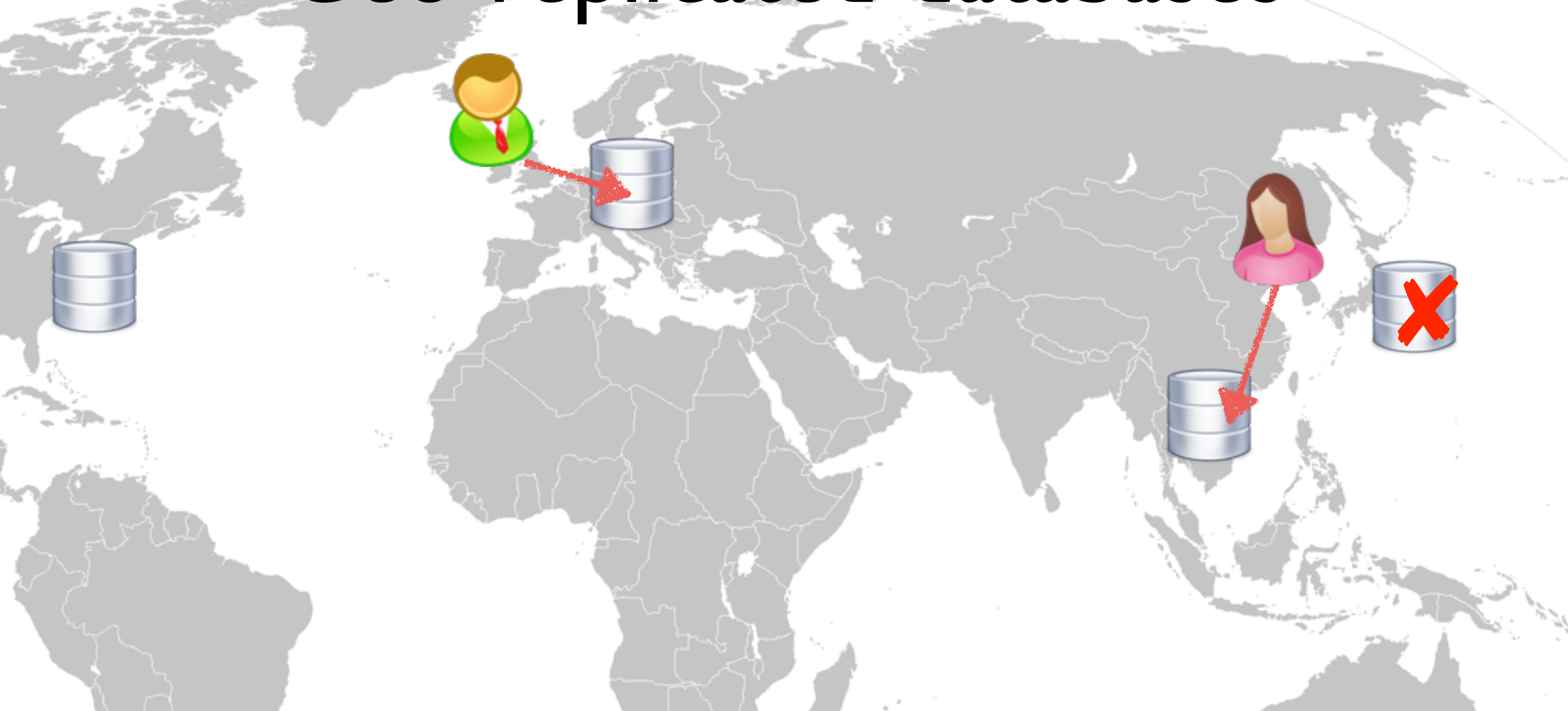

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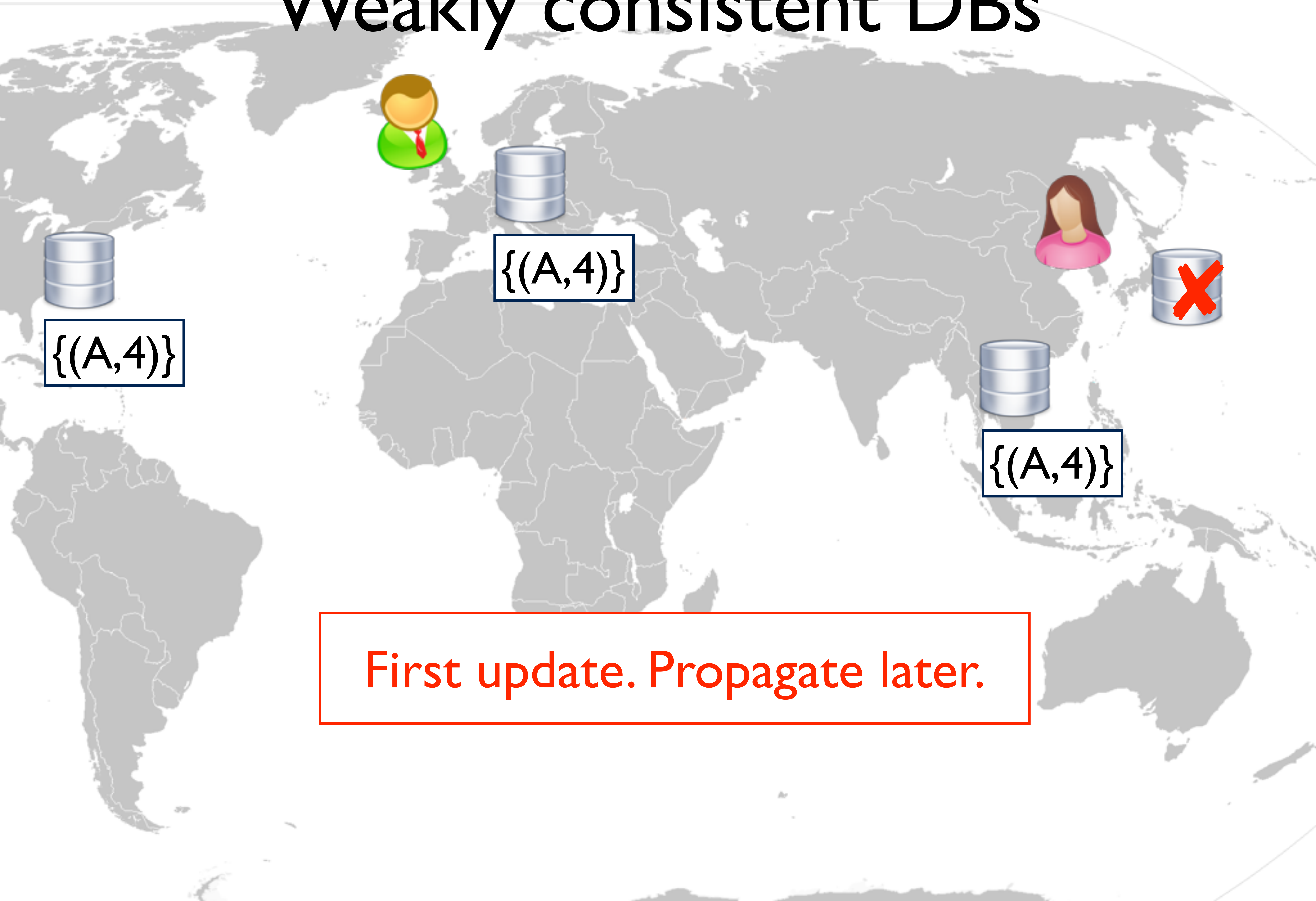
Weakly consistent DBs



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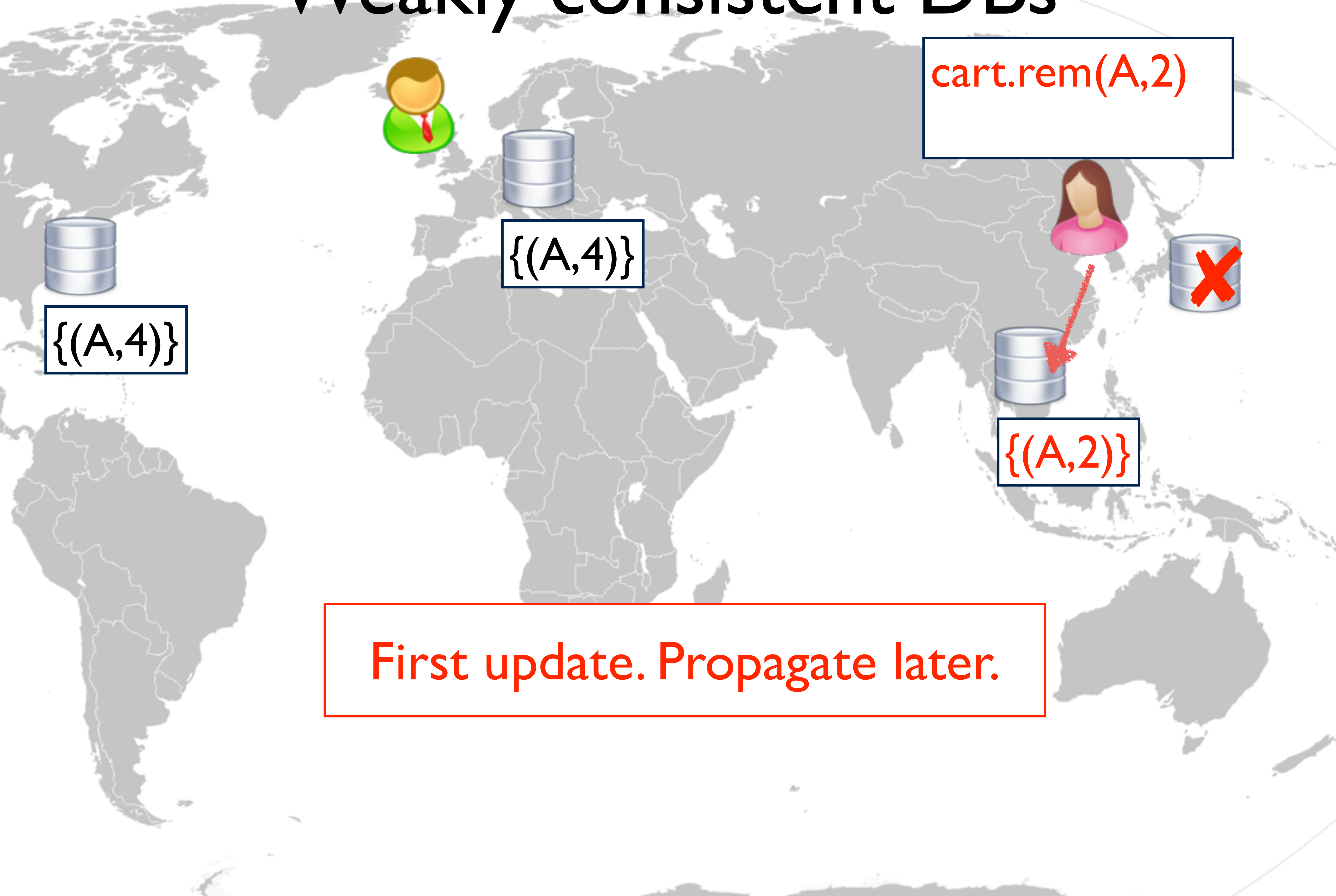

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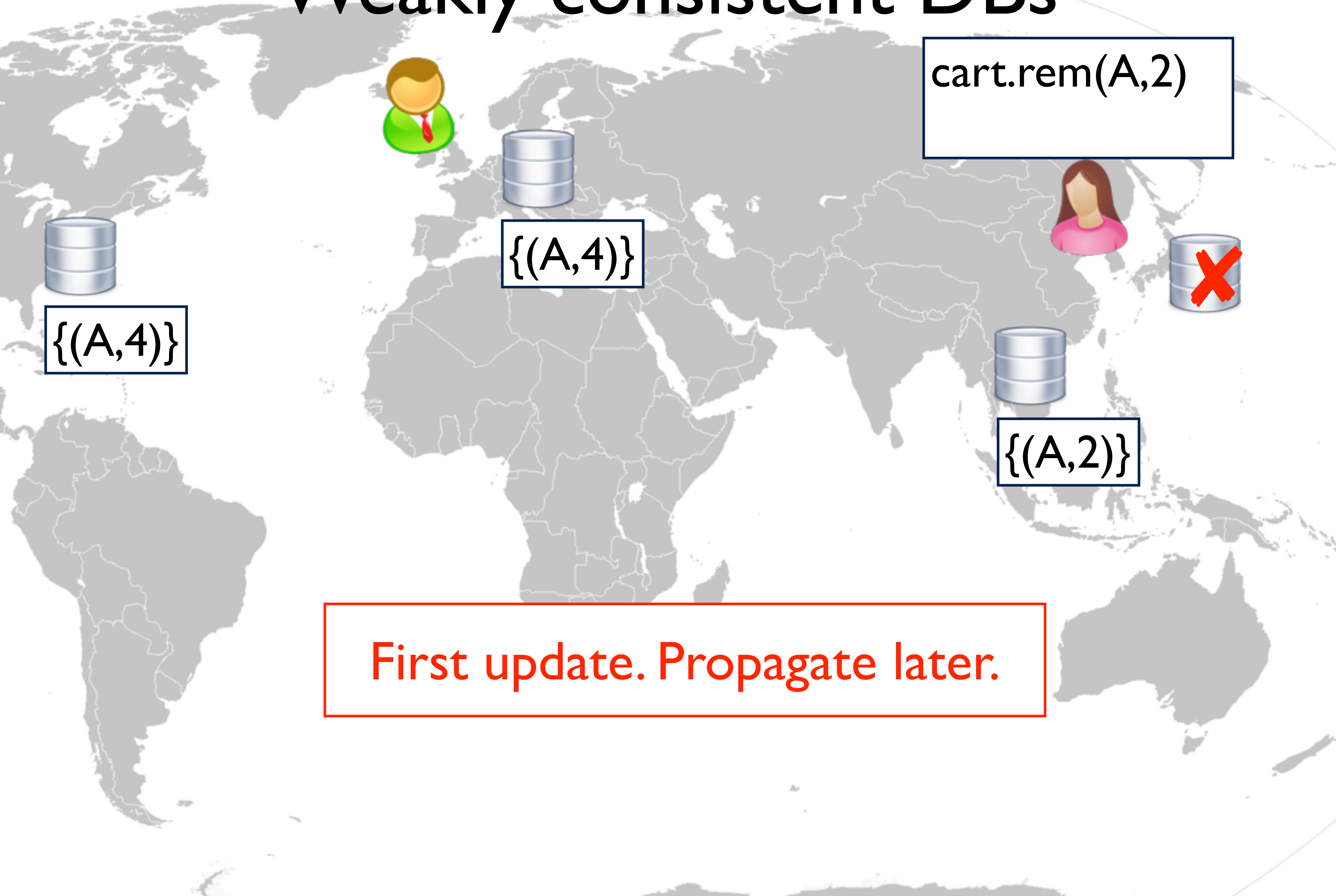

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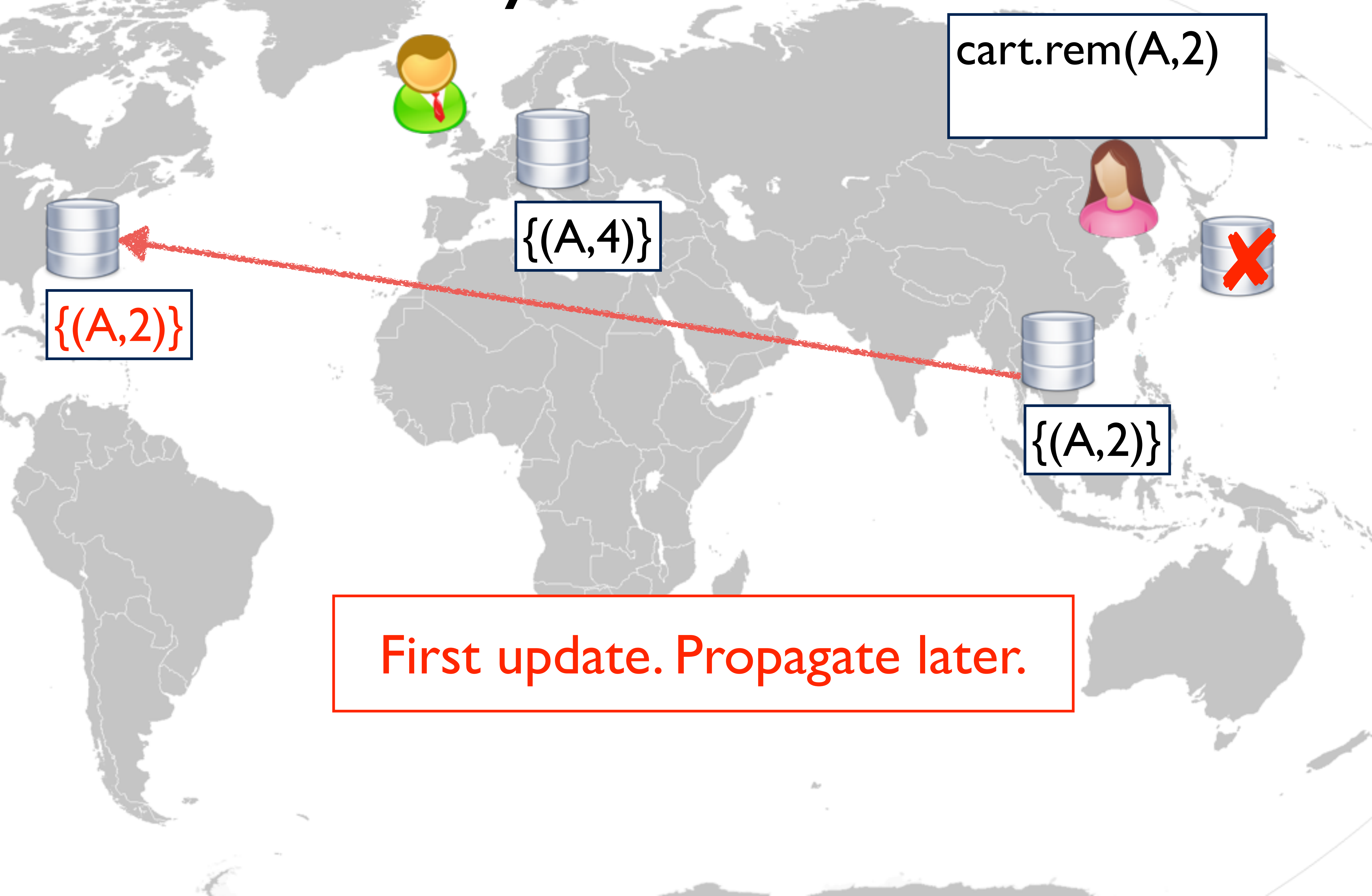

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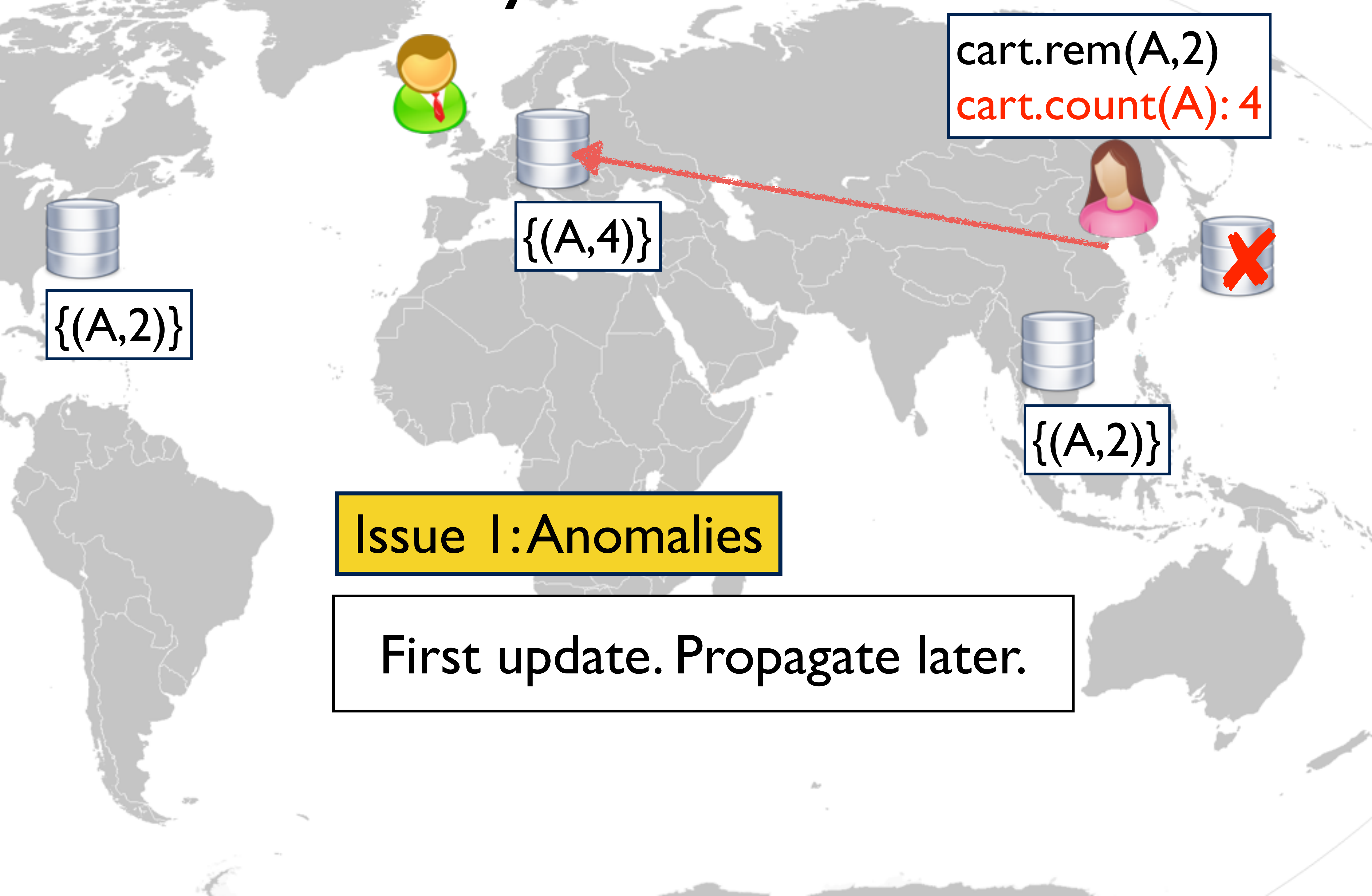

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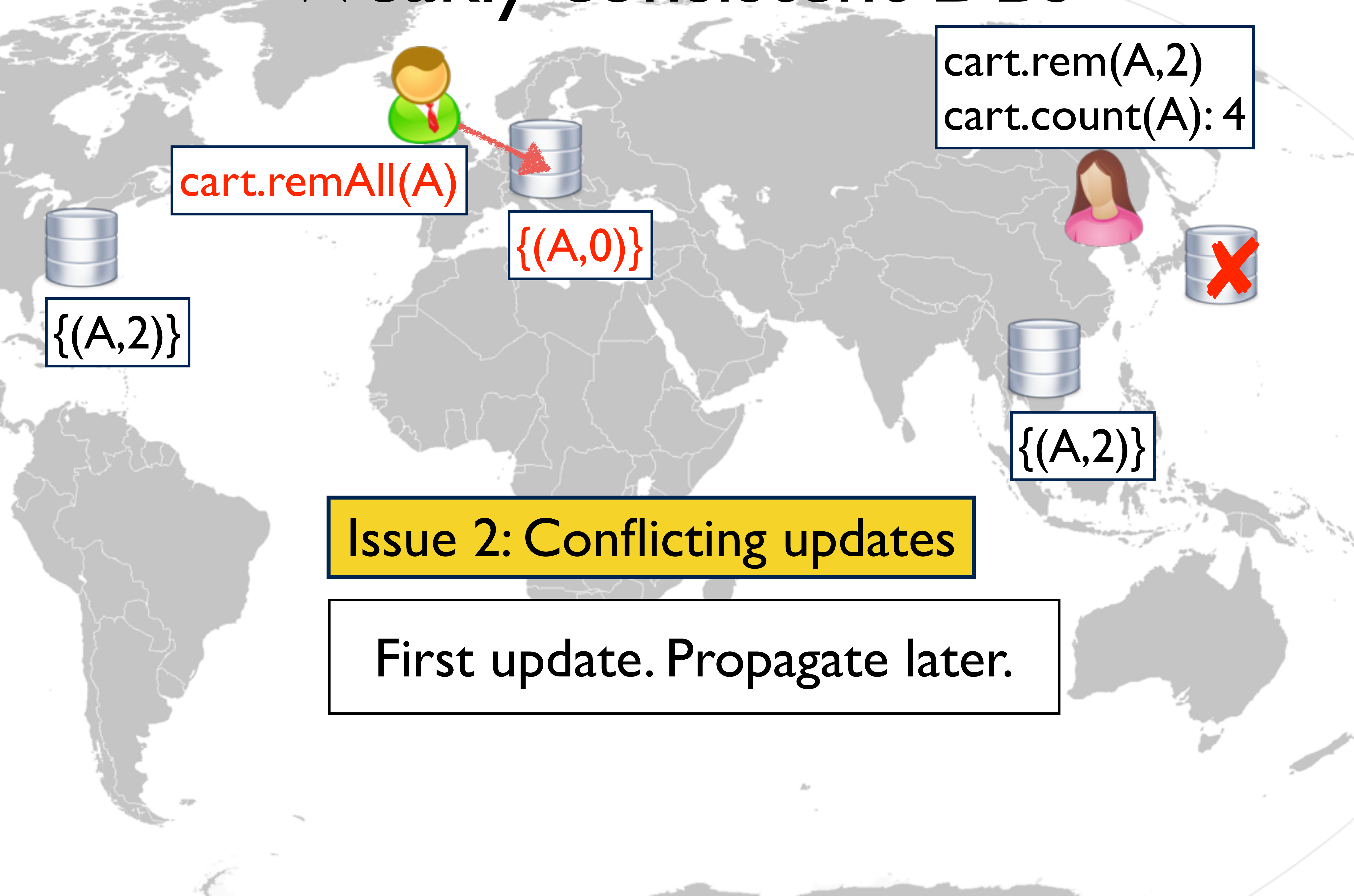
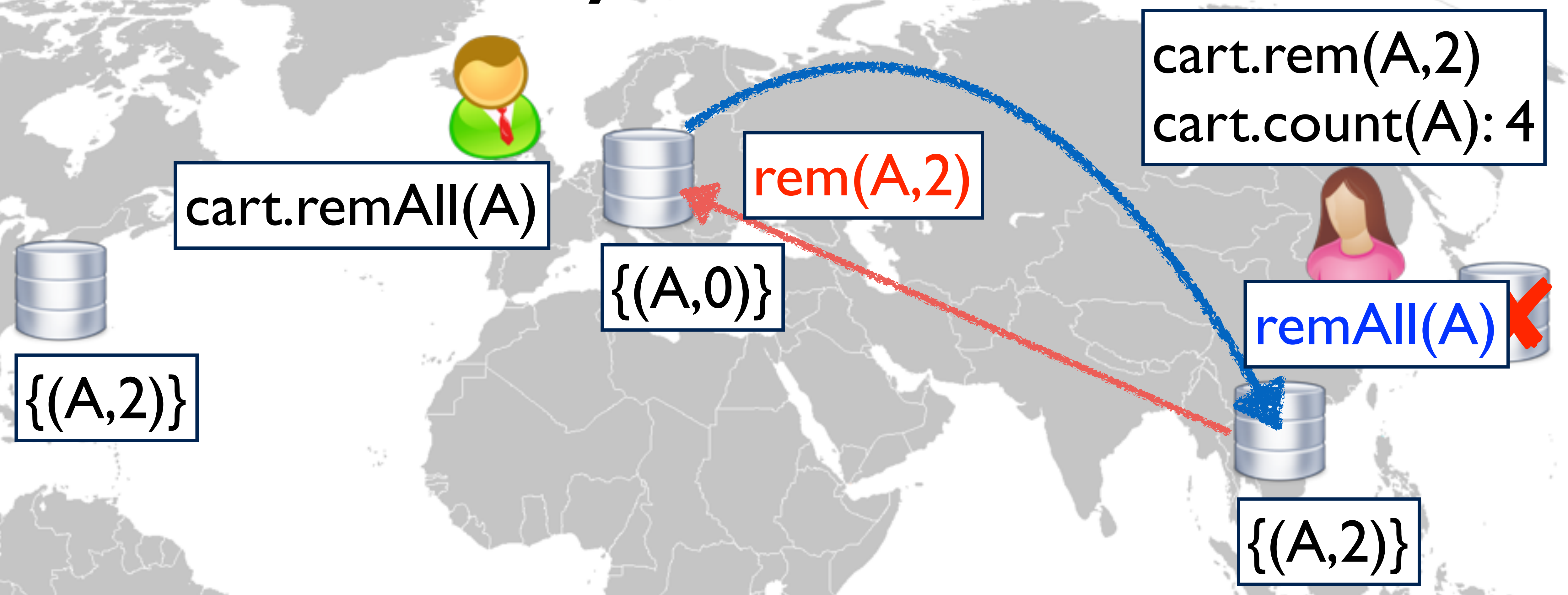


Issue 2: Conflicting updates

First update. Propagate later.

Weakly consistent DBs



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How to develop correct programs running on top of weakly consistent distributed databases?

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1. Strengthen consistency selectively.
2. Prove the correctness of a program.

Simple bank account

```
class account {
  // invariant: amount >= 0
  var amount = 0

  def query() = { return amount }

  def inc() = {
    amount = amount+1; return true
  }

  def dec() = {
    if (amount > 0) {
      amount = amount-1; return true
    }
    else { return false }
  }
}
```

Distributed bank account

```
class account {
  // invariant: amount >= 0
  var[dis] amount = 0

  def query() = { return (amount, (a)=>a) }

  def inc() = {
    amount = amount+1; return (true, (a)=>a+1)
  }

  def dec() = {
    if (amount > 0) {
      amount = amount-1; return (true, (a)=>a-1)
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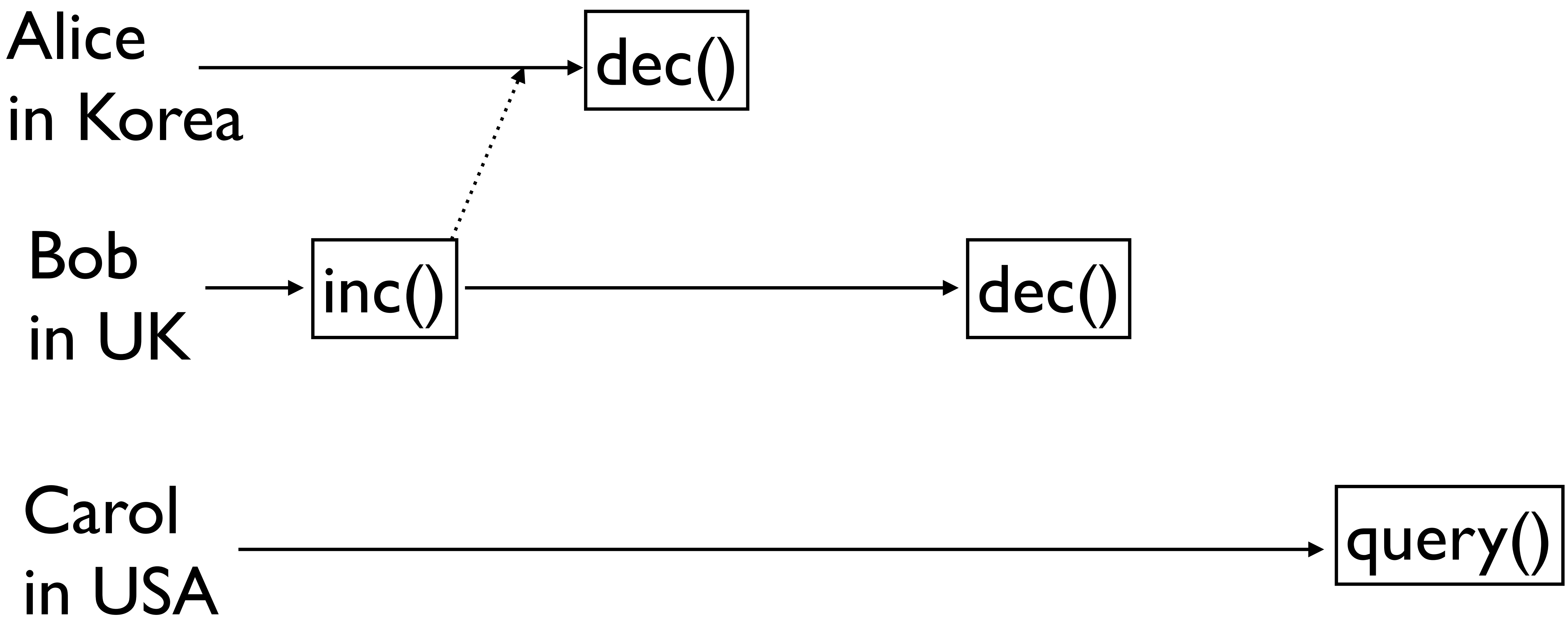
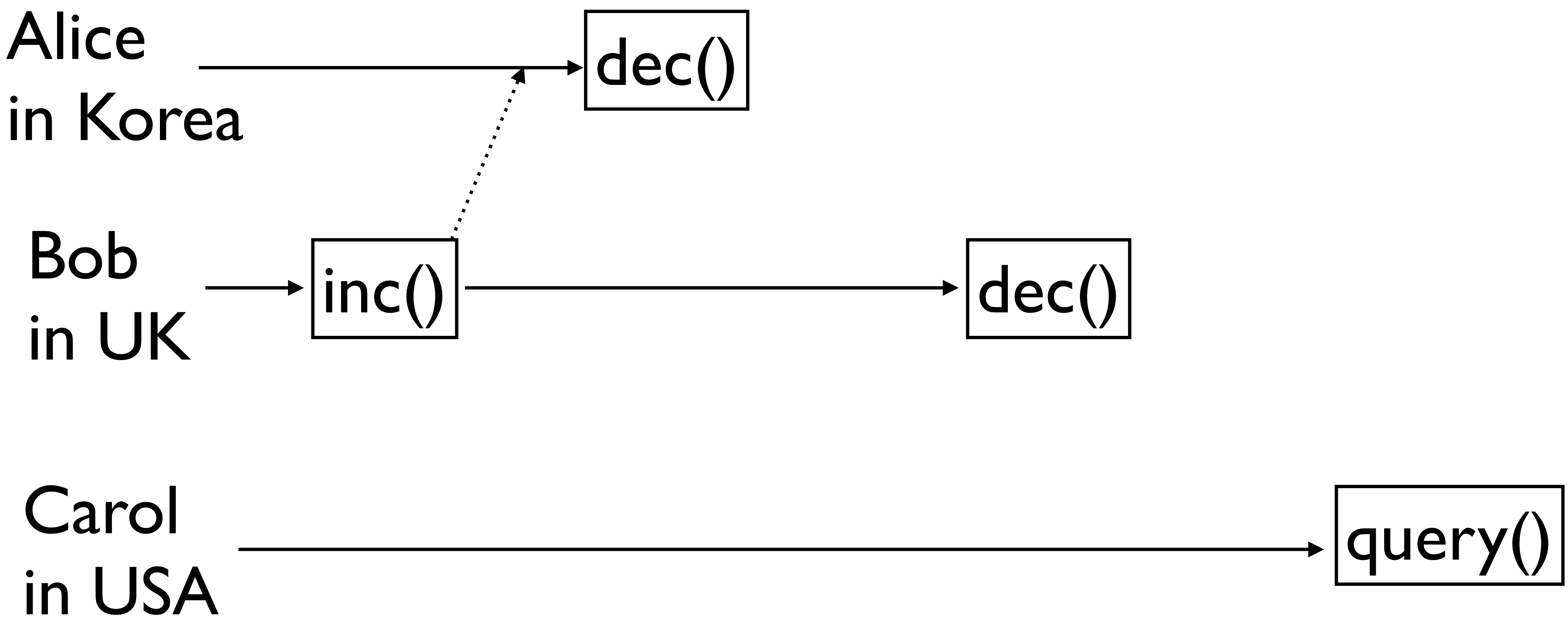


[Q] What cannot be the result of query()?
(a) 0 (b) -1 (c) -2 (d) all possible



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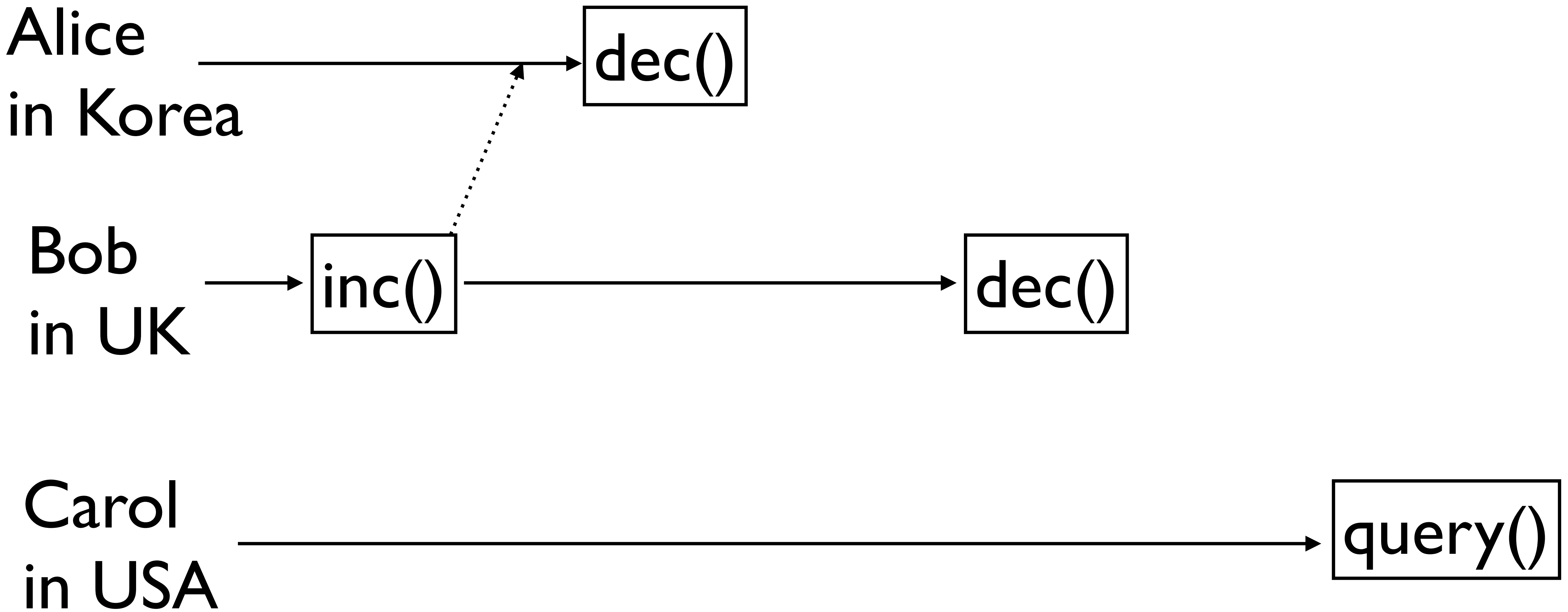

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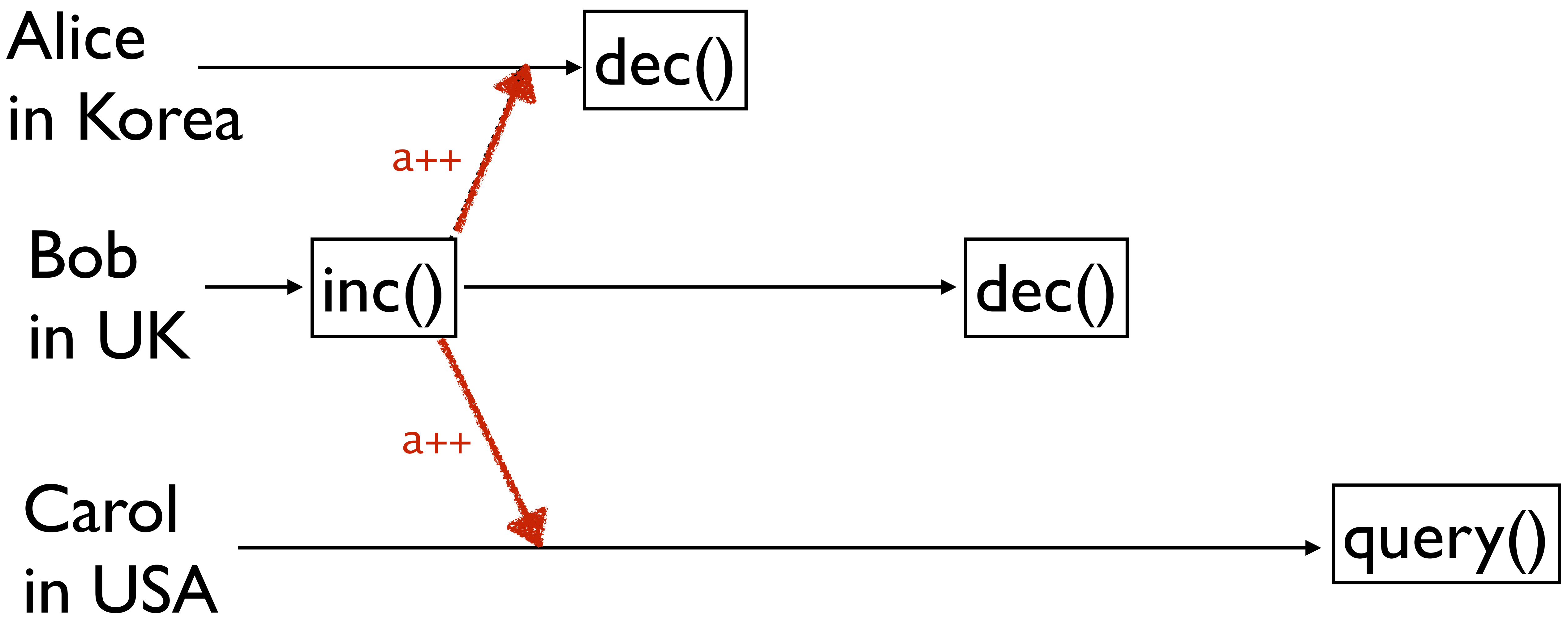

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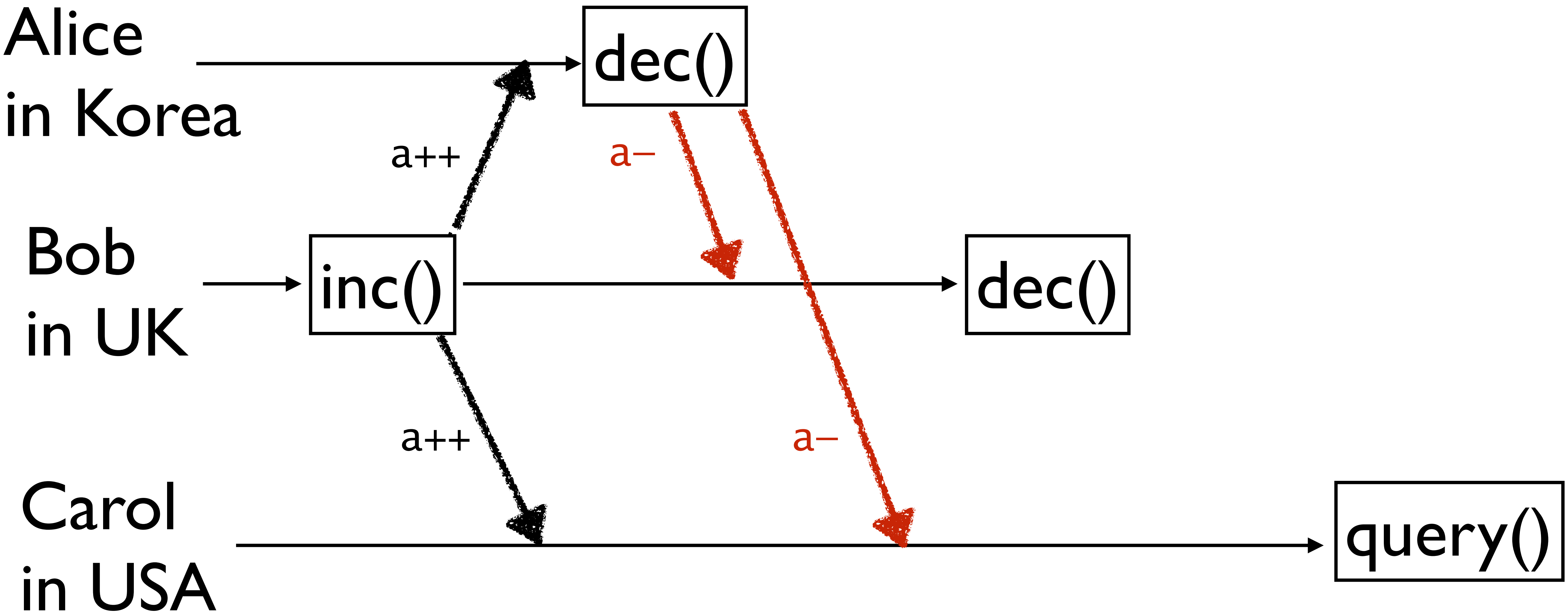

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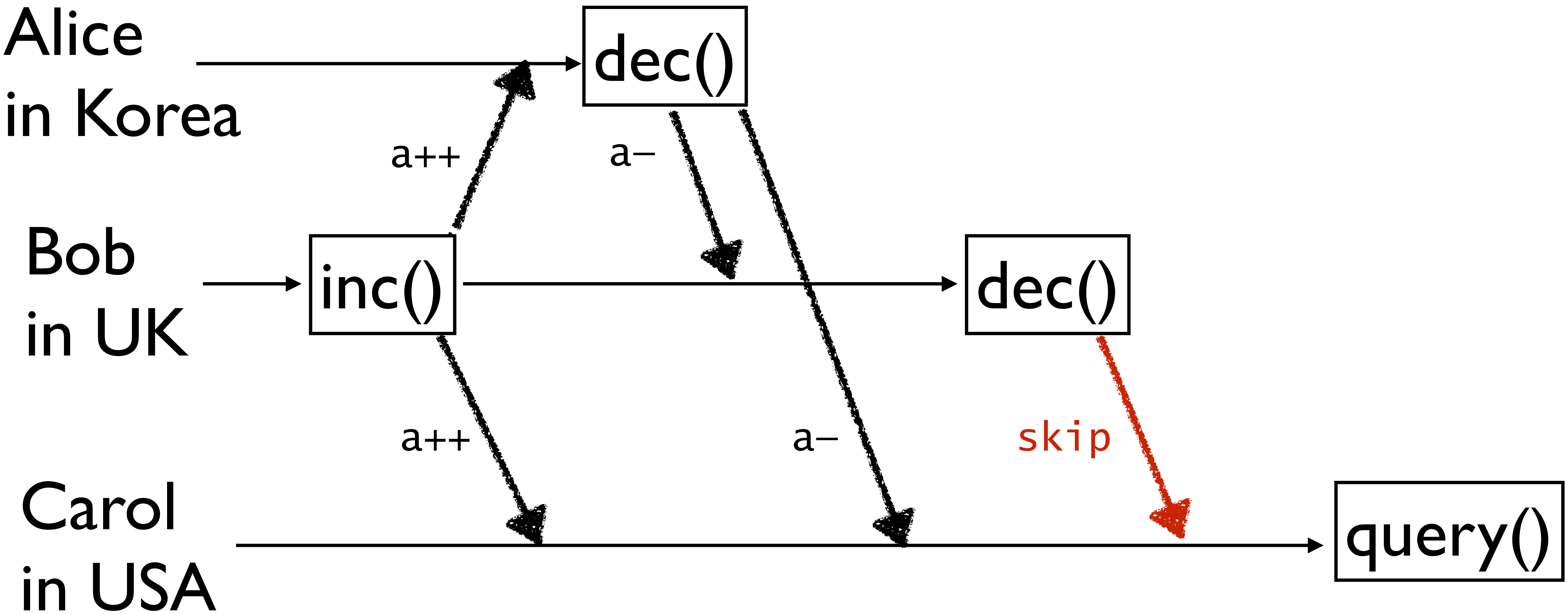

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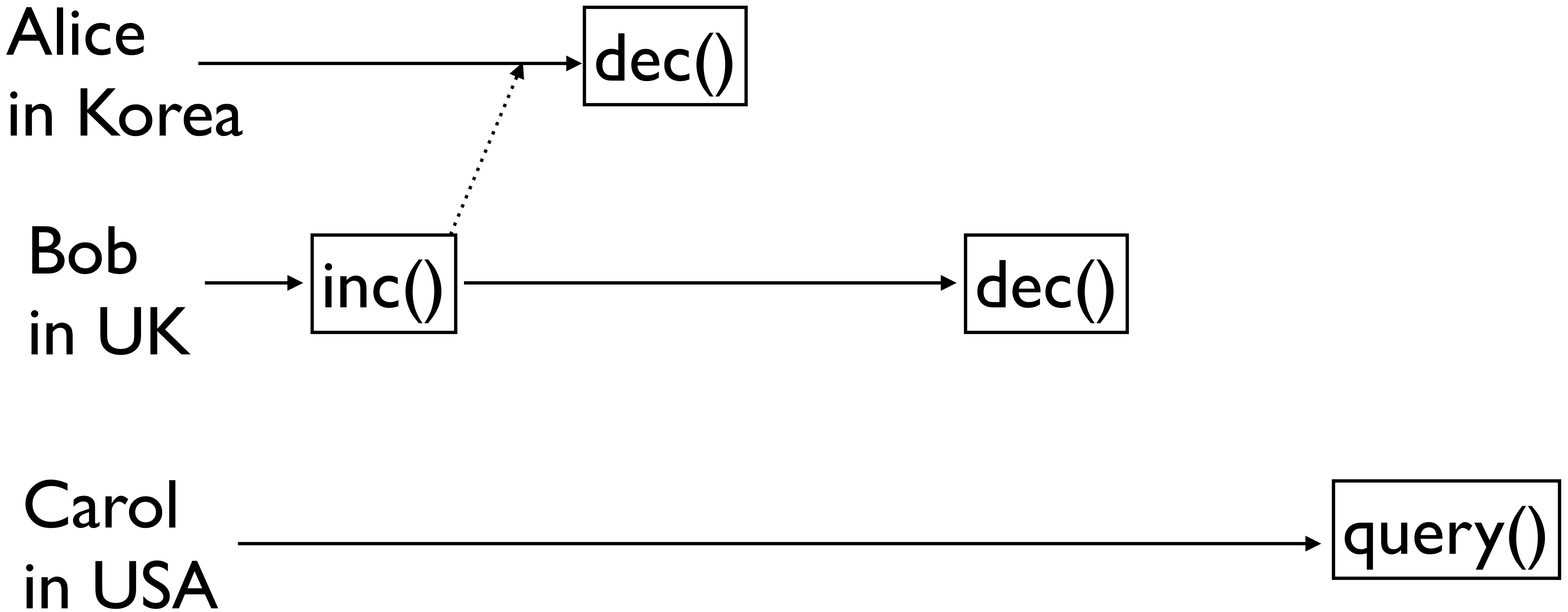

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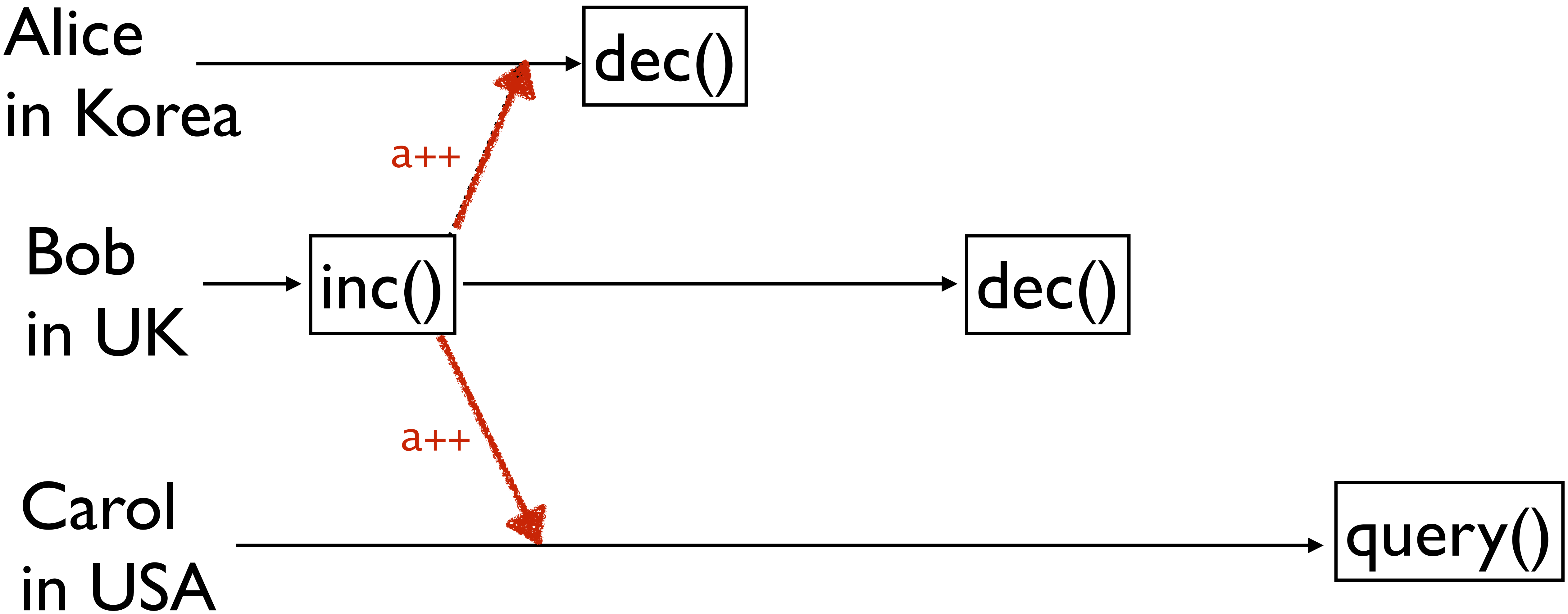

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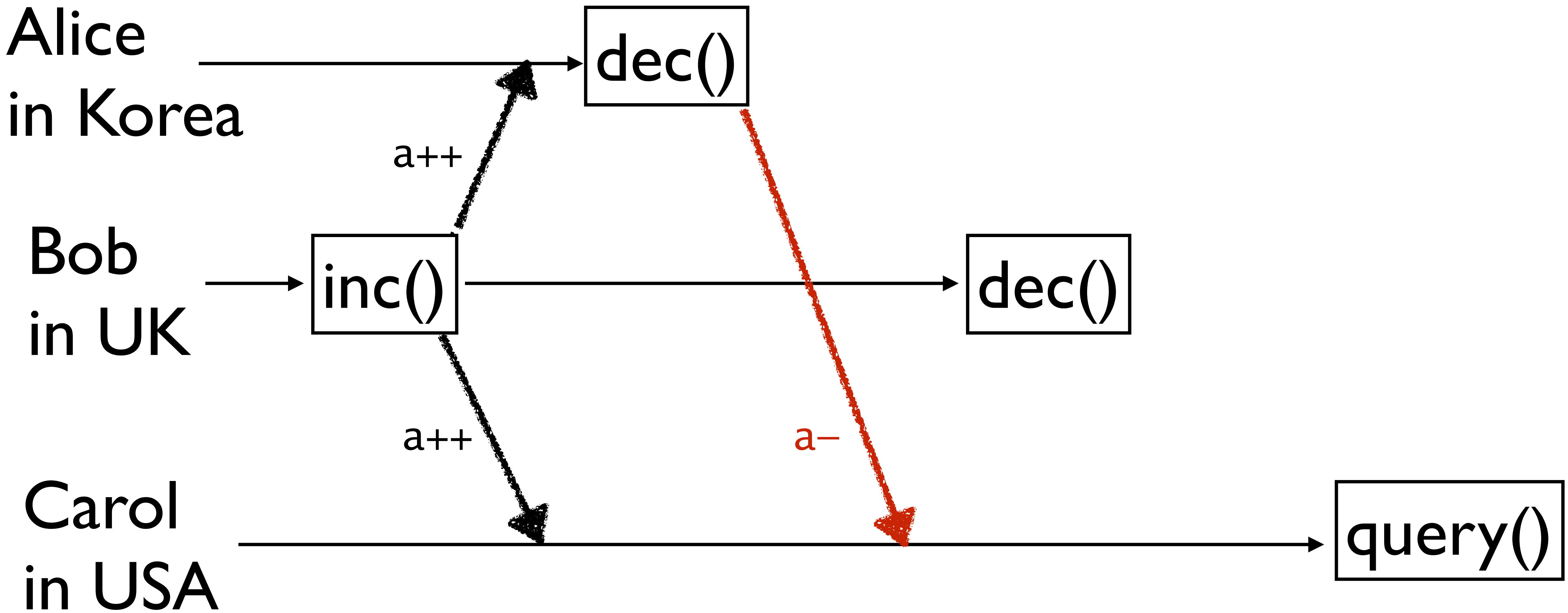

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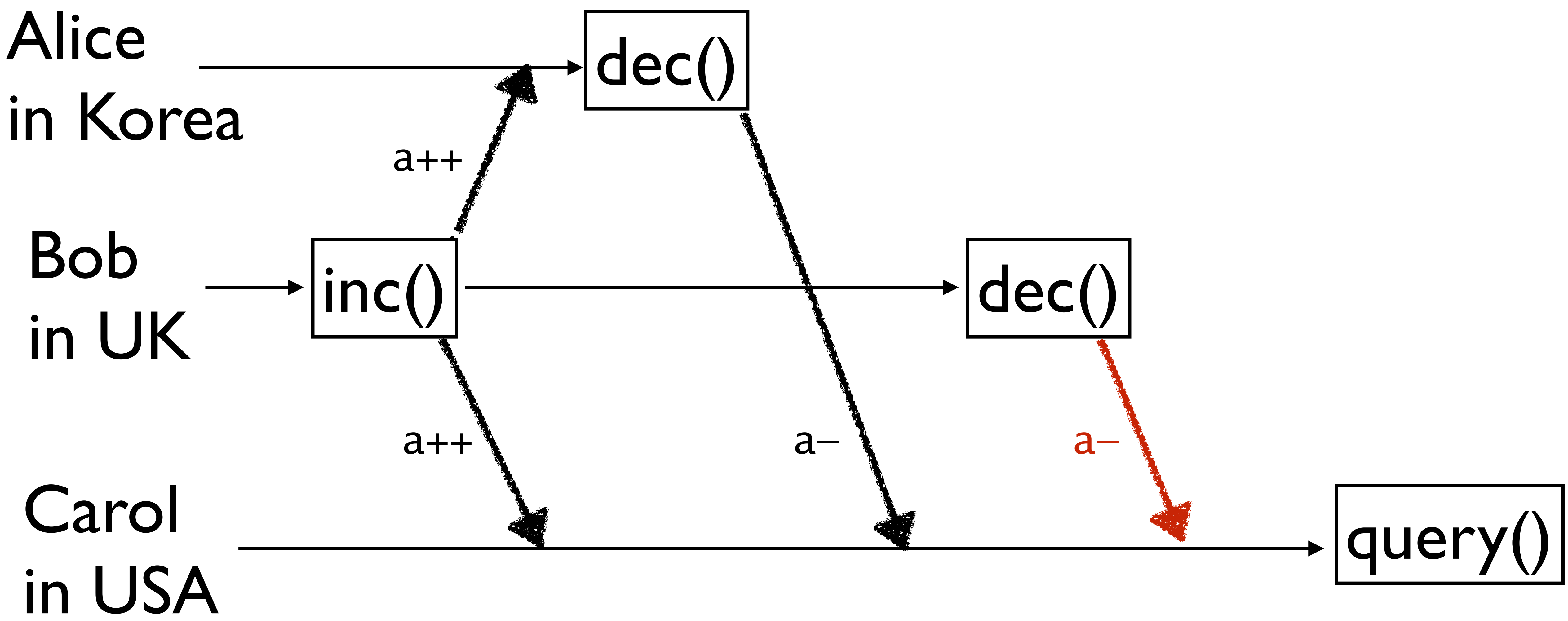

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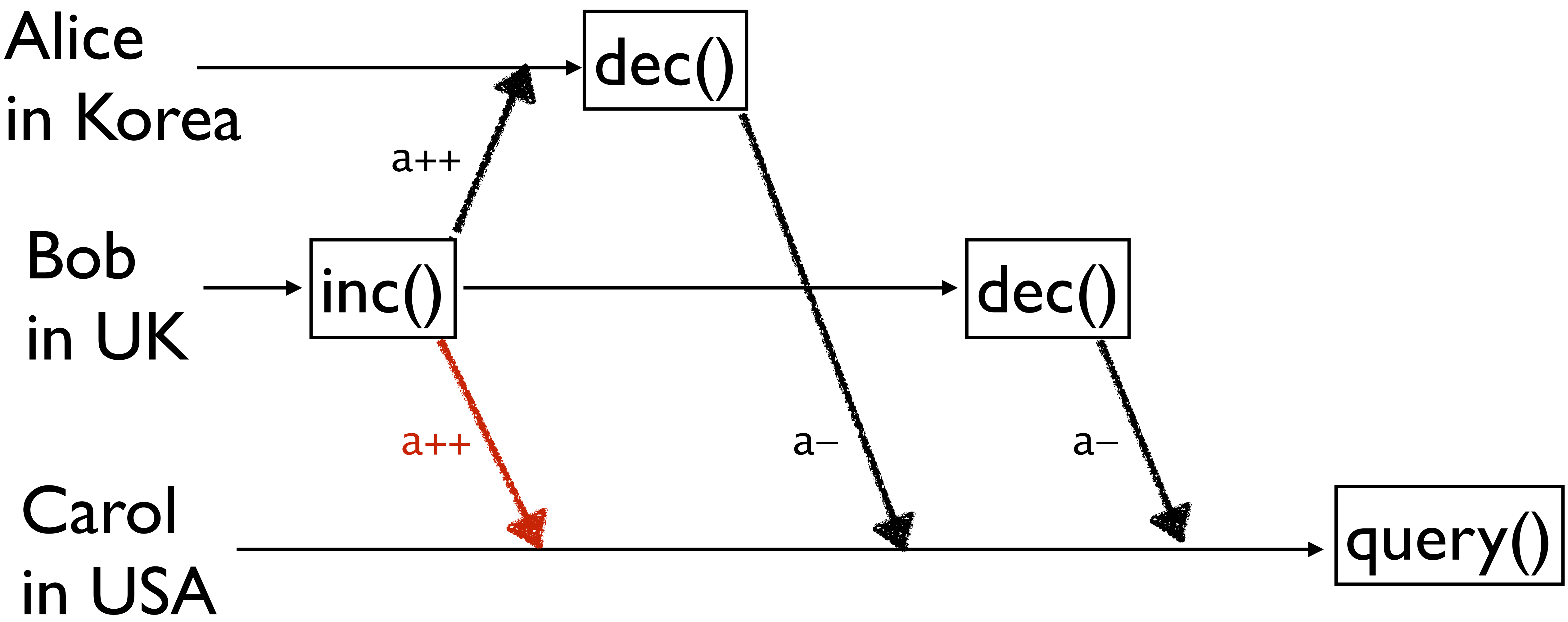

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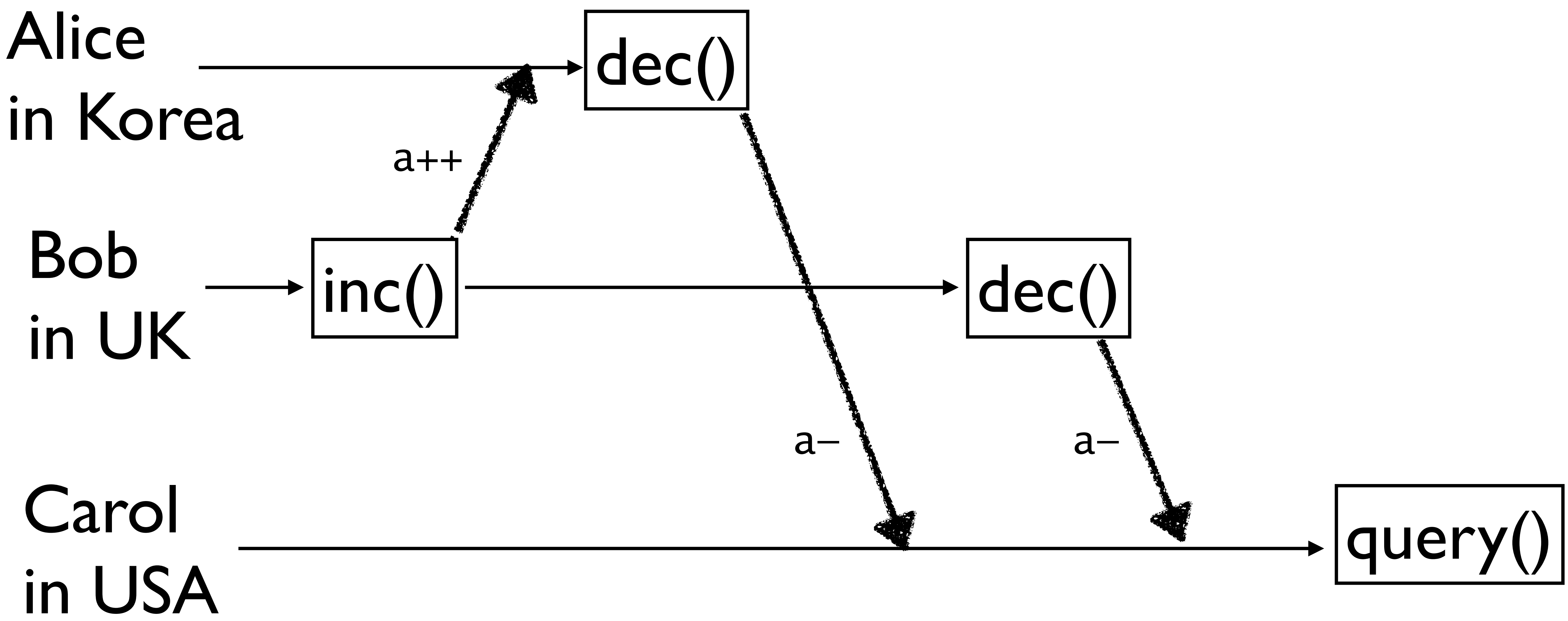

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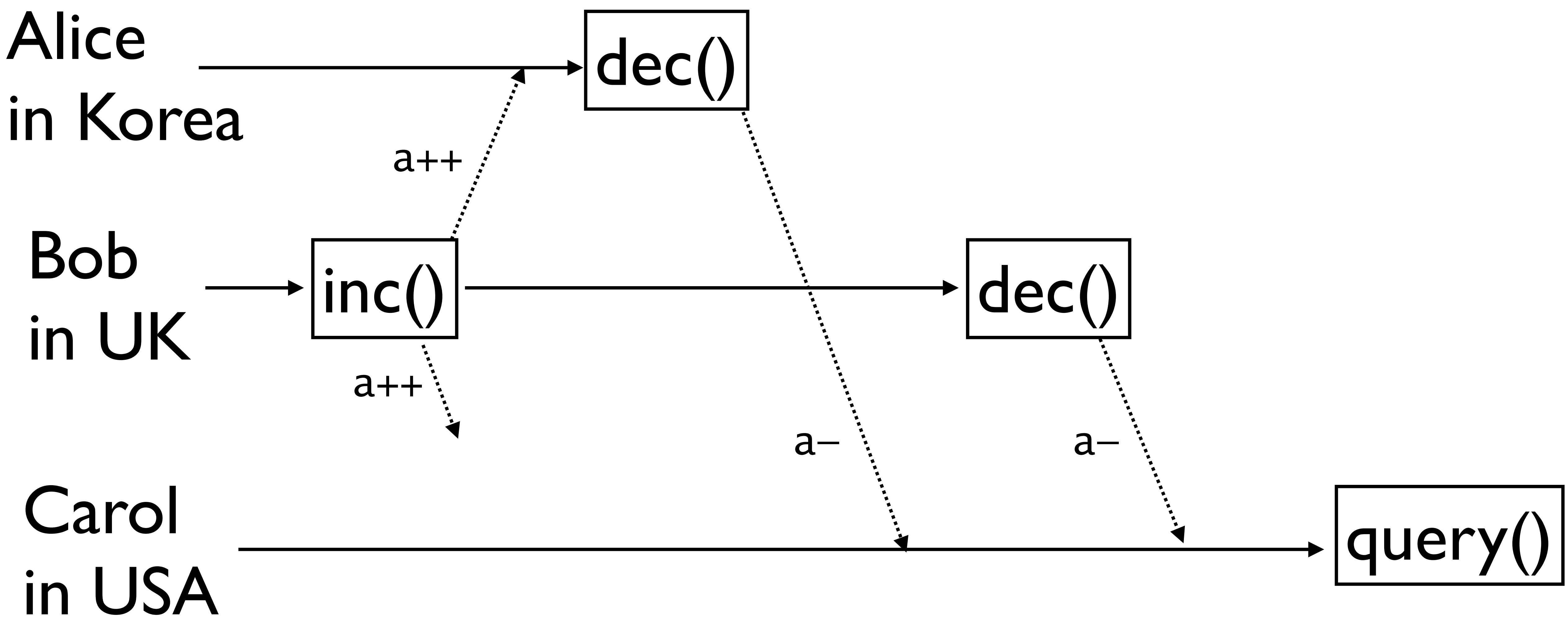
How to write correct prog.?

1. Strengthen consistency selectively.
2. Prove the correctness of a program.

Causal consistency

- Message delivery preserves the dependency of events.

Axiom: HB is transitive.



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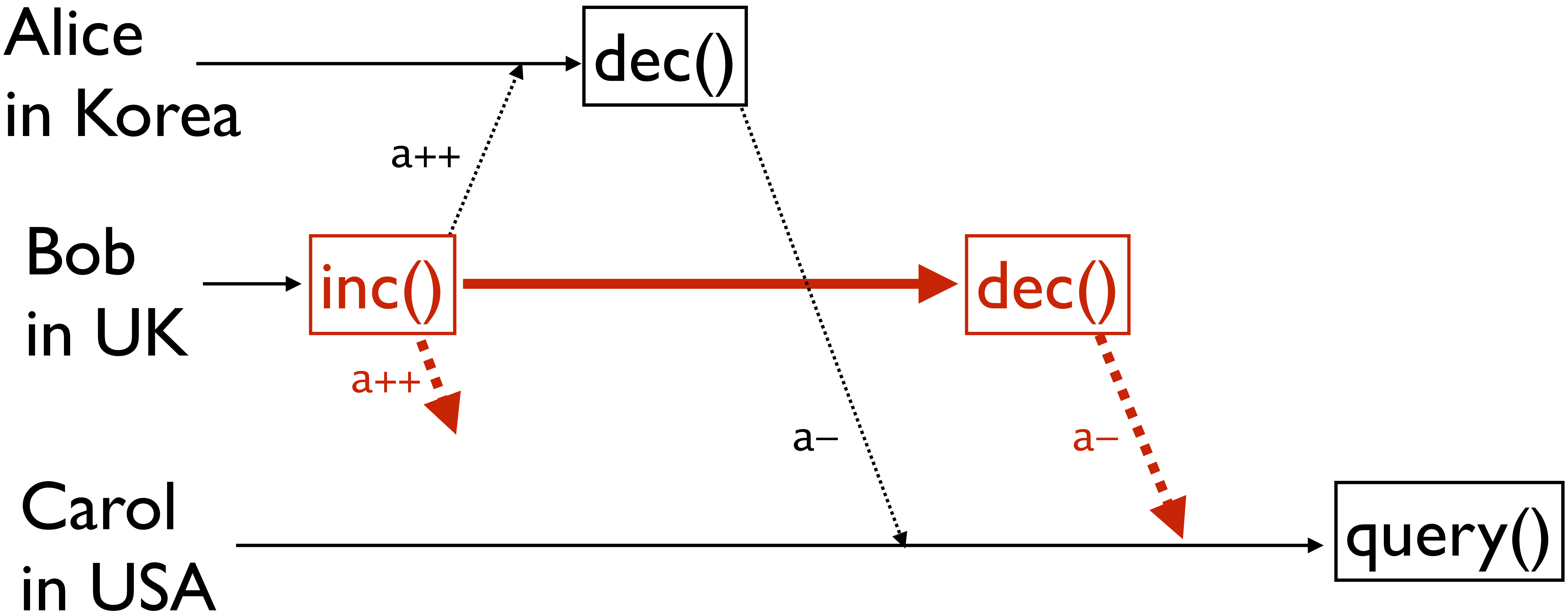

Not causally consistent.

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```
use causality

class account {
  // invariant: amount >= 0
  var[dis] amount = 0

  def query() = { return (amount, (a)=>a) }

  def inc() = {
    amount = amount+1; return (true, (a)=>a+1)
  }

  def dec() = {
    if (amount > 0) {
      amount = amount-1; return (true, (a)=>a-1)
    }
    else { return (false, (a)=>a) }
  }
}
```

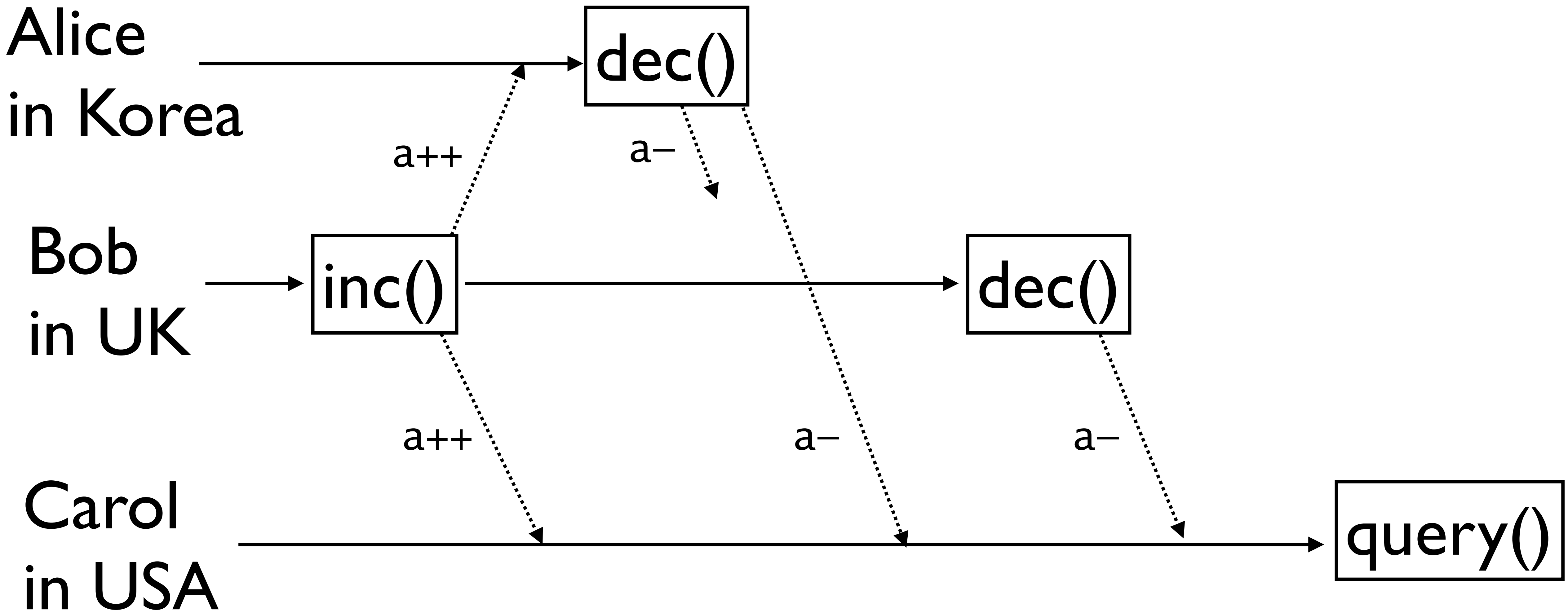
Token system

- $(\mathcal{T}, $💔$)$ where 💔 is a symmetric rel. on $\mathcal{T}$.
- Examples:

1. $\mathcal{T}$ = {lock}, 💔 = {(lock,lock)}
2. $\mathcal{T}$ = {rd,wr}, 💔 = {(rd,wr), (wr,wr), (wr,rd)}

On-demand consistency using a token system ($\mathcal{T}$, 💔)

- Each operation acquires a set of tokens.
- Operations with conflicting tokens cannot be run concurrently.



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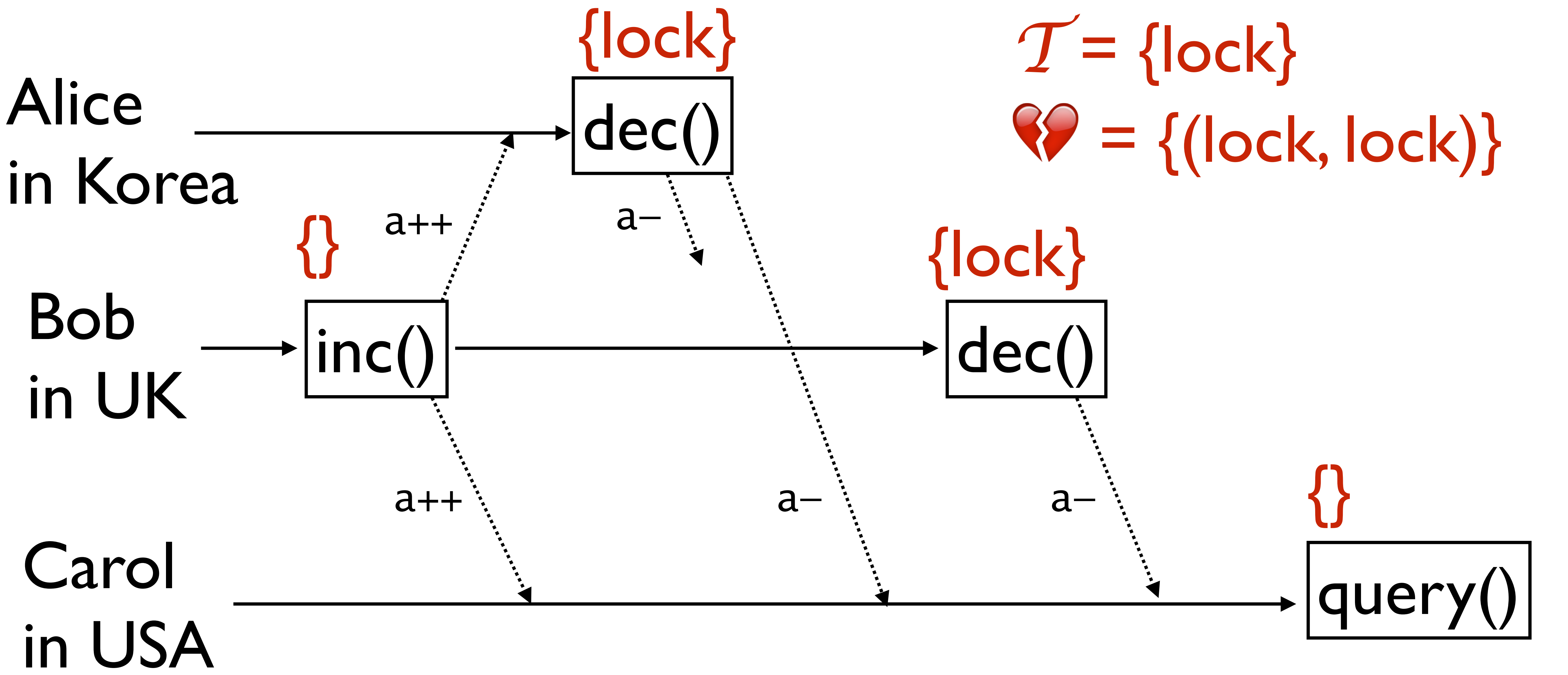

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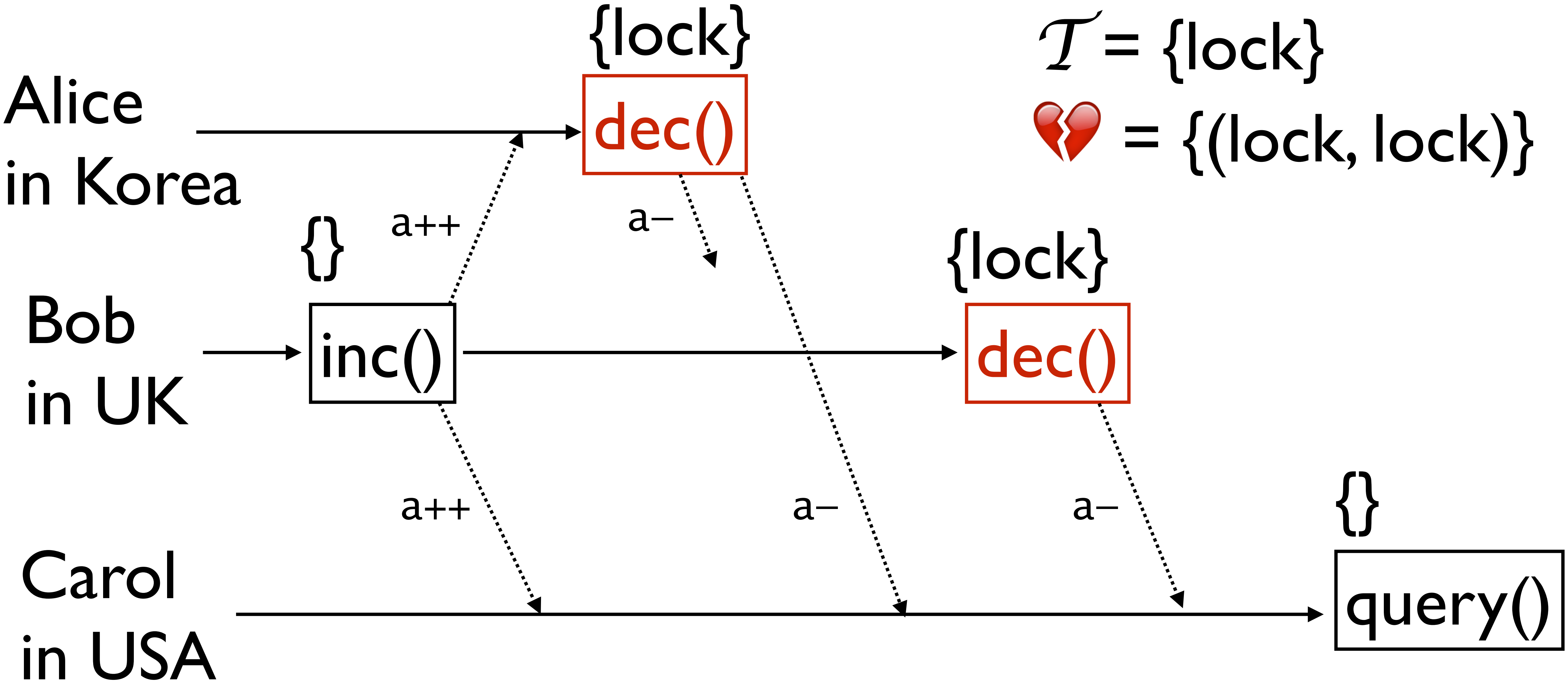

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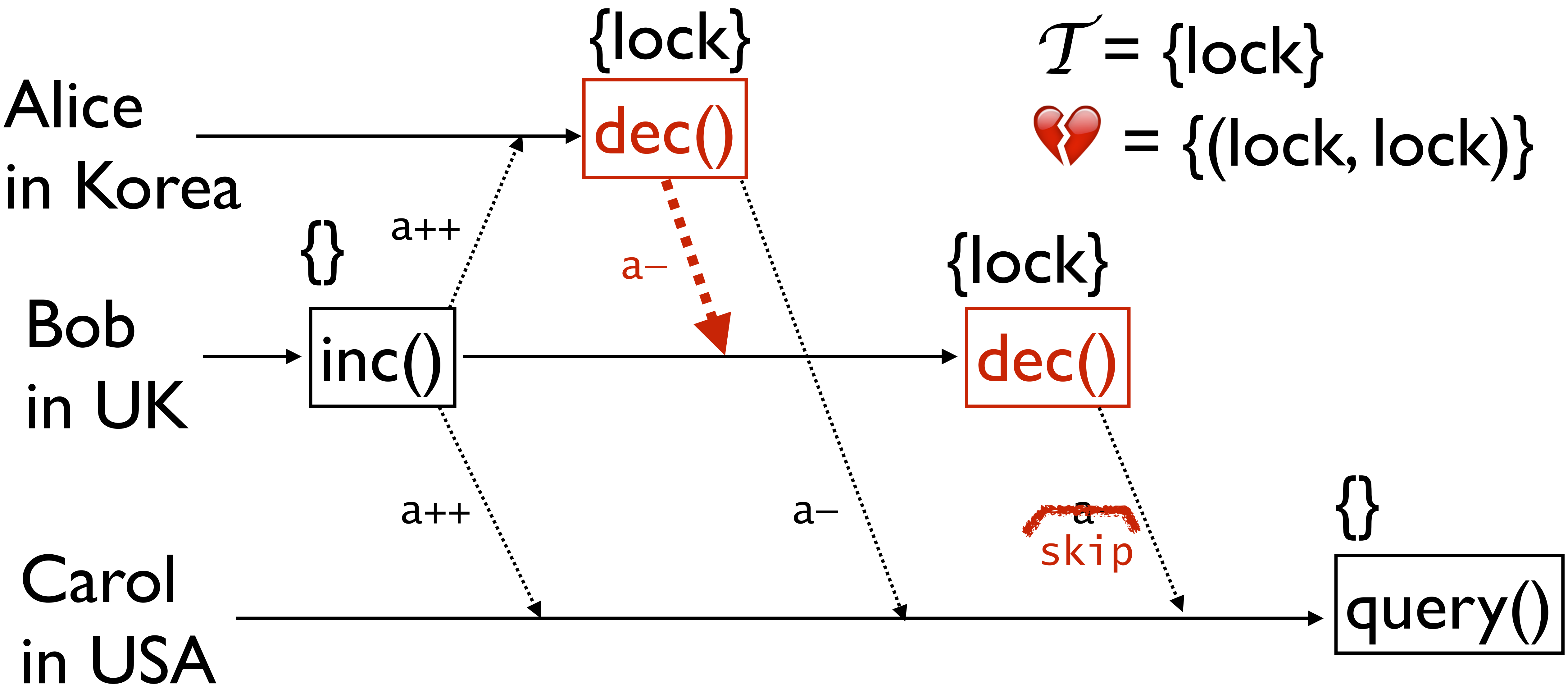

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```
use causality

class account {
  // invariant: amount >= 0
  var[dis] amount = 0
  use-token-system({lock},{(lock,lock)})

  def query() with {} =
  { return (amount, (a)=>a) }

  def inc() with {} = {
    amount = amount+1; return (true, (a)=>a+1)
  }

  def dec() with {lock} = {
    if (amount > 0) {
      amount = amount-1; return (true, (a)=>a-1)
    }
    else { return (false, (a)=>a) }
  }
}
```

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Our proof rule

- Based on rely-guarantee.
- Incorporates guarantees from causal and on-demand consistency.

To prove that I is an invariant

To prove that $\mathcal{I}$ is an invariant

$$\exists G_0 \in \mathcal{P}(\mathsf{State} \times \mathsf{State}), G \in \mathsf{Token} \to \mathcal{P}(\mathsf{State} \times \mathsf{State})$$

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S2. $G_0(I) \subseteq I \land \forall \tau.\ G(\tau)(I) \subseteq I$

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S3. $\forall o, \sigma, \sigma'.\ (\sigma \in I \wedge (\sigma, \sigma') \in (G_0 \cup G((\mathcal{F}_o^{\mathsf{tok}}(\sigma))^{\perp}))^*)$
$\implies (\sigma', \mathcal{F}_o^{\mathsf{eff}}(\sigma)(\sigma')) \in G_0 \cup G(\mathcal{F}_o^{\mathsf{tok}}(\sigma))$

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$$\implies (\sigma', \mathcal{F}_o^{\textsf{eff}}(\sigma)(\sigma')) \in G_0 \cup G(\mathcal{F}_o^{\textsf{tok}}(\sigma))$$

T$^{\perp}$ = {**τ** | ∄**τ**'∈T. (**τ**, **τ**') ∈ 💔}

$$\mathcal{I} = \{ \sigma \mid 0 \le \sigma \}$$
$$G_0 = \{(\sigma, \sigma') \mid \sigma \le \sigma'\}$$
$$G_1(\text{lock}) = \{(\sigma, \sigma') \mid 0 < \sigma \wedge \sigma' \le \sigma\}$$

$$\text{S3. } \forall o, \sigma, \sigma'. \ (\sigma \in I \wedge (\sigma, \sigma') \in (G_0 \cup G((\mathcal{F}_o^{\mathsf{tok}}(\sigma))^{\perp}))^*) \\ \implies (\sigma', \mathcal{F}_o^{\mathsf{eff}}(\sigma)(\sigma')) \in G_0 \cup G(\mathcal{F}_o^{\mathsf{tok}}(\sigma))$$

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$$\text{S3. } \forall o, \sigma, \sigma'.\ (\sigma \in I \wedge (\sigma, \sigma') \in (G_0 \cup G((\mathcal{F}_o^{\mathsf{tok}}(\sigma))^{\perp}))^*)$$

dec()

$$\implies (\sigma', \mathcal{F}_o^{\mathsf{eff}}(\sigma)(\sigma')) \in G_0 \cup G(\mathcal{F}_o^{\mathsf{tok}}(\sigma))$$

$\mathcal{I}$ = { σ | 0 ≤ σ }
G_0 = {(σ, σ') | σ ≤ σ'}
G_1(lock) = {(σ, σ') | 0 < σ ∧ σ' ≤ σ}

{}

$$\text{S3. } \forall o, \sigma, \sigma'.\ (\sigma \in I \wedge (\sigma, \sigma') \in (G_0 \cup G(((\mathcal{F}_o^{\mathsf{tok}}(\sigma))^{\perp}))^*)$$

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G_0^*

$\{\}$

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$\{\text{lock}\}$

$G_0 \cup G_1(\text{lock})$

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dec()

$$\implies (\sigma', \underline{\mathcal{F}_o^{\text{eff}}(\sigma)(\sigma')}) \in \square$$

if $0 < \sigma$ then σ'-1 else σ'

$G_0 \cup G_1(\text{lock})$

What if no on-demand consistency?

$$\text{S3. } \forall o, \sigma, \sigma'.\ (\sigma \in I \wedge (\sigma, \sigma') \in (G_0 \cup G((\mathcal{F}_o^{\mathsf{tok}}(\sigma))^{\perp}))^*) \implies (\sigma', \mathcal{F}_o^{\mathsf{eff}}(\sigma)(\sigma')) \in G_0 \cup G(\mathcal{F}_o^{\mathsf{tok}}(\sigma))$$

What if no on-demand consistency?

G*

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G

What if no causality?

G*

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G

What if no causality?

$\sigma' \in \mathcal{I}$

$$\text{S3. } \forall o, \sigma, \sigma'.\ (\sigma \in I \wedge \sout{(\sigma, \sigma') \in (G_0 \cup G((\mathcal{F}_o^{\mathsf{tok}}(\sigma))^{\perp}))^{*}})$$

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$\mathcal{F}_o^{\mathsf{eff}}(\sigma)(\sigma') \in \mathcal{I}$