Replicated Data Types: Specification, Verification, Optimality

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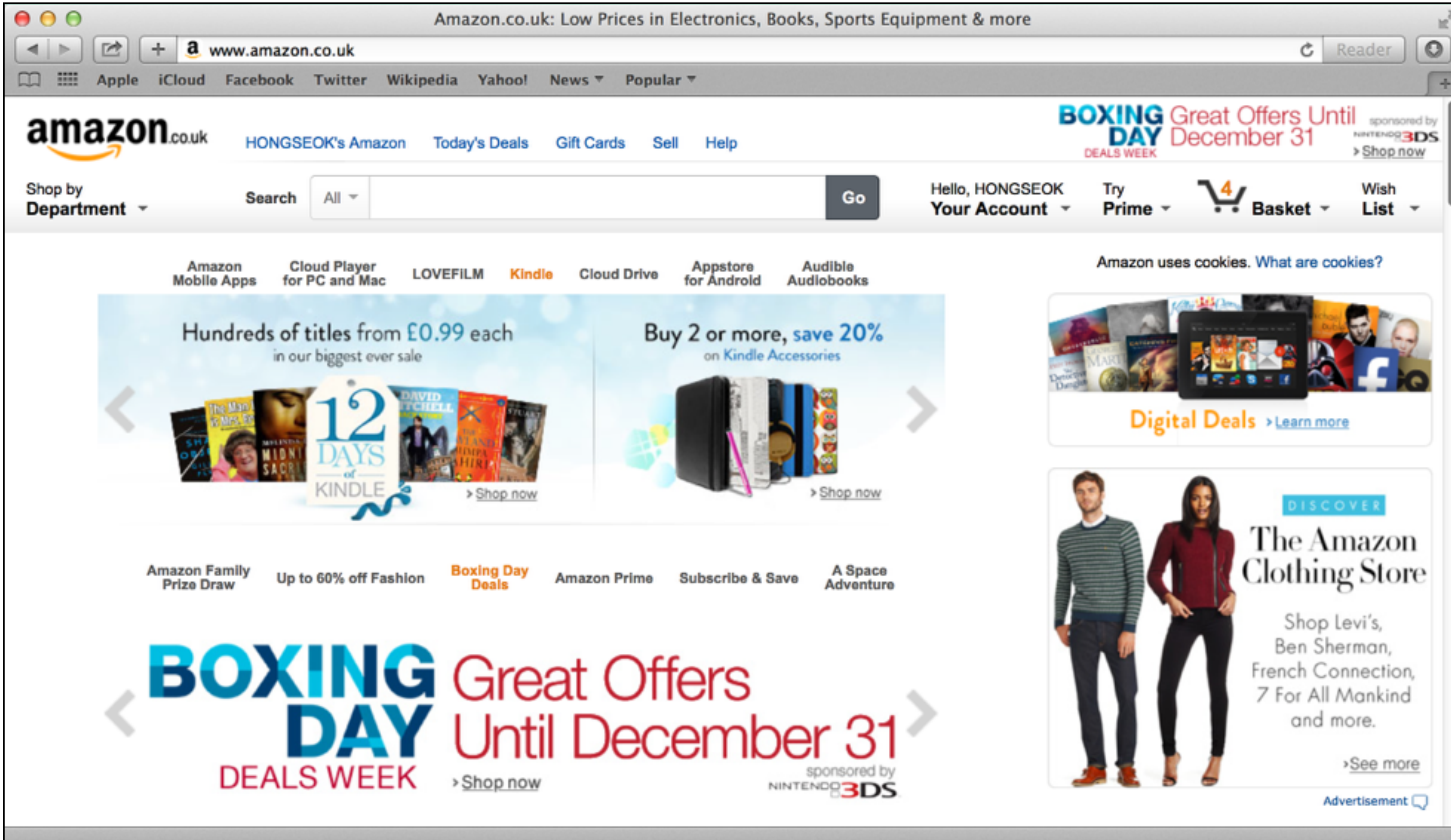
University of Oxford

Joint work with Sebastian Burckhardt (Microsoft Research),
Alexey Gotsman (IMDEA) and Marek Zawirski (UPMC & INRIA)

What is it like writing programs running on top of weakly consistent distributed stores?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Information about each customer

Shopping cart (Shapiro et al.)

Bob
in UK

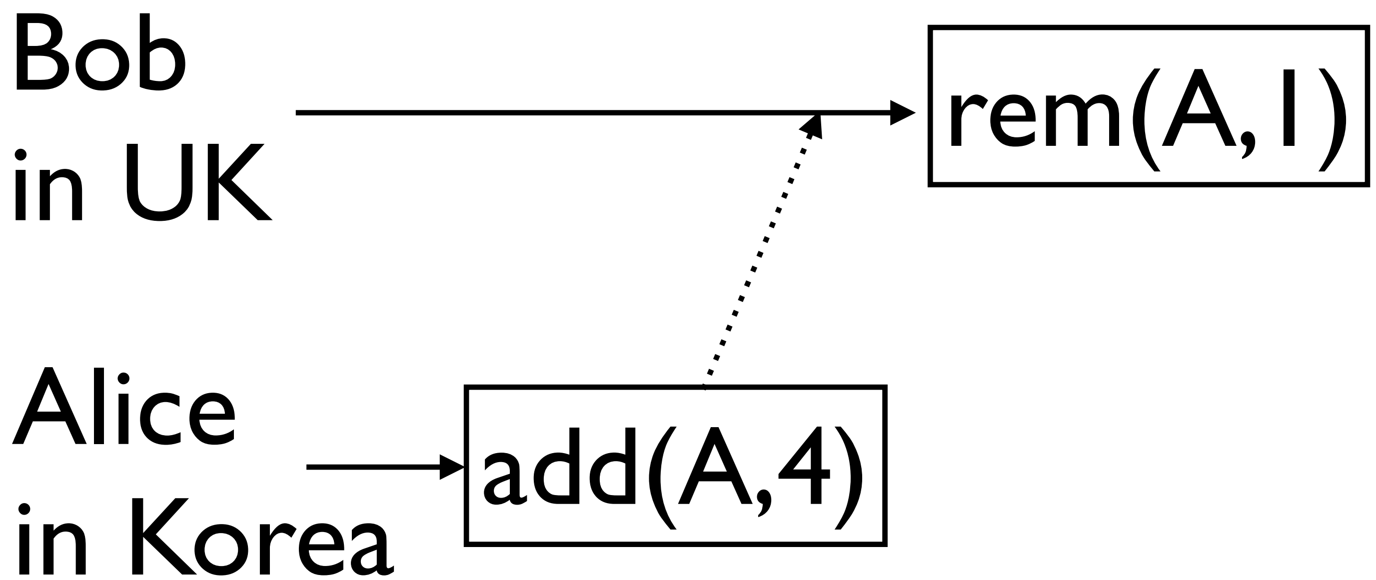
Alice
in Korea

Shopping cart (Shapiro et al.)

Bob
in UK

Alice
in Korea → $\boxed{\text{add(A,4)}}$

Shopping cart (Shapiro et al.)



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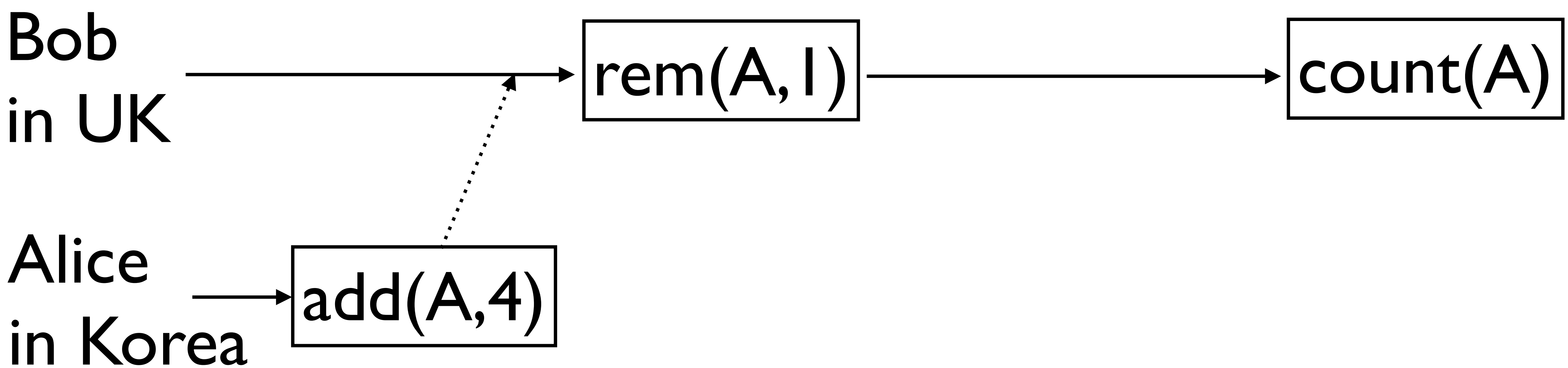

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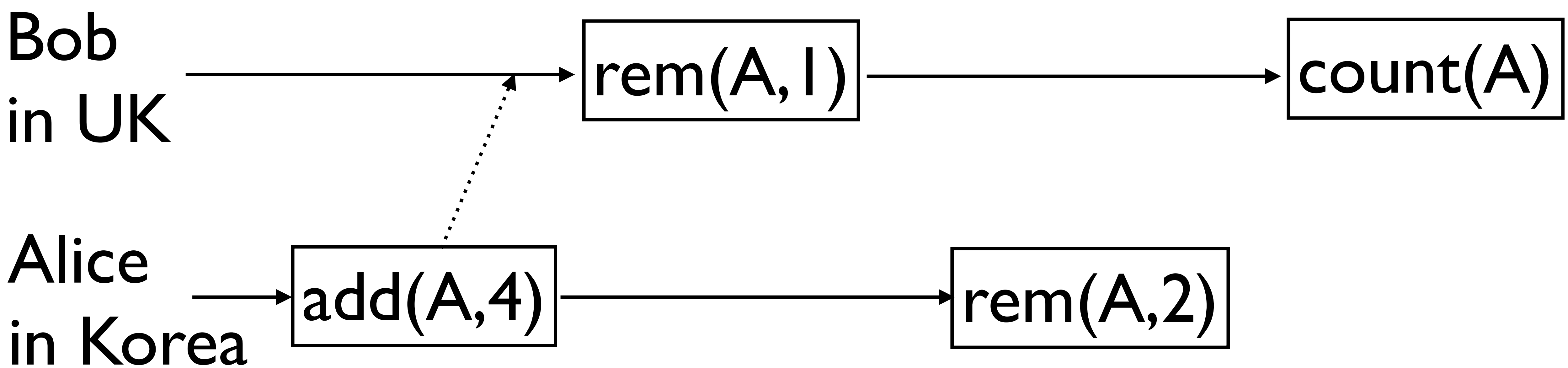

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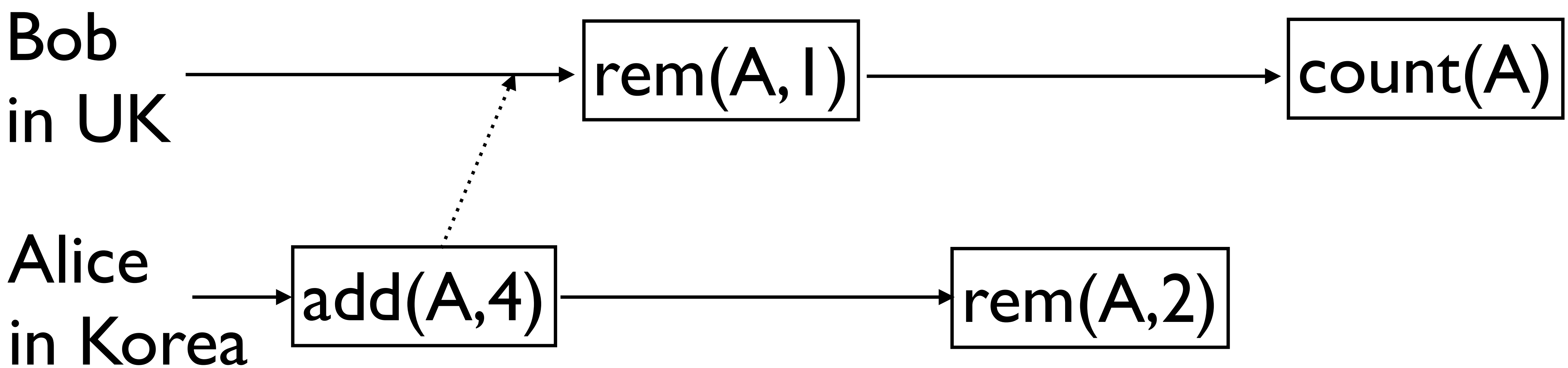
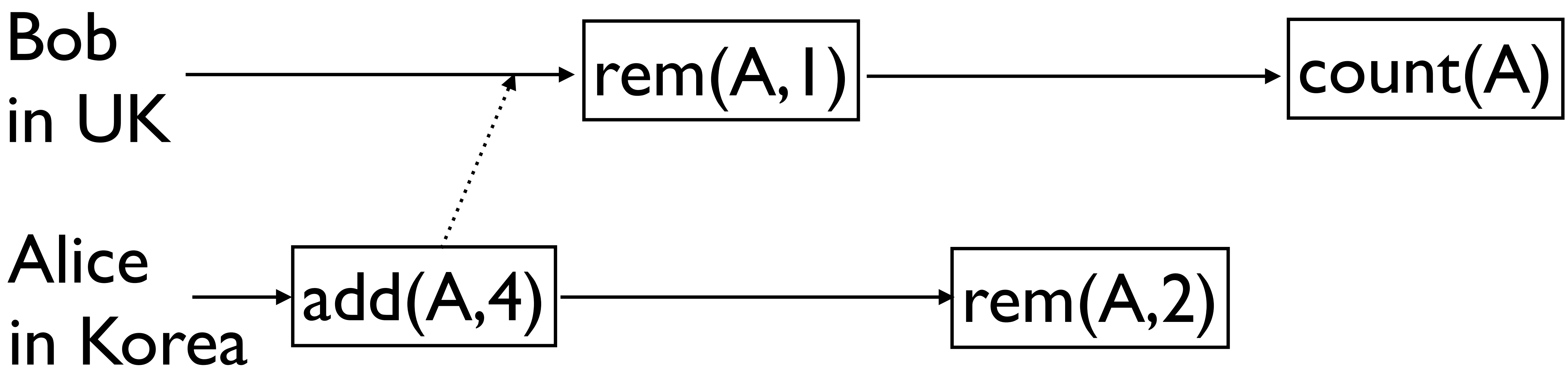


[Q] What cannot be the result of count(A)?
(a) 1 (b) 3 (c) 5 (d) all possible

Shopping cart (Shapiro et al.)



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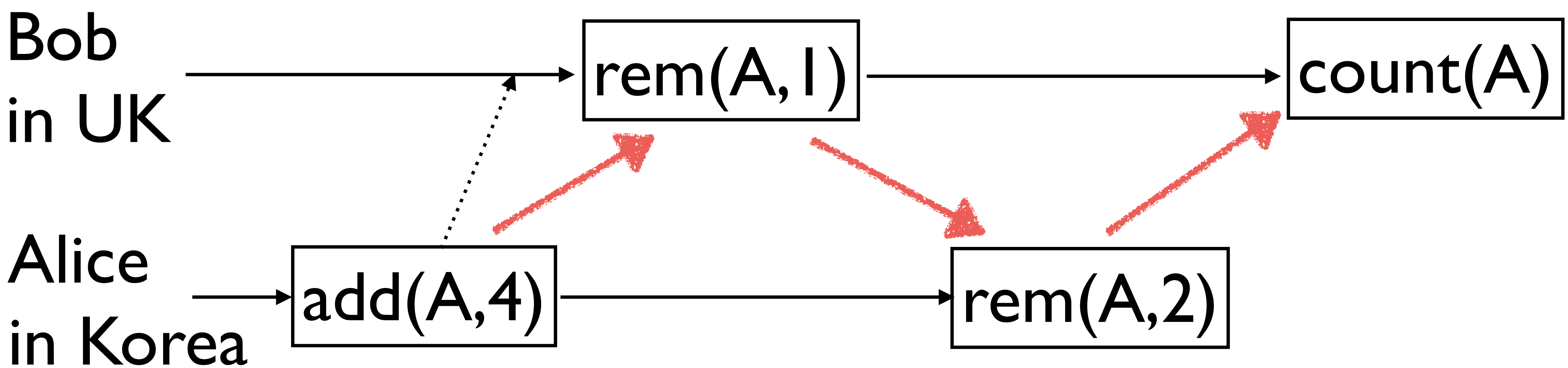

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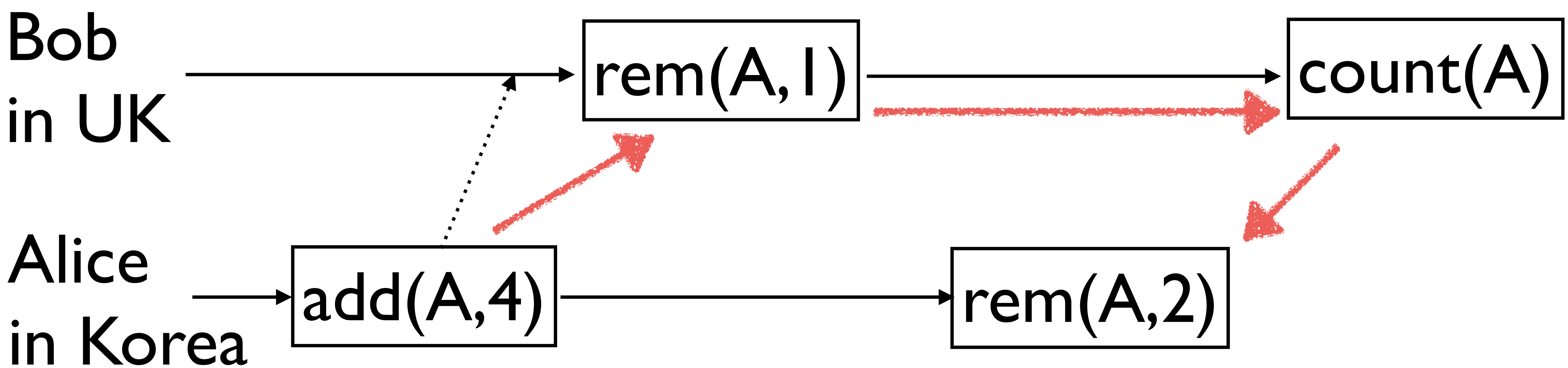

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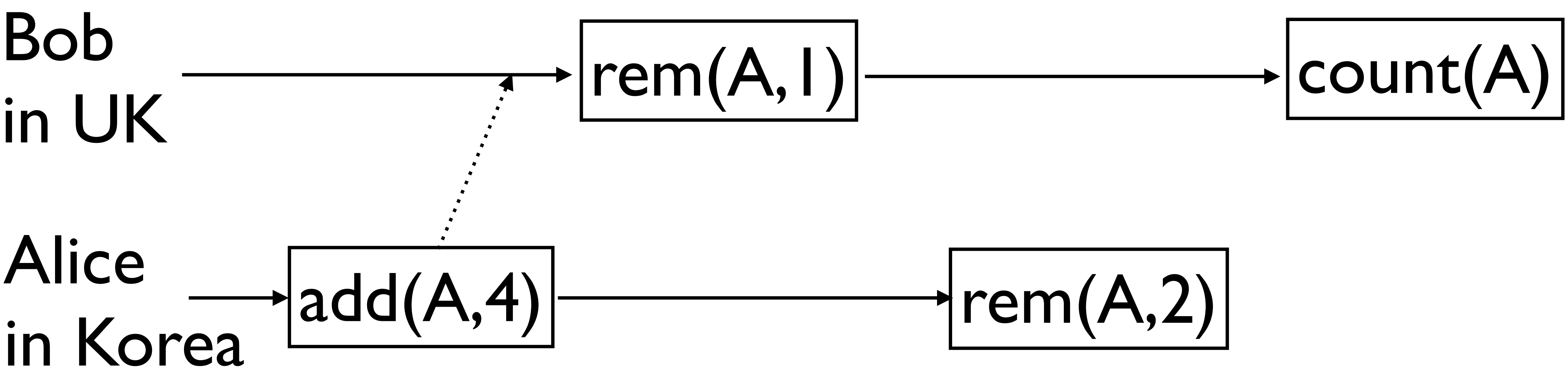


[Q] What cannot be the result of count(A)?

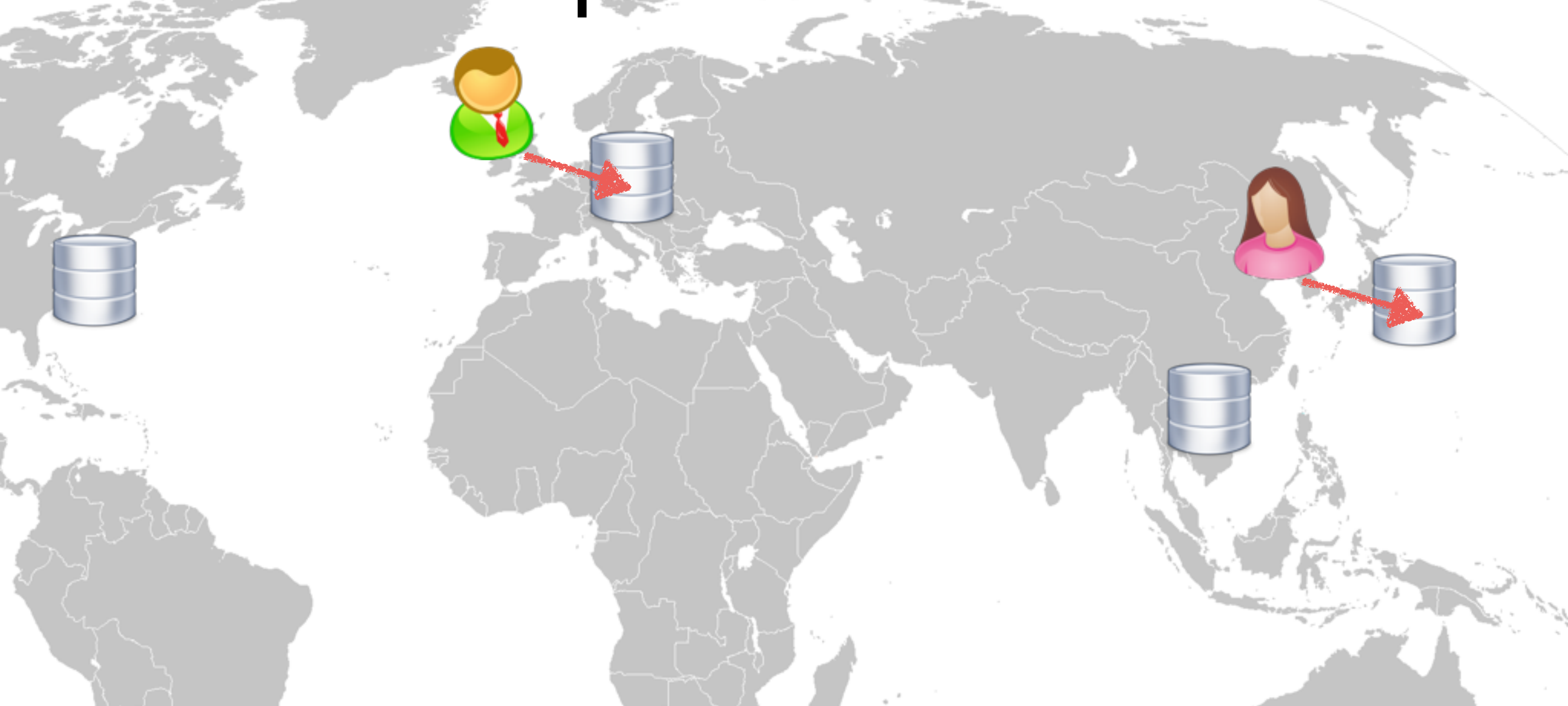
(a) 1 (b) 3 (c) 5 (d) all possible

Because of weak consistency of the store.

Geo-replicated databases

- Every data centre stores a complete replica of data
- Purpose: Minimising latency. Fault tolerance.

Geo-replicated databases



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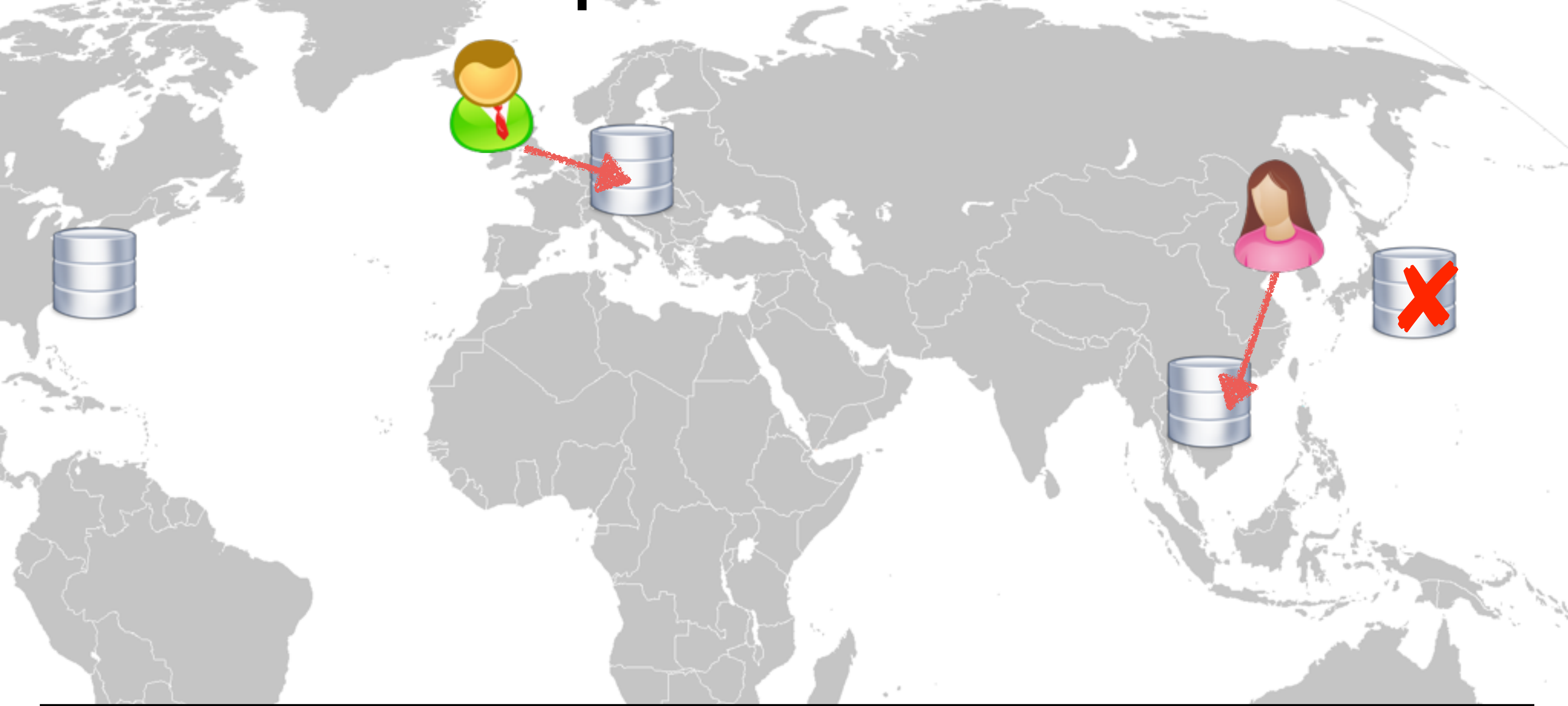

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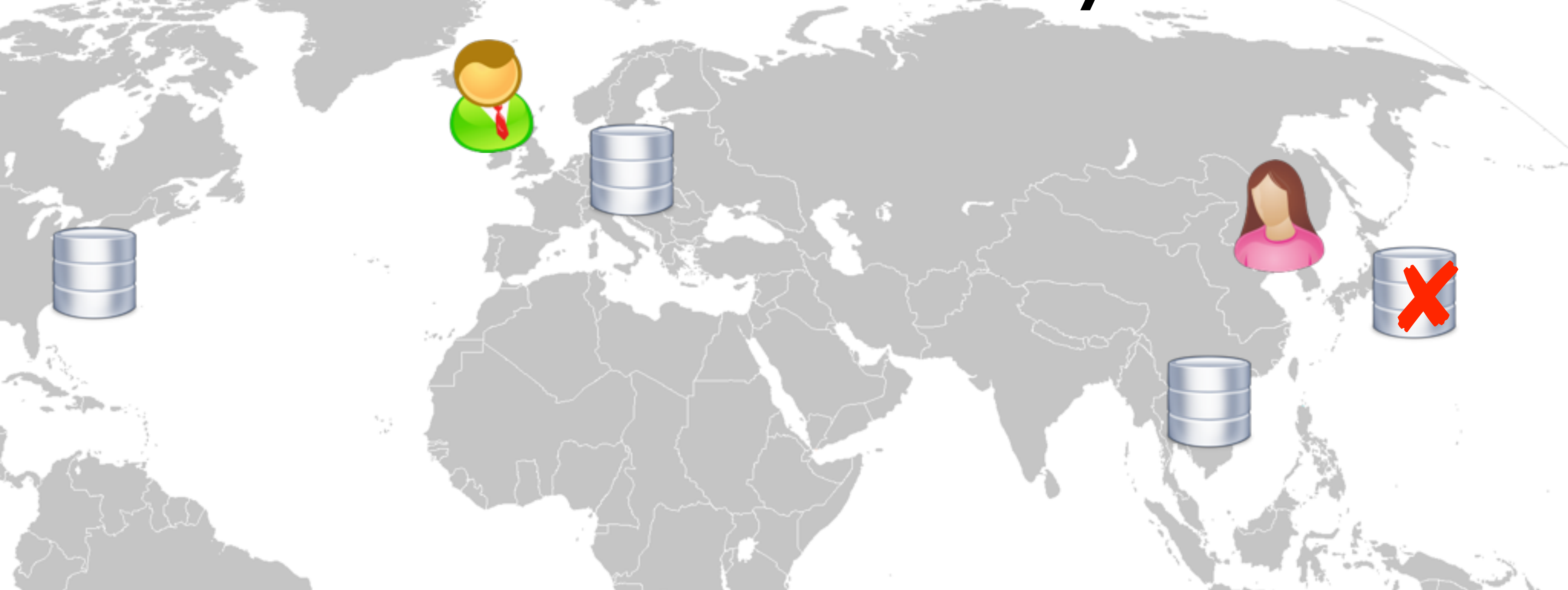
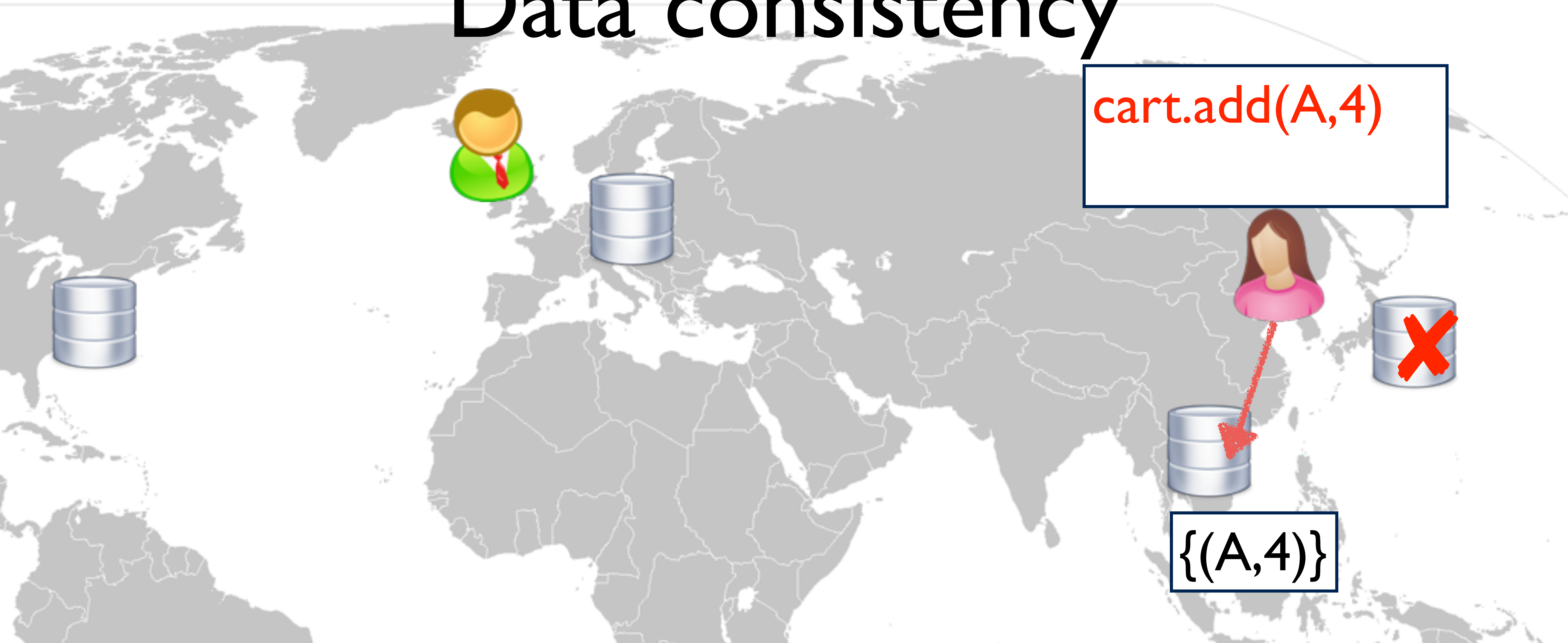
Data consistency



- Do data centre stores behave as if a single store?
- Strong consistency: Yes.
- Weak consistency: No.

Data consistency



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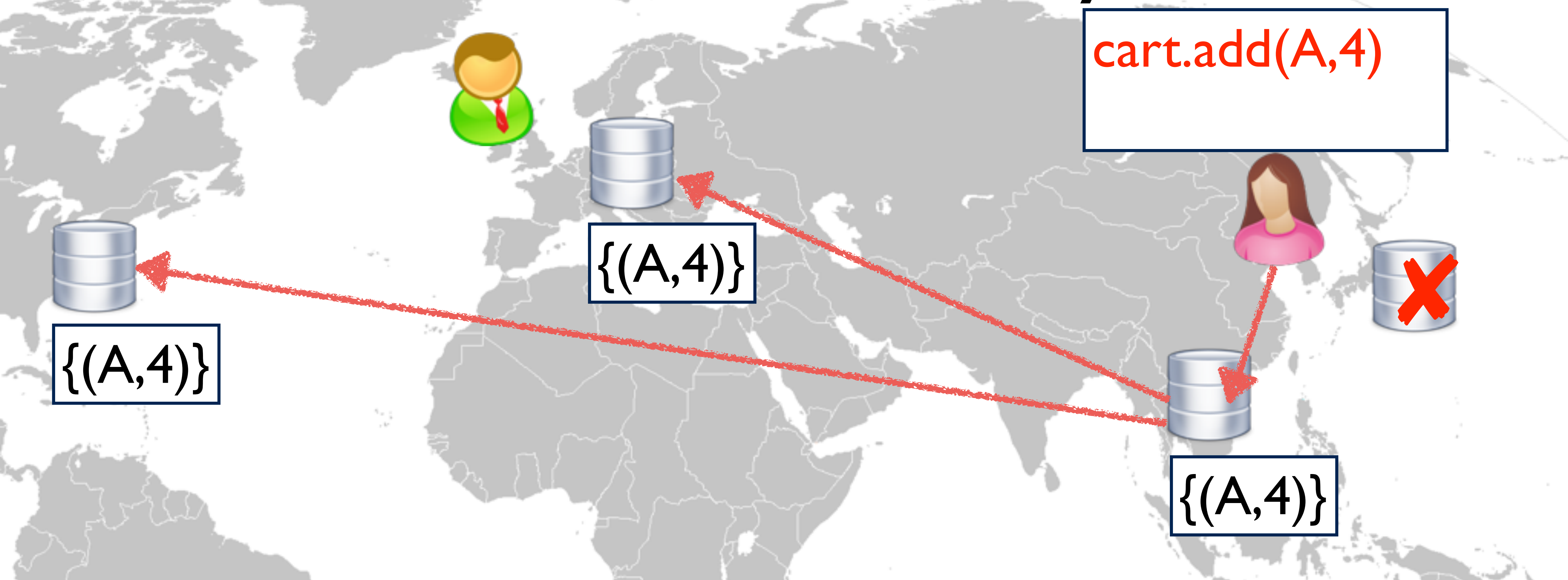

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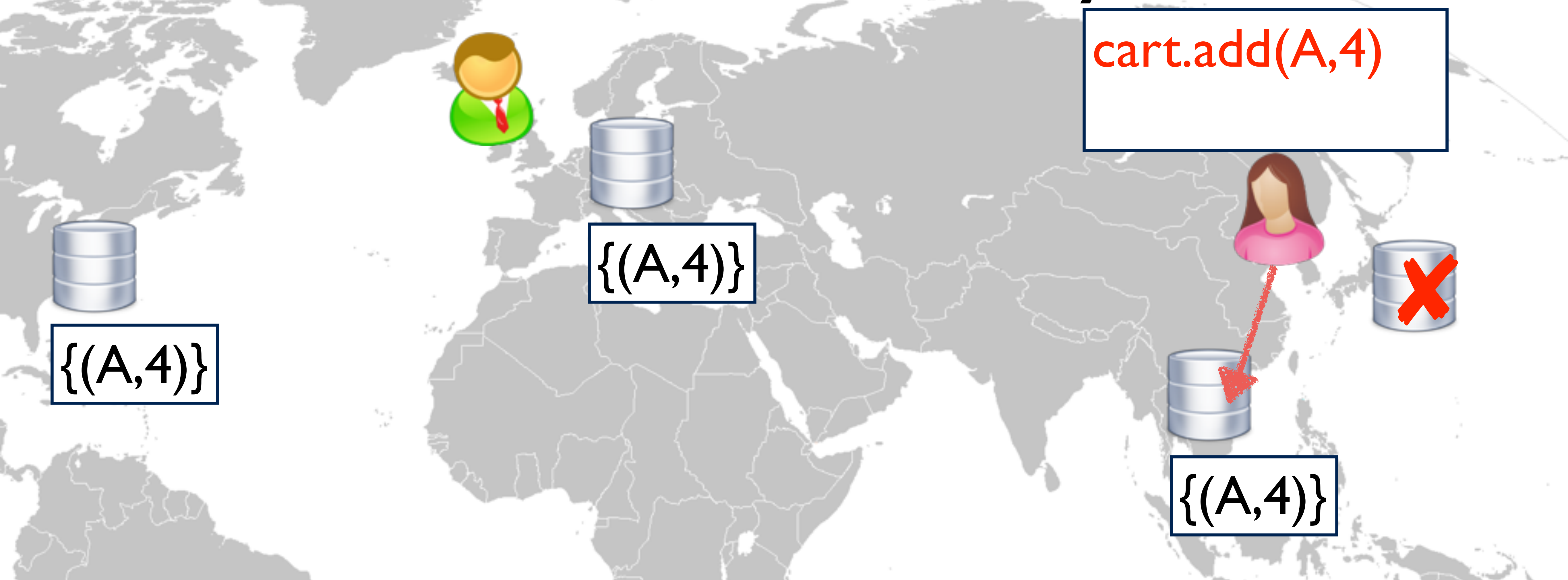

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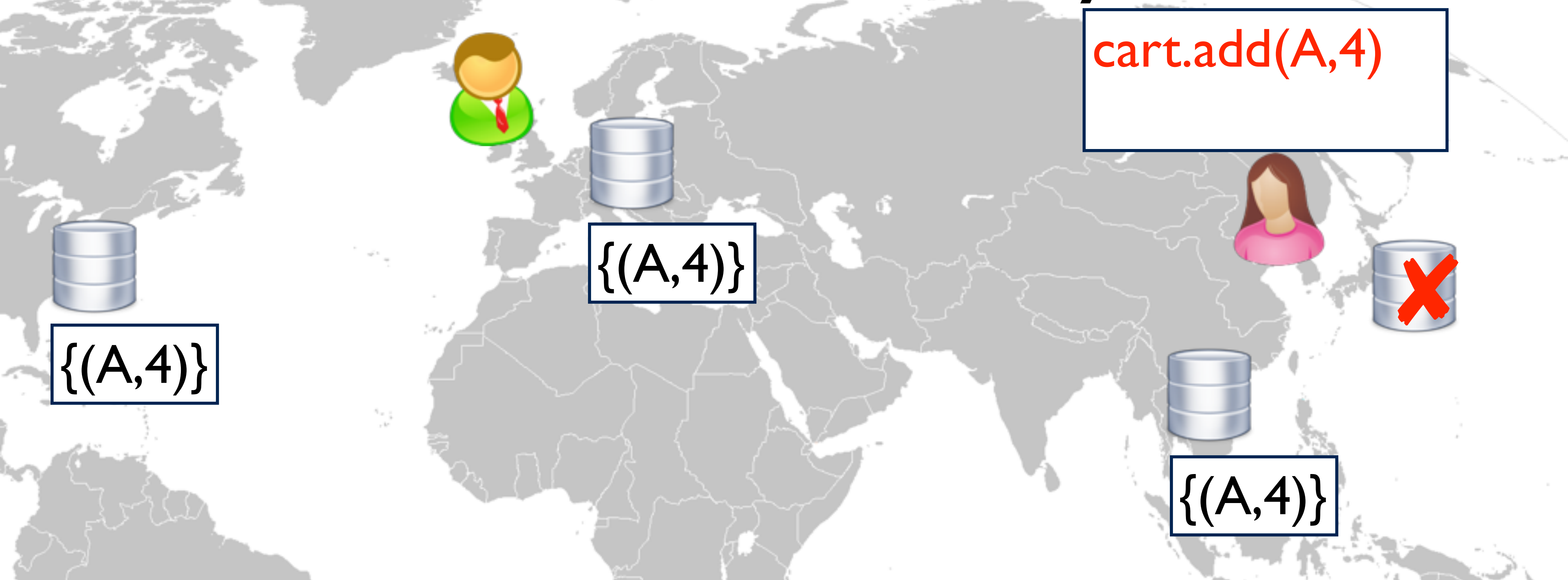

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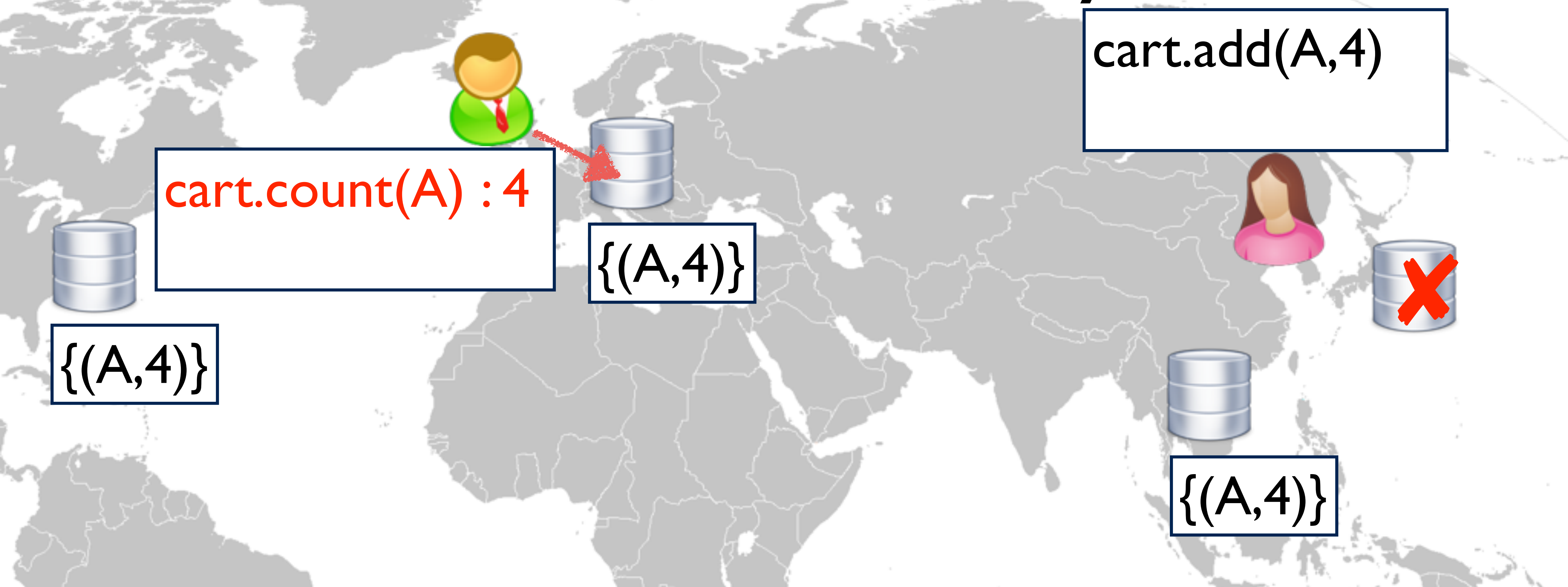

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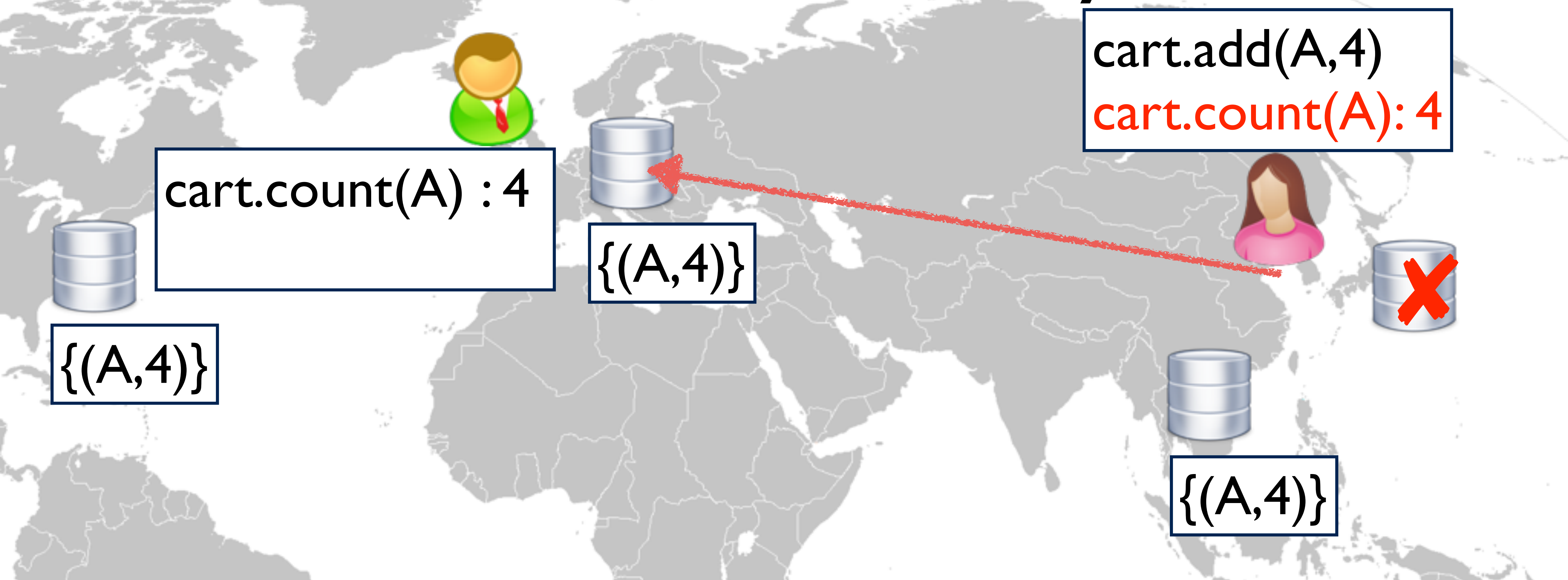

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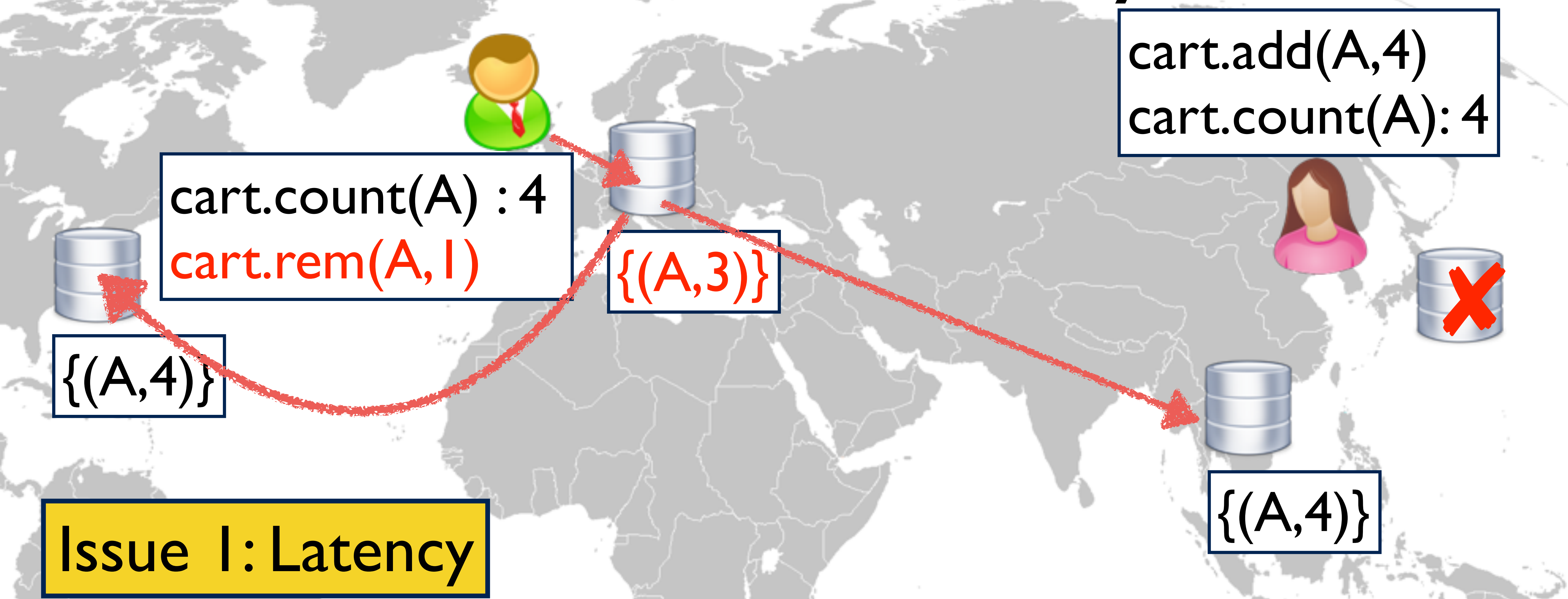

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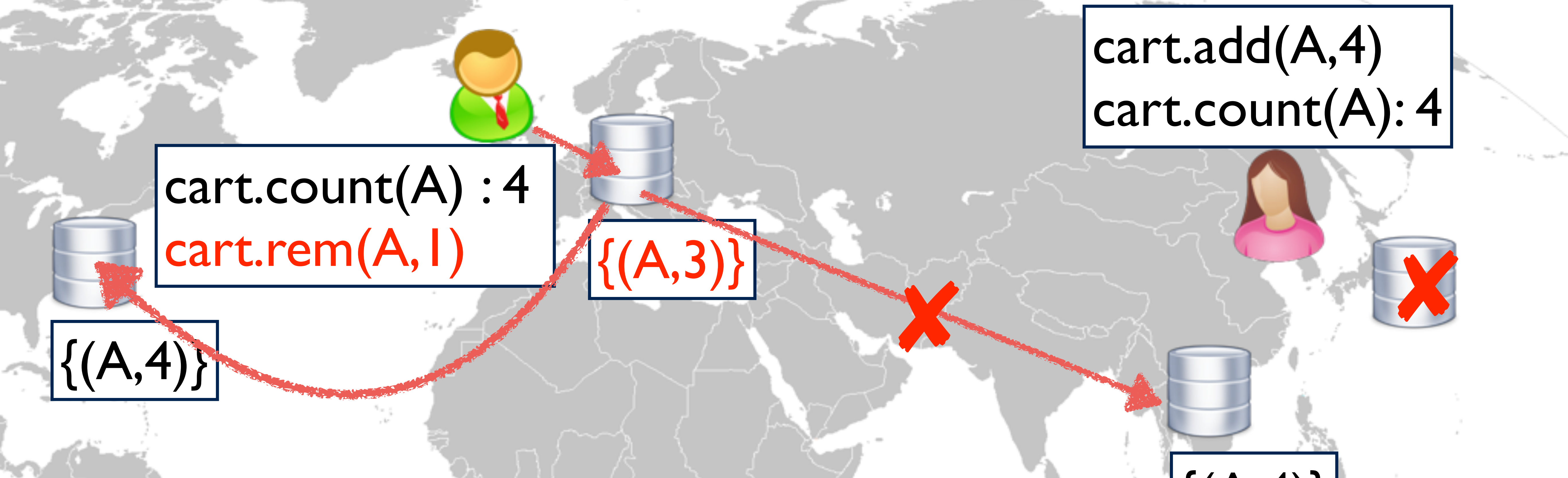

Issue 2: Cannot tolerate network partition

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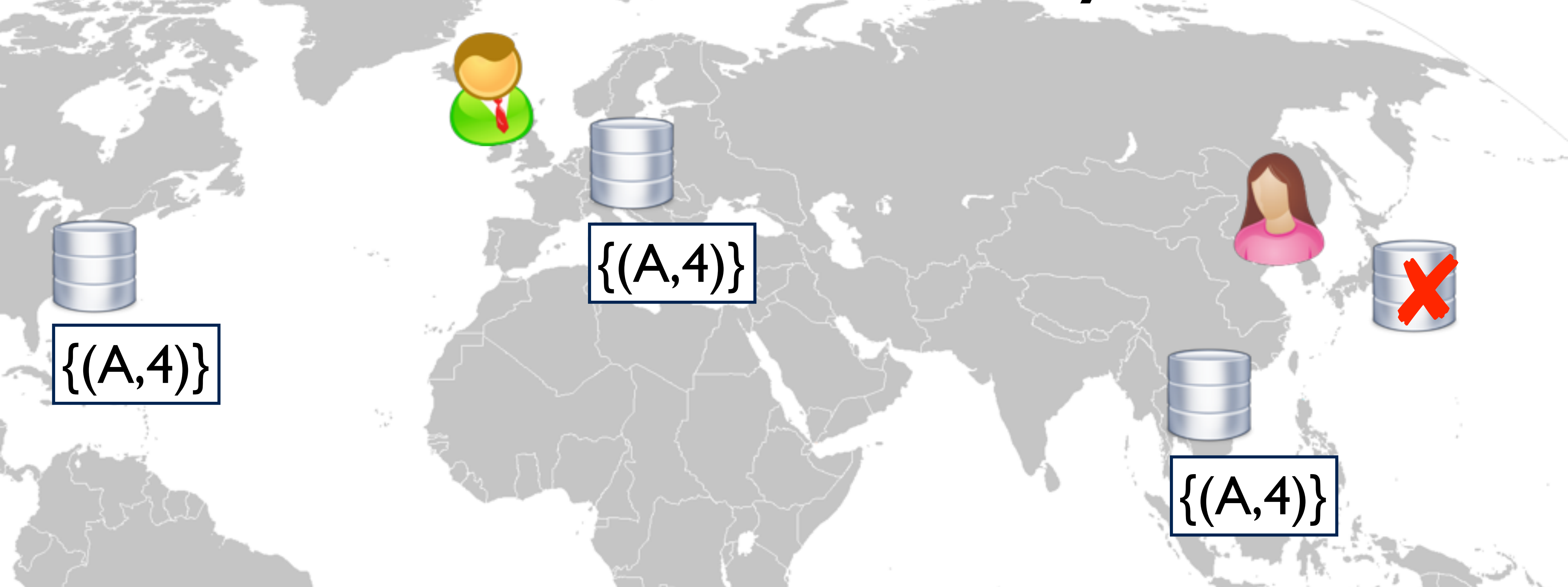
Data consistency



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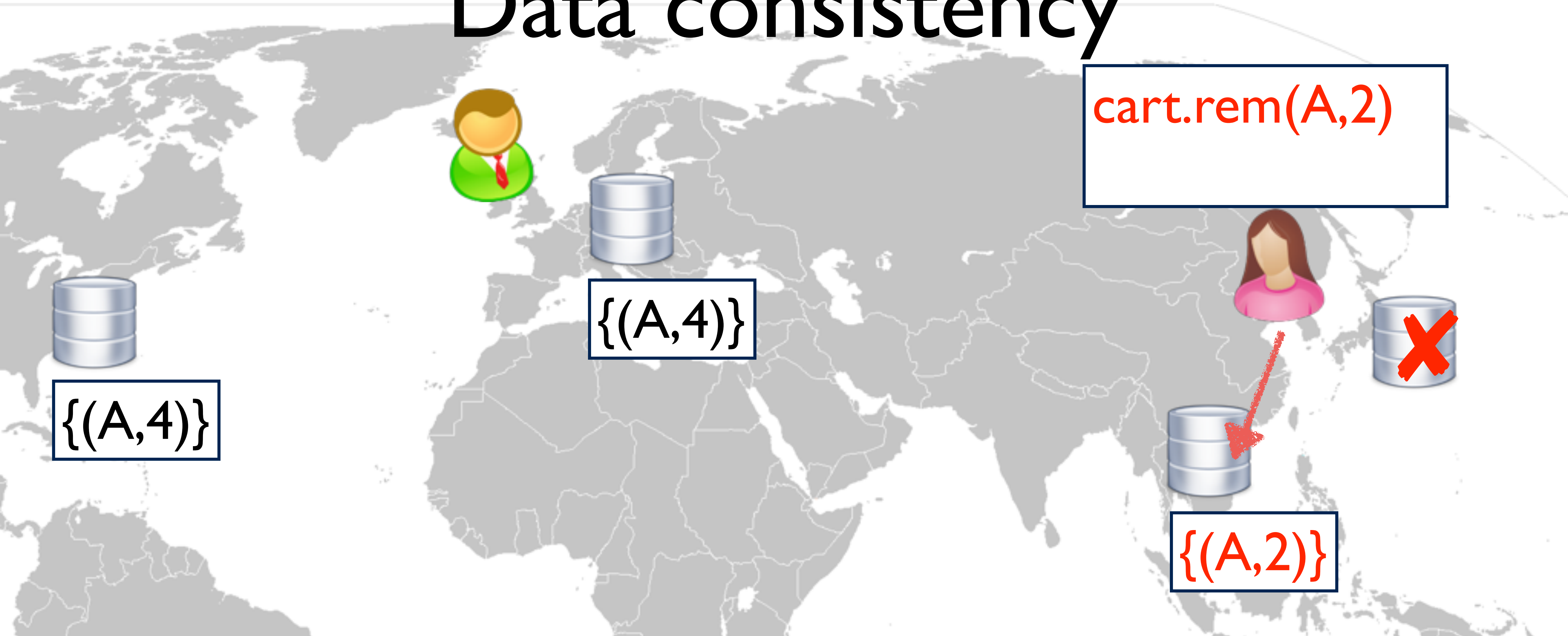

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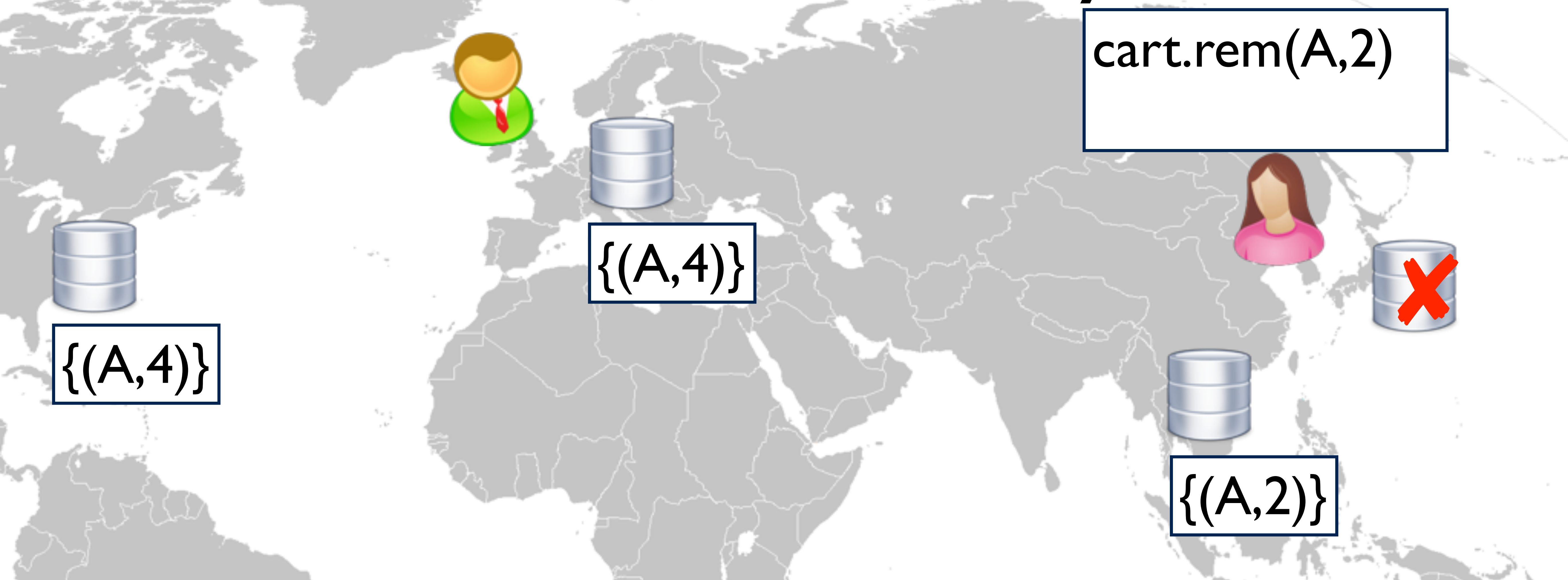

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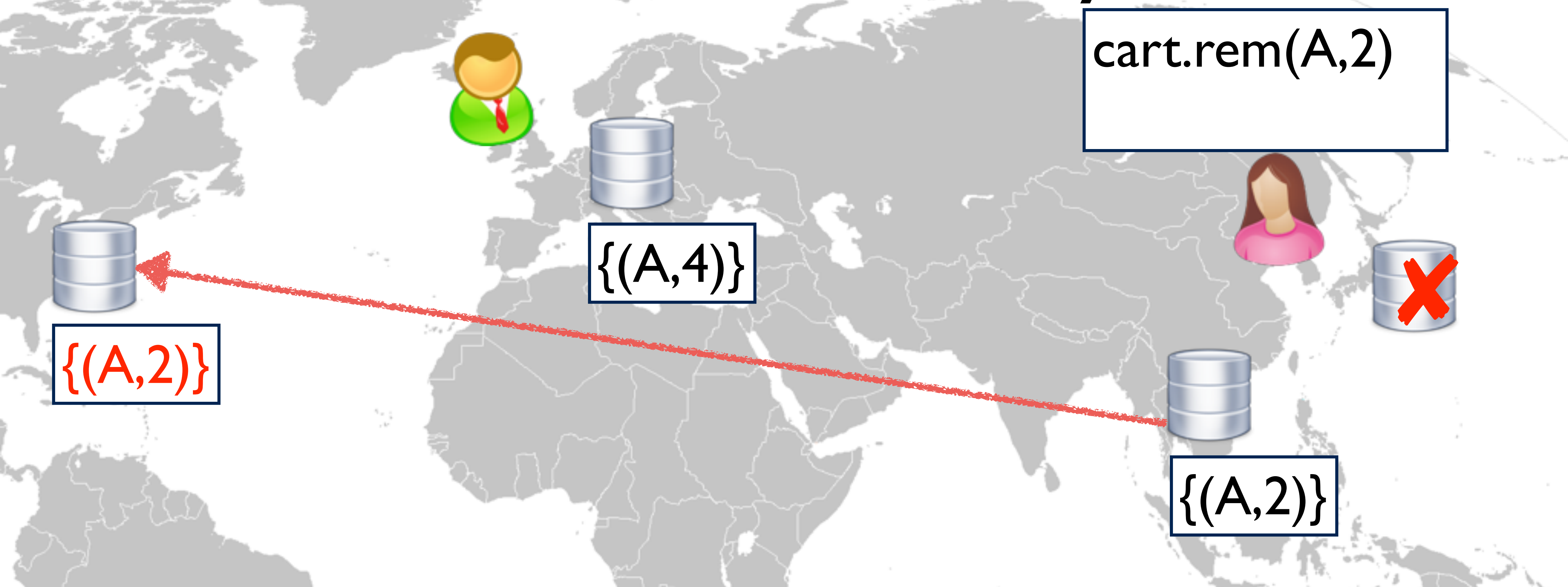

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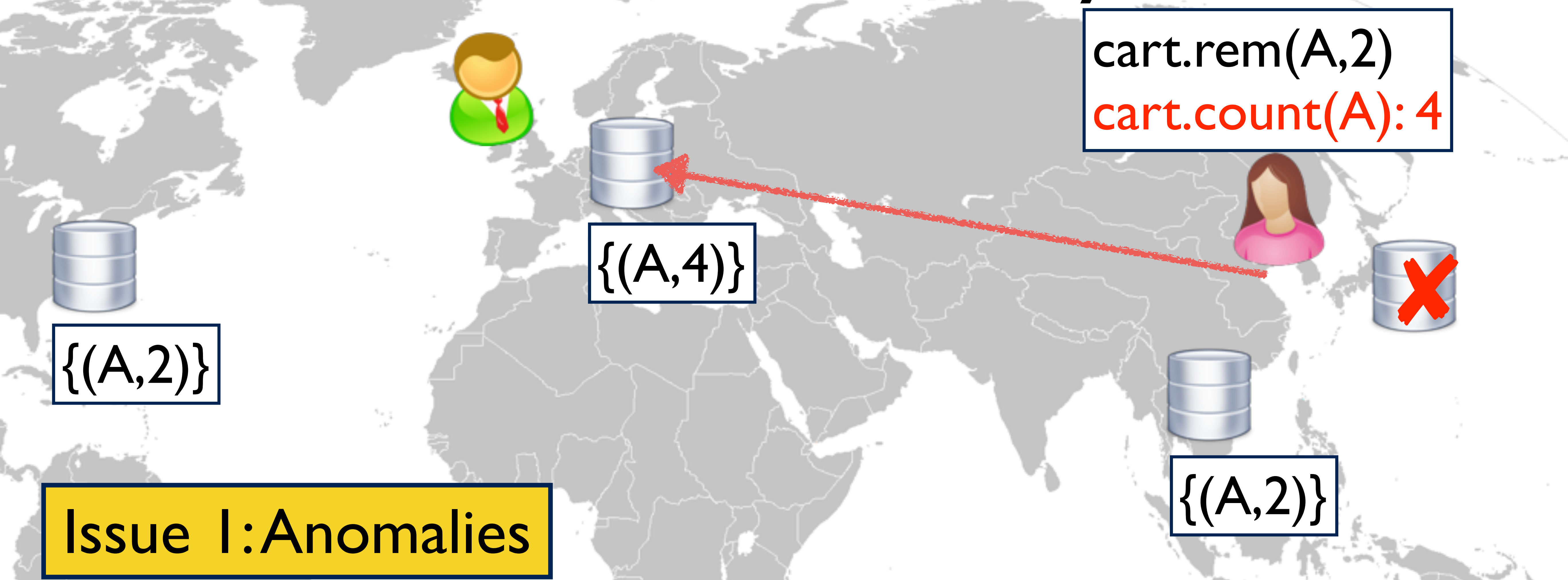

Issue 1: Anomalies

- Do data centre stores behave as if a single store?
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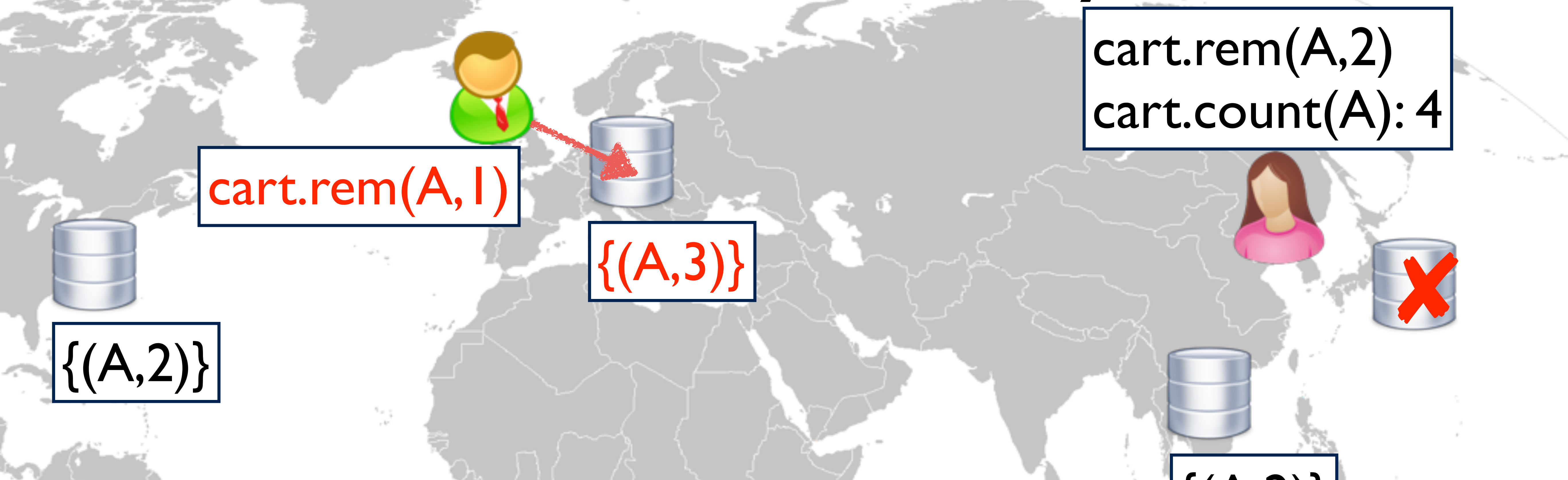
Data consistency



Issue 2: Conflicting updates

- Do data centre stores behave as if a single store?
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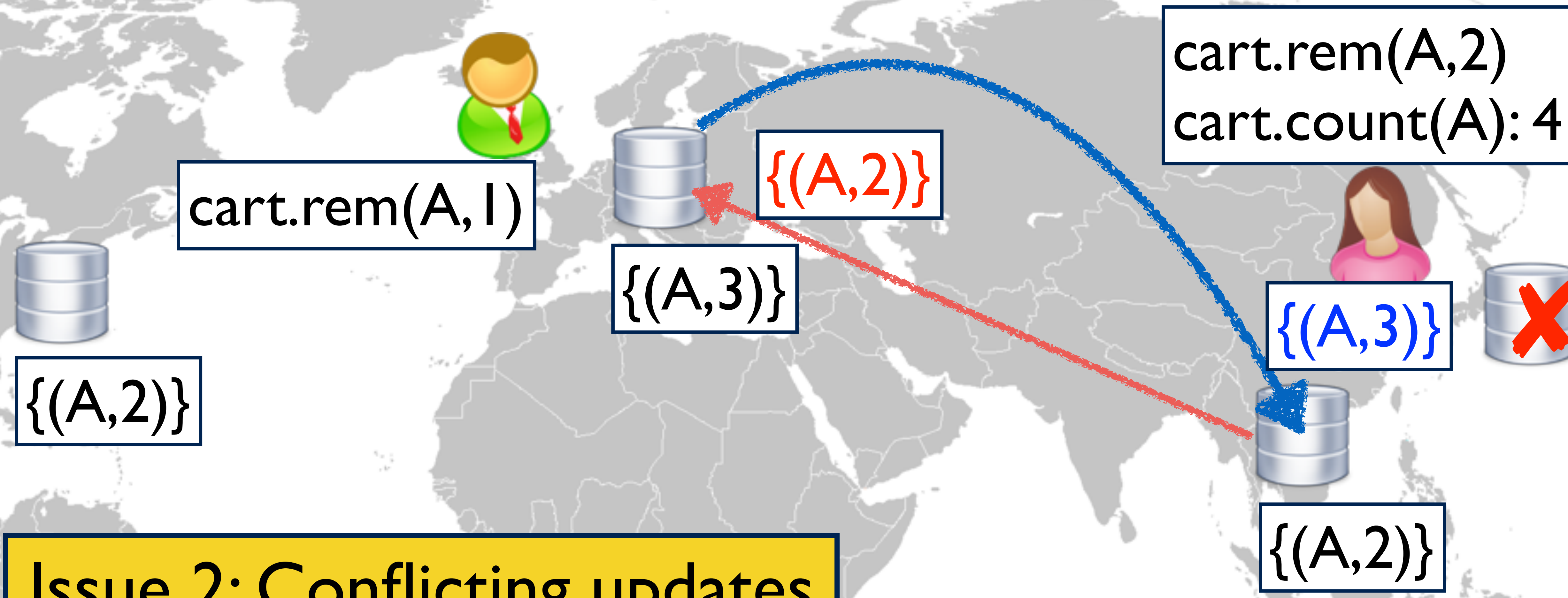

Issue 2: Conflicting updates

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Popularity of weak consistency

- Dynamo/Simple DB (Amazon), Riak, Cassandra (Facebook/Twitter), CouchDB, Google Doc,etc
- Due to better responsiveness and availability.
- But created a programming challenge.

Shopping cart

- Map from book ids to natural numbers.
- Initially, constant-0 function.
- add : $\{0,\ldots,N\text{-}1\} \times \mathrm{Nat} \longrightarrow \mathrm{Unit}$
- rem : $\{0,\ldots,N\text{-}1\} \times \mathrm{Nat} \longrightarrow \mathrm{Unit}$
- count : $\{0,\ldots,N\text{-}1\} \longrightarrow \mathrm{Nat}$

Sequential data type

```
class Cart {
  val map = new Array[Nat](N)

  def add(b,n) = { val m = map(b); map(b) = m+n }
  def rem(b,n) = { val m = map(b); map(b) = max(m-n,0) }
  def count(b) = { val m = map(b); return m }
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Replicated

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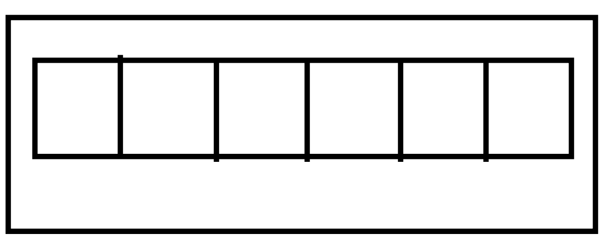
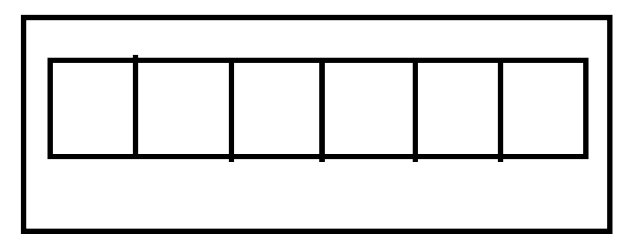
cart



cart



Things to be added:

1. Asynchronous message sending & receiving.
2. Conflict detection and resolution.

Primitive replicated data types (AKA CRDT) [Shapiro+ 2011]

- Designed for weakly consistent replicated stores.
- Send & receive update messages asynchronously.
- Detect and resolve conflicts.
- Register, set, dag, graph, etc.
- Conceptually, implement a weak shared memory on top of a replicated store.

Multi-valued register

$$\text{set} : T \longrightarrow \text{Unit} \qquad \text{get} : \text{Unit} \longrightarrow \text{Set}[T]$$

- Implements a memory cell of type T.

Multi-valued register

$\text{set} : T \longrightarrow \text{Unit} \qquad \text{get} : \text{Unit} \longrightarrow \text{Set}[T]$

- Implements a memory cell of type T.
- Resolves conflicts by taking union.

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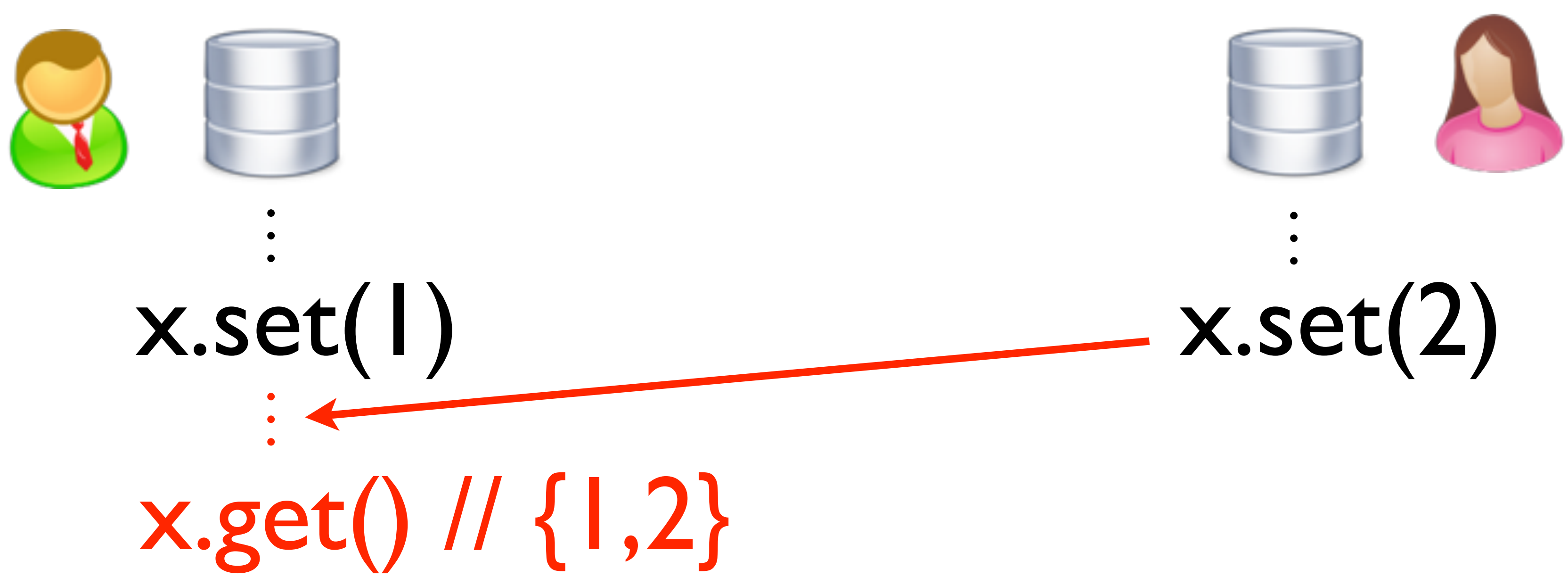

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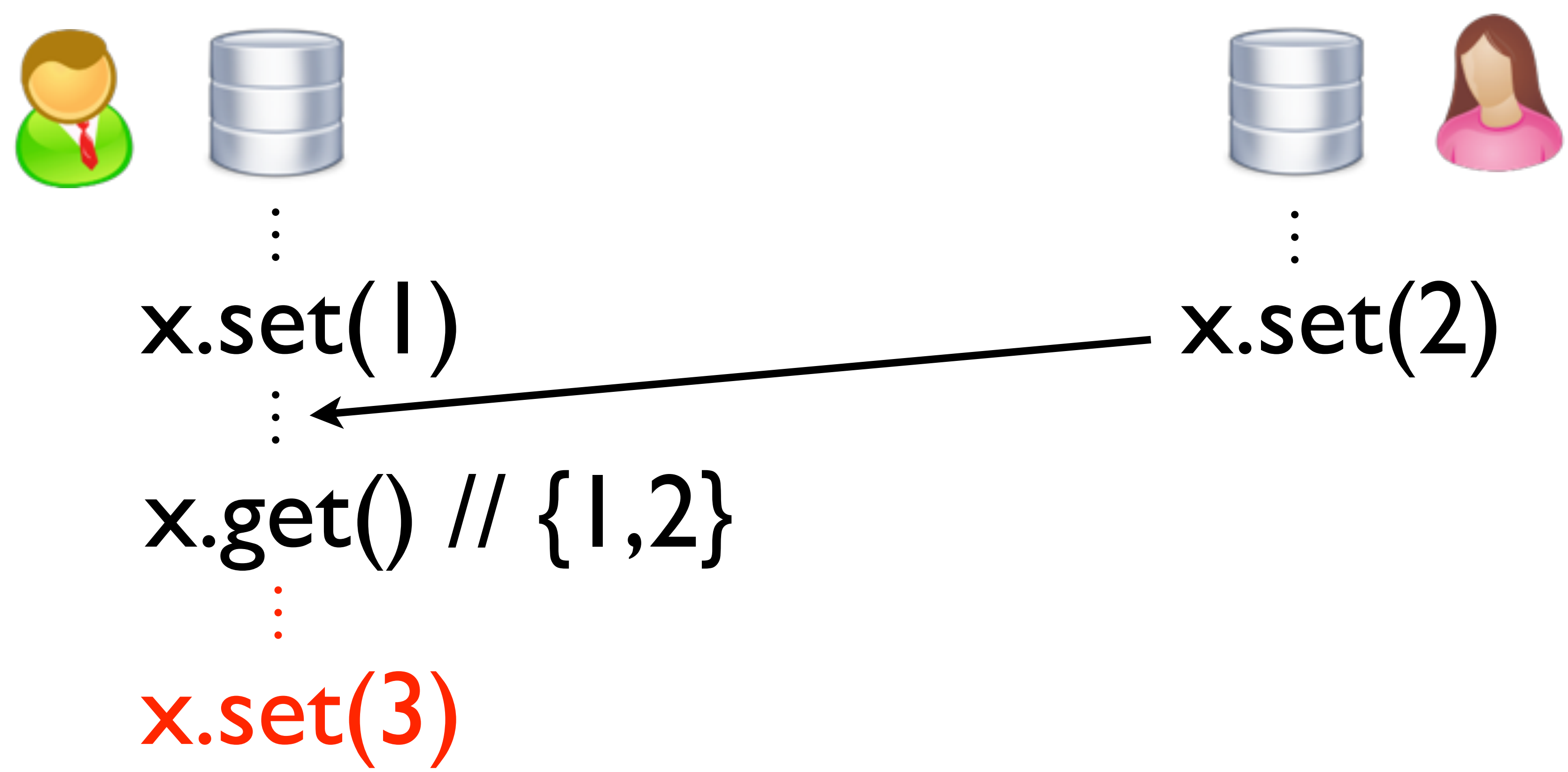

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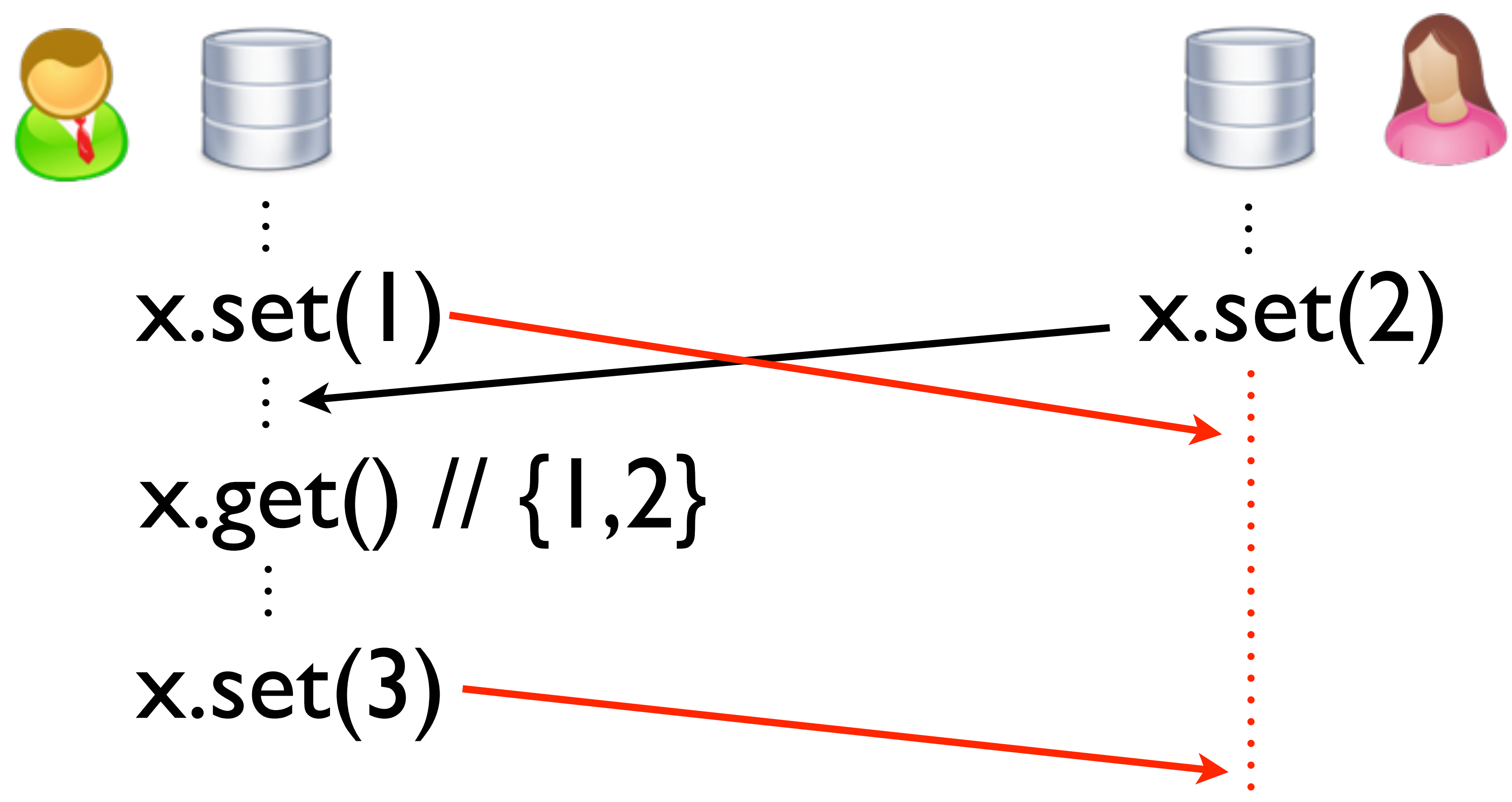

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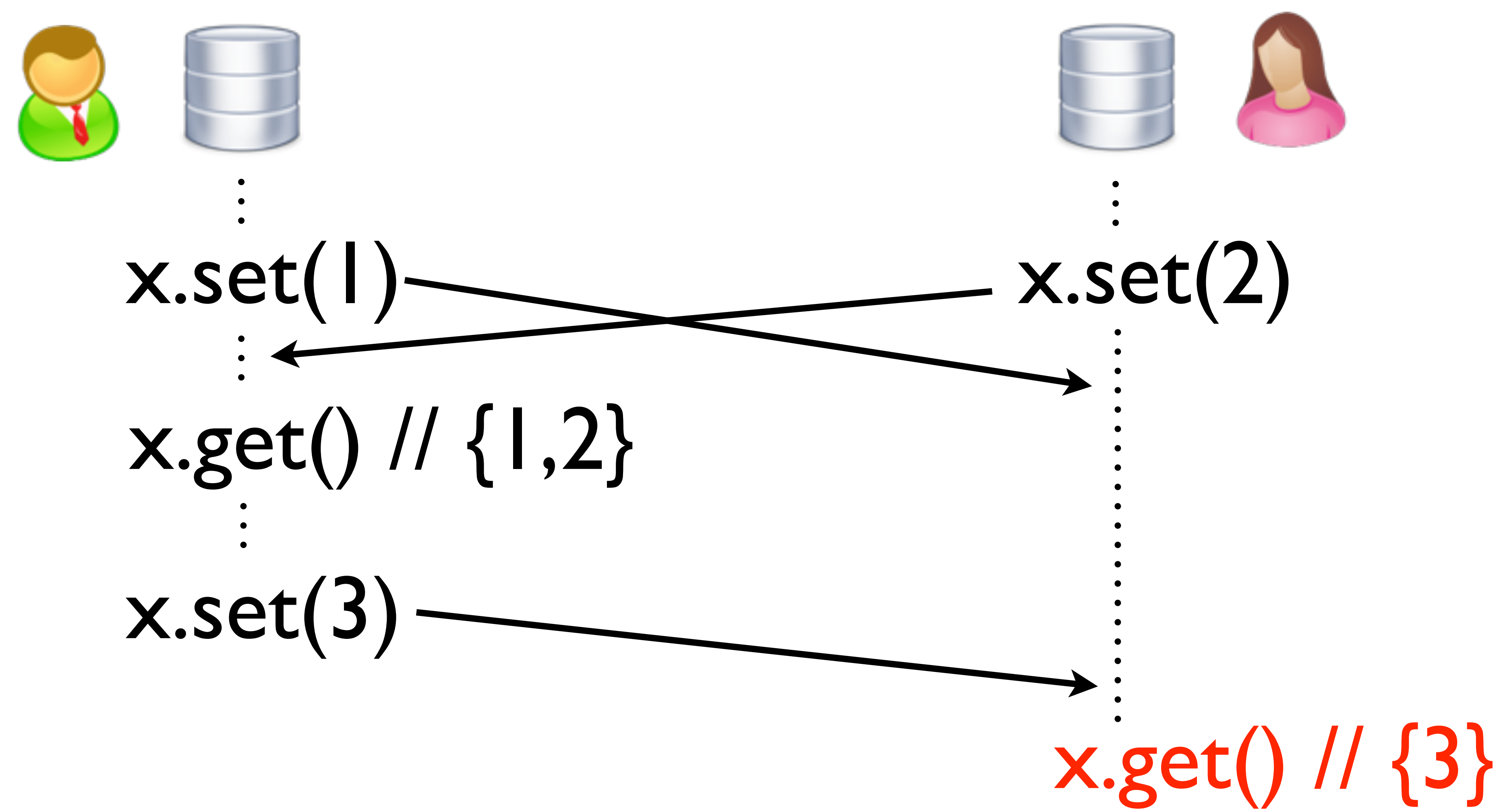

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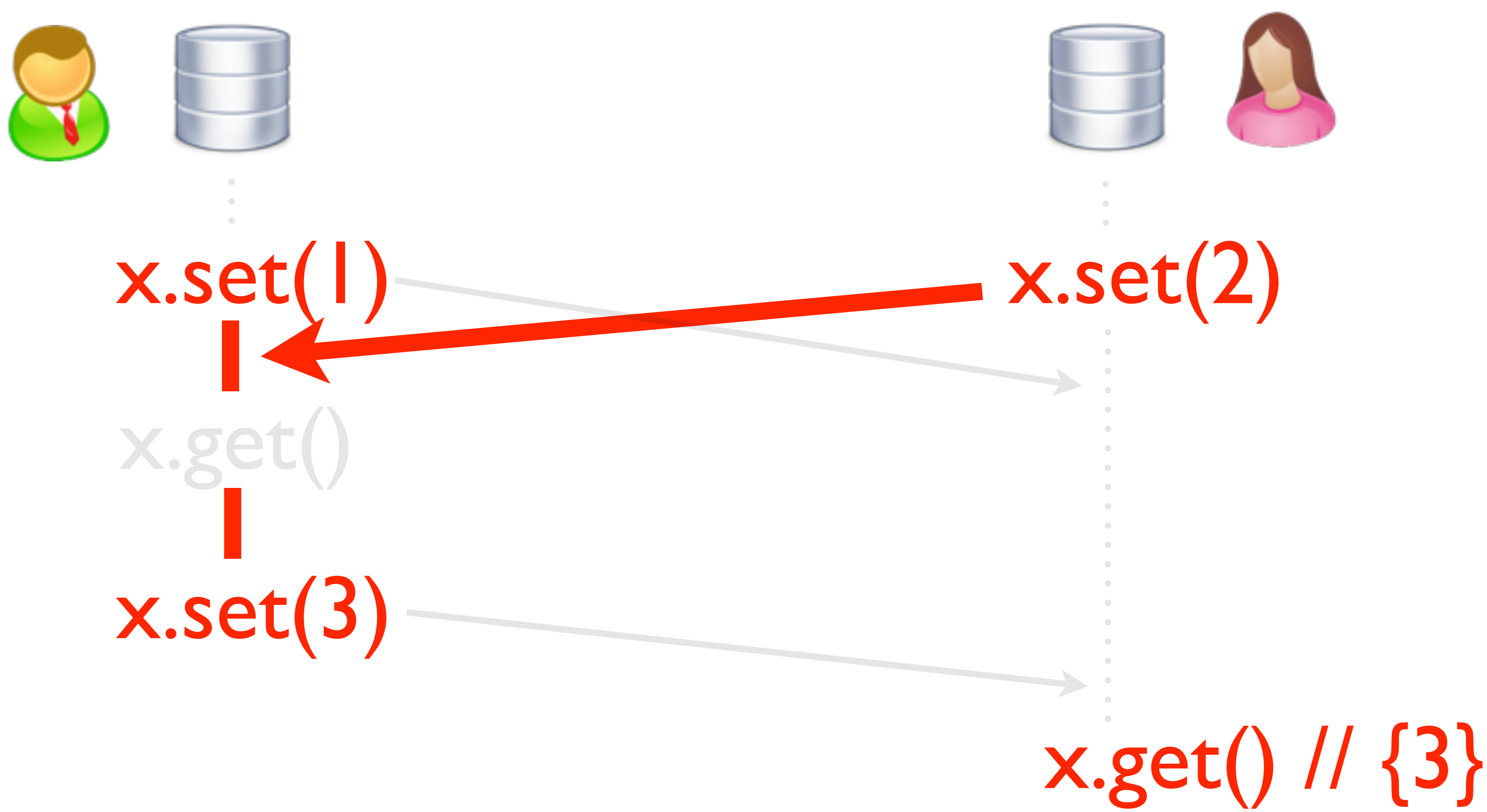

Shopping cart for replicated stores

```
class Cart {
  val map = new Array[Nat](N)


  def add(b,n) = {
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    map(b) = m+n
  }
  def rem(b,n) = {
    val m = map(b)
    map(b) = max(m-n,0)
  }
  def count(b) = {
    val m = map(b)
    return m
  }
}
```

Shopping cart for replicated stores

```
class Cart {
  val map = new Array[MVRegister[Nat]](N)
  for(i <- 0 until N) map(i) = new MVRegister[Nat]

  def add(b,n) = {
    val m = map(b).get()
    map(b).set(m+n)
  }
  def rem(b,n) = {
    val m = map(b).get()
    map(b).set(max(m-n,0))
  }
  def count(b) = {
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}
```

1. Use MVRegister.

Shopping cart for replicated stores

```
class Cart {
  val map = new array[MVRegister[Nat]](N)
  for(i <- 0 until N) map(i) = new MVRegister[Nat]

  def add(b,n) = {
    val m = sum(map(b).get())
    map(b).set(m+n)
  }
  def rem(b,n) = {
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  }
  def count(b) = {
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1. Use MVRegister.
2. Add conflict resolution.

Shopping cart for replicated stores

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class Cart {
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1. Use MVRegister.
2. Add conflict resolution.

Semantics of replicated data types

Dag[Op] = Set of Op-labelled finite dags

$$F : \text{Dag[Op]} \times \text{Op} \longrightarrow_{\text{partial}} \text{Value}$$

$$F_m\left(\begin{tikzcd} & \text{set}(2) \arrow[dl] \arrow[dd] \\ \text{set}(3) \arrow[dr] & \\ & \text{set}(4) \end{tikzcd} ,\quad \text{get}() \right) = \{4\}$$

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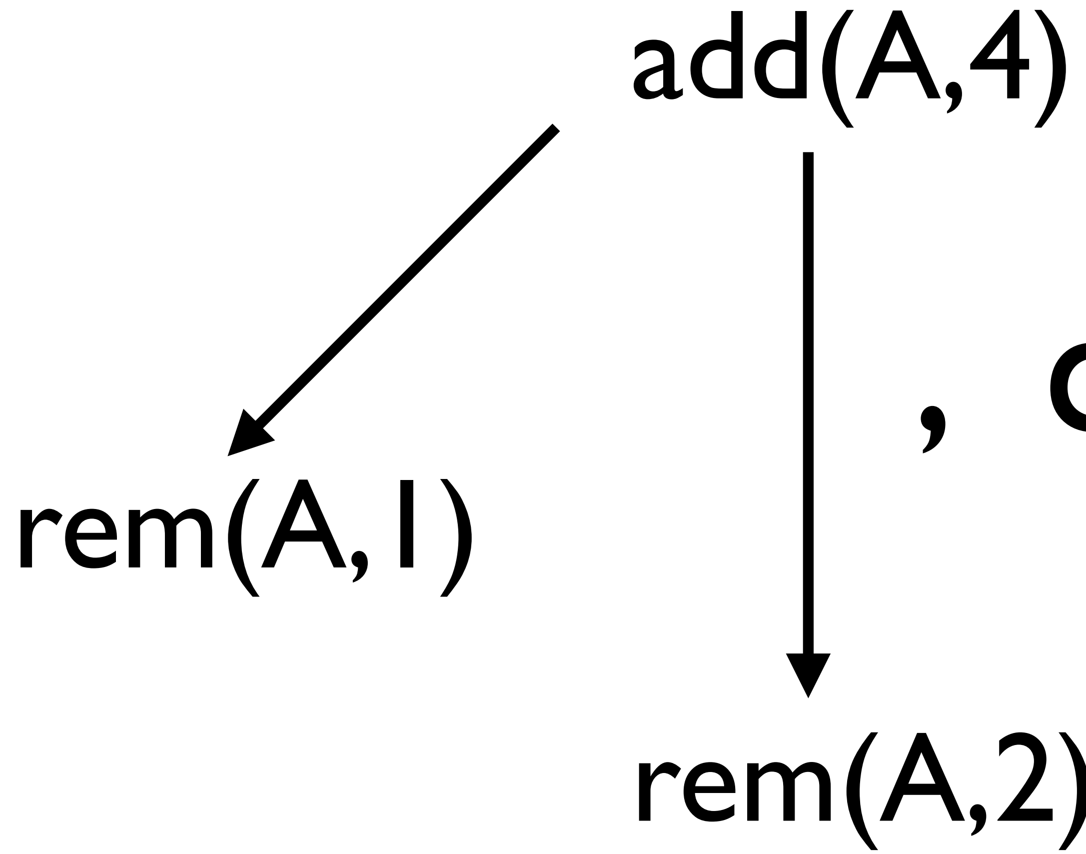
```
val map = new array[MVRegister[Nat]](N)
. . . . .
def add(b,n)={ val m = max(map(b).get()); map(b).set(m+n) }
def rem(b,n)={ val m = max(map(b).get()); map(b).set(max(m-n,0)) }
def count(b)={ val m = max(map(b).get()); return m }
```

$F_s($ 

$, \text{count}()) = ?$

```
val map = new array[MVRegister[Nat]](N)
. . . . .
def add(b,n)={ val m = max(map(b).get()); map(b).set(m+n) }
def rem(b,n)={ val m = max(map(b).get()); map(b).set(max(m-n,0)) }
def count(b)={ val m = max(map(b).get()); return m }
```

$$F_s\left(\begin{tikzcd} & \text{add(A,4)} \arrow[dl] \arrow[d] \\ \text{rem(A,1)} & \text{rem(A,2)} \end{tikzcd},\ \text{count()}\right) = ?$$

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$$\max\left(F_m\left(\begin{tikzcd} & \mathrm{add}(A,4) \arrow[dl] \arrow[d] \\ \mathrm{rem}(A,1) & \mathrm{rem}(A,2) \end{tikzcd},\ \mathrm{get}()\right)\right) = ?$$

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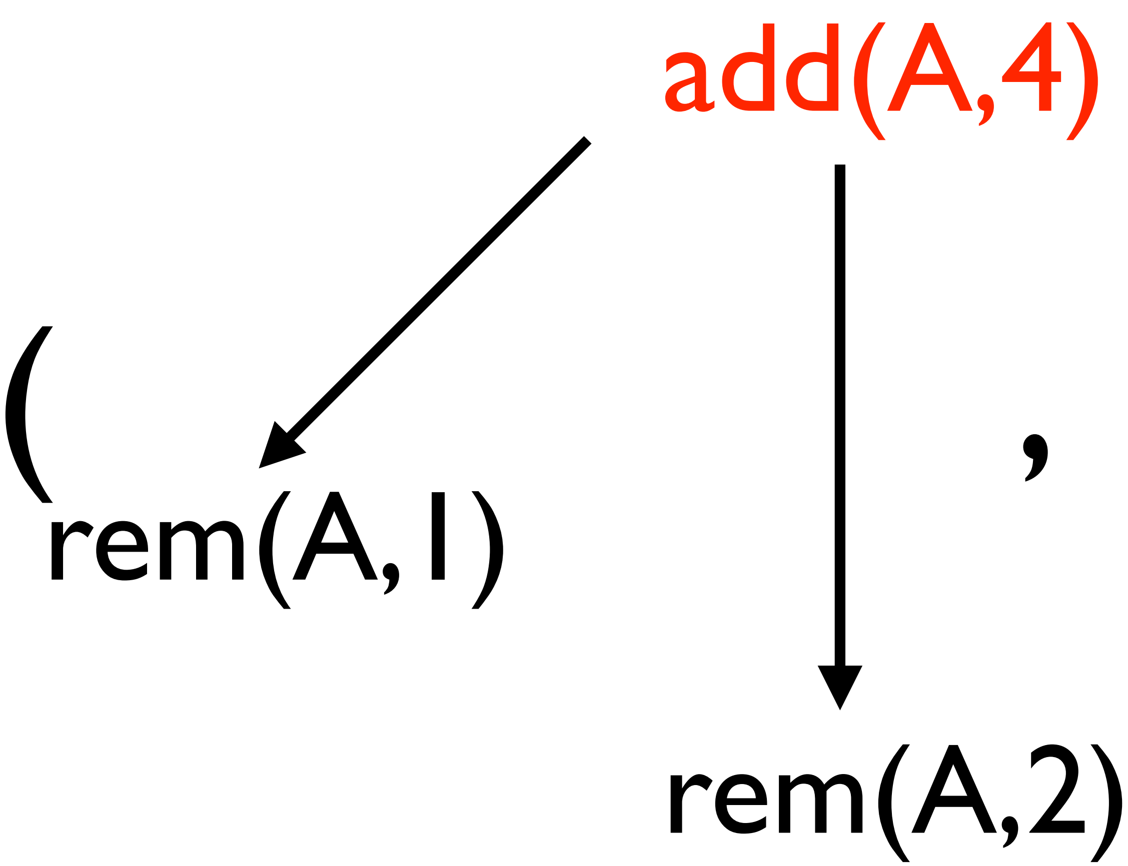
$\max(F_m($ 

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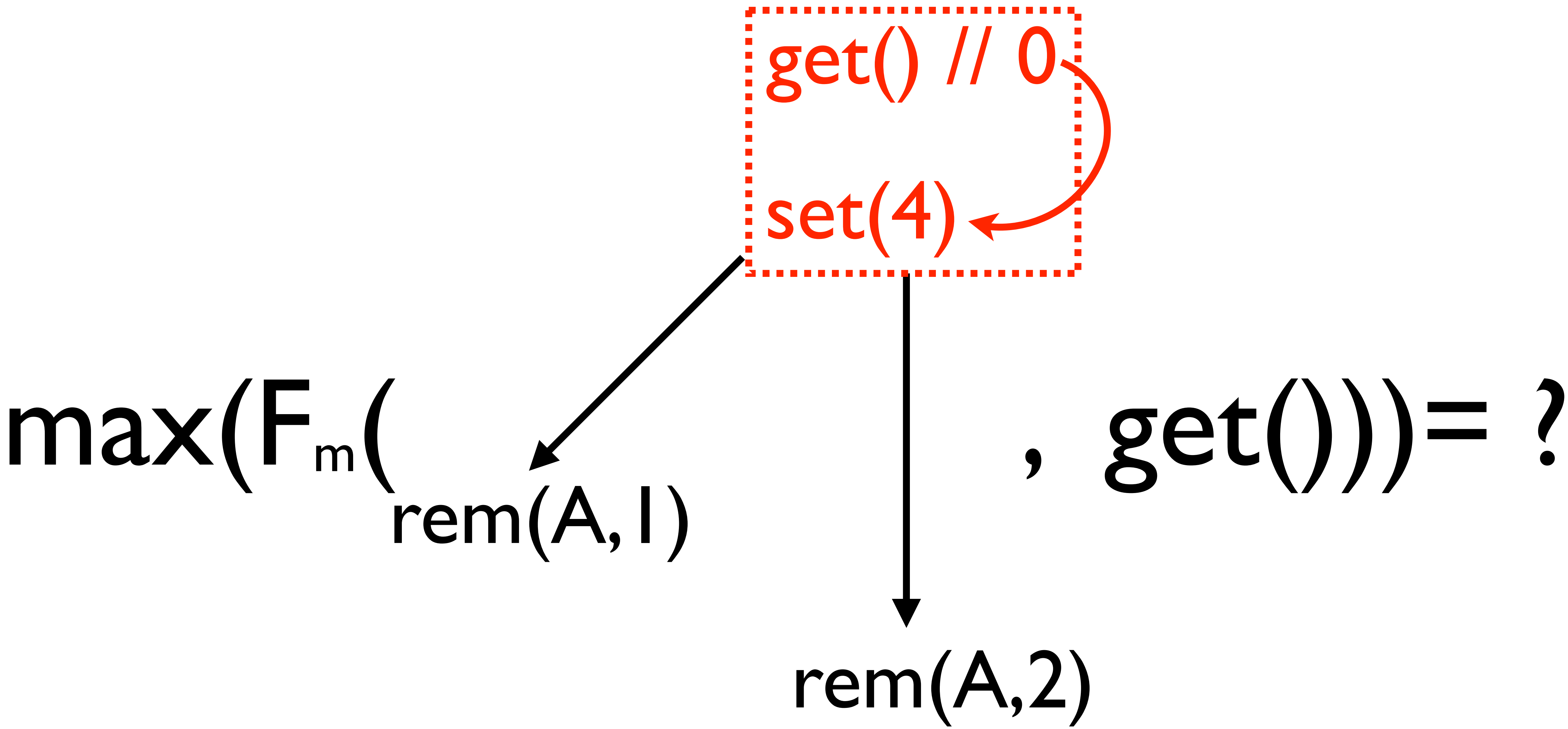

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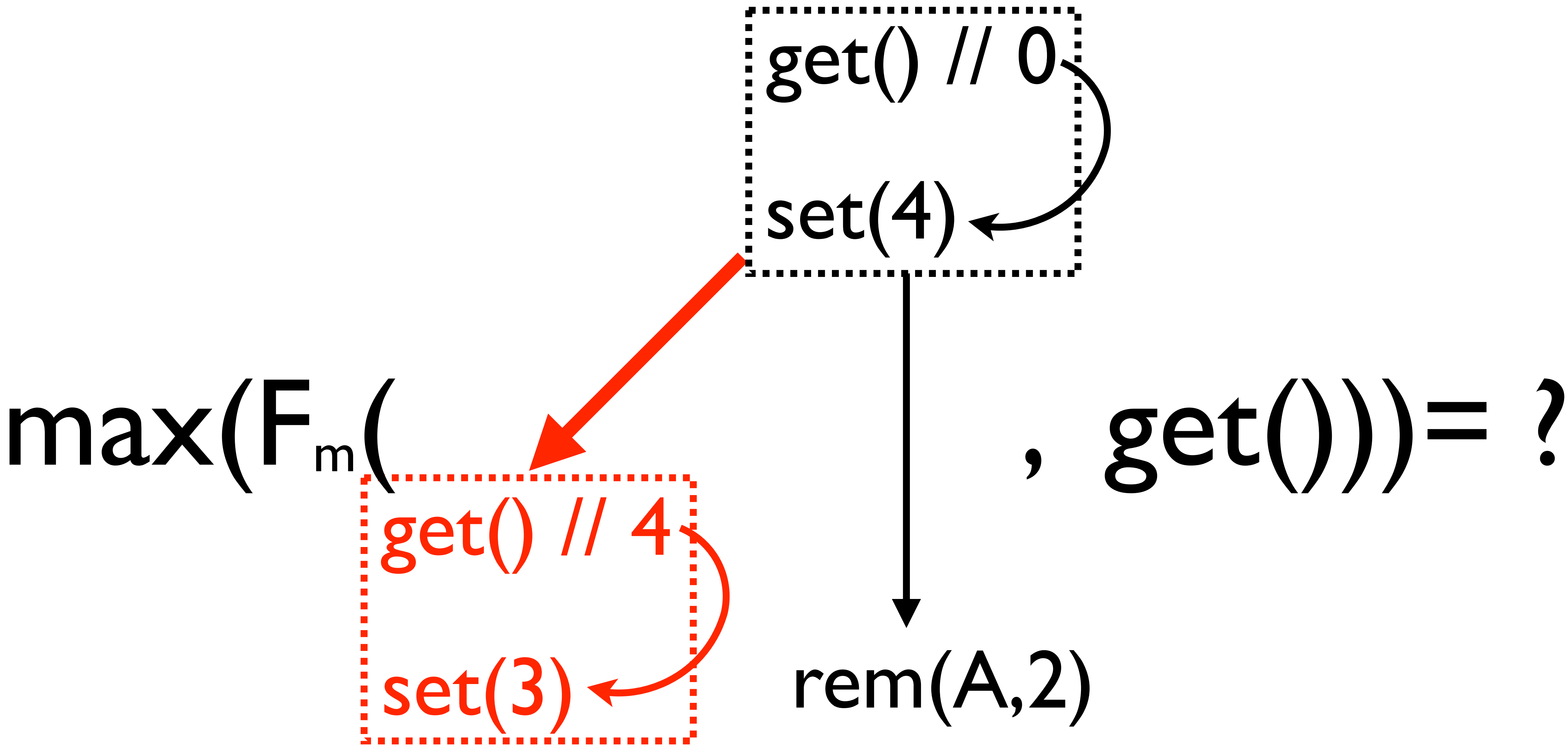

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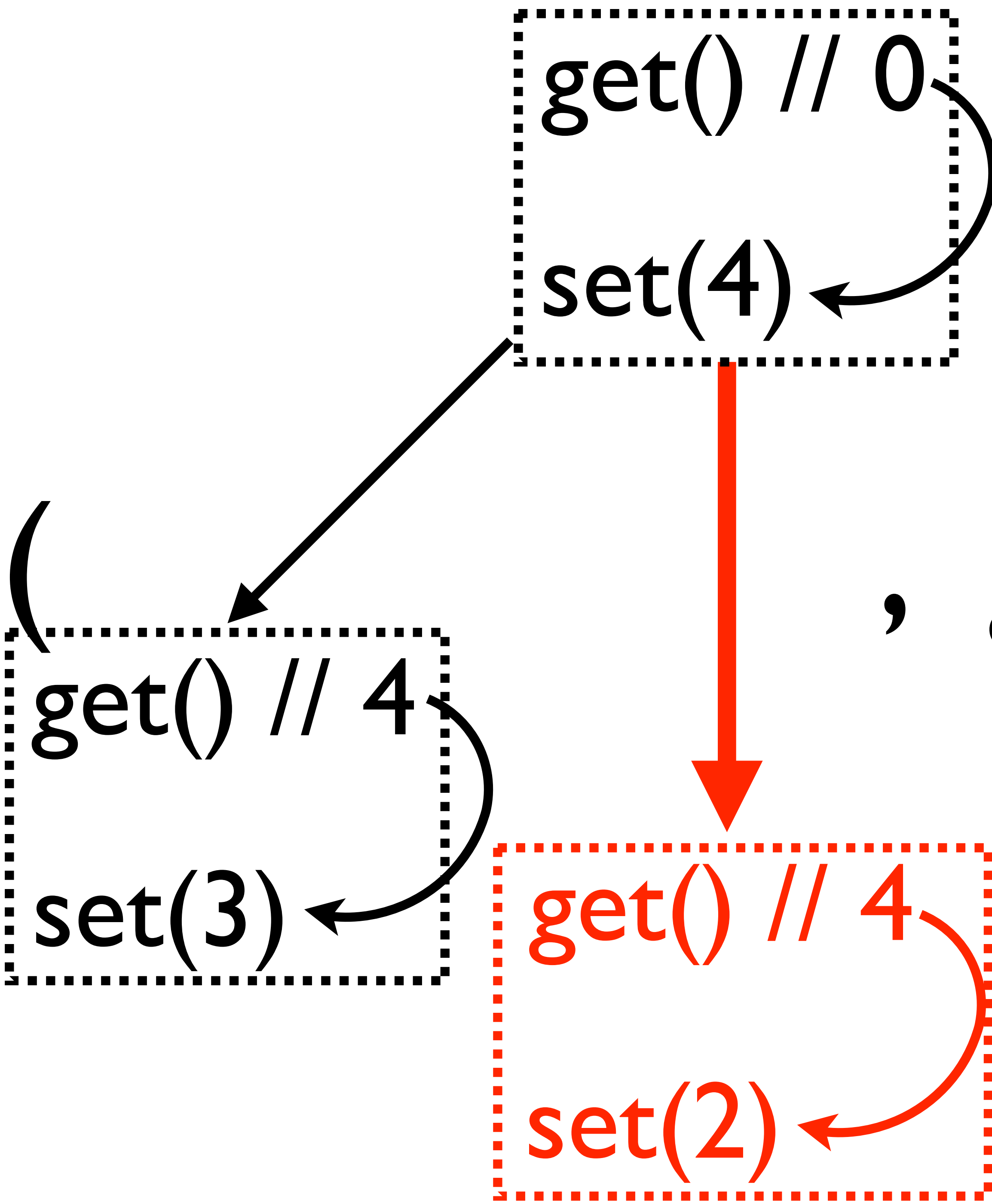
def count(b)={ val m = max(map(b).get()); return m }
```

$$\max(F_m(\quad\quad , \; get())) = \,?$$



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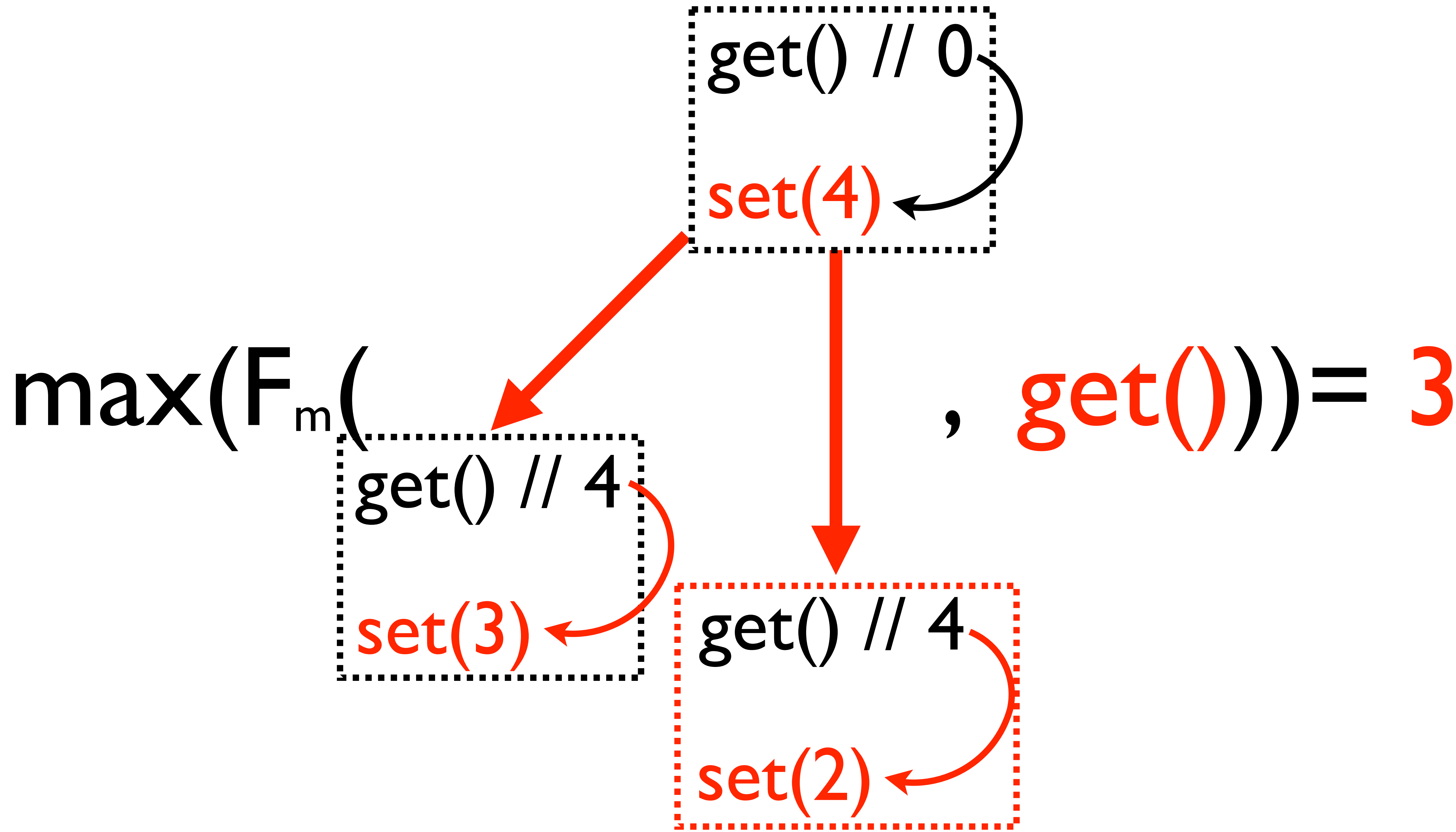

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```

$$F_s\left(\begin{tikzcd} & \mathrm{add}(A,4) \arrow[dl] \arrow[d] \\ \mathrm{rem}(A,1) & \mathrm{rem}(A,2) \end{tikzcd},\ \mathrm{count}()\right) = 3$$

```
val map = new array[MVRegister[Nat]](N)
. . . . .
def add(b,n)={ val m = max(map(b).get()); map(b).set(m+n) }
def rem(b,n)={ val m = max(map(b).get()); map(b).set(max(m-n,0)) }
def count(b)={ val m = max(map(b).get()); return m }
```

$$F_s\left(\begin{tikzcd} & \mathrm{add}(A,4) \arrow[dl] \arrow[d] \\ \mathrm{rem}(A,1) & \mathrm{rem}(A,2) \end{tikzcd},\ \mathrm{count}()\right) = 3$$

3 copies

```
val map = new array[MVRegister[Nat]](N)
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```

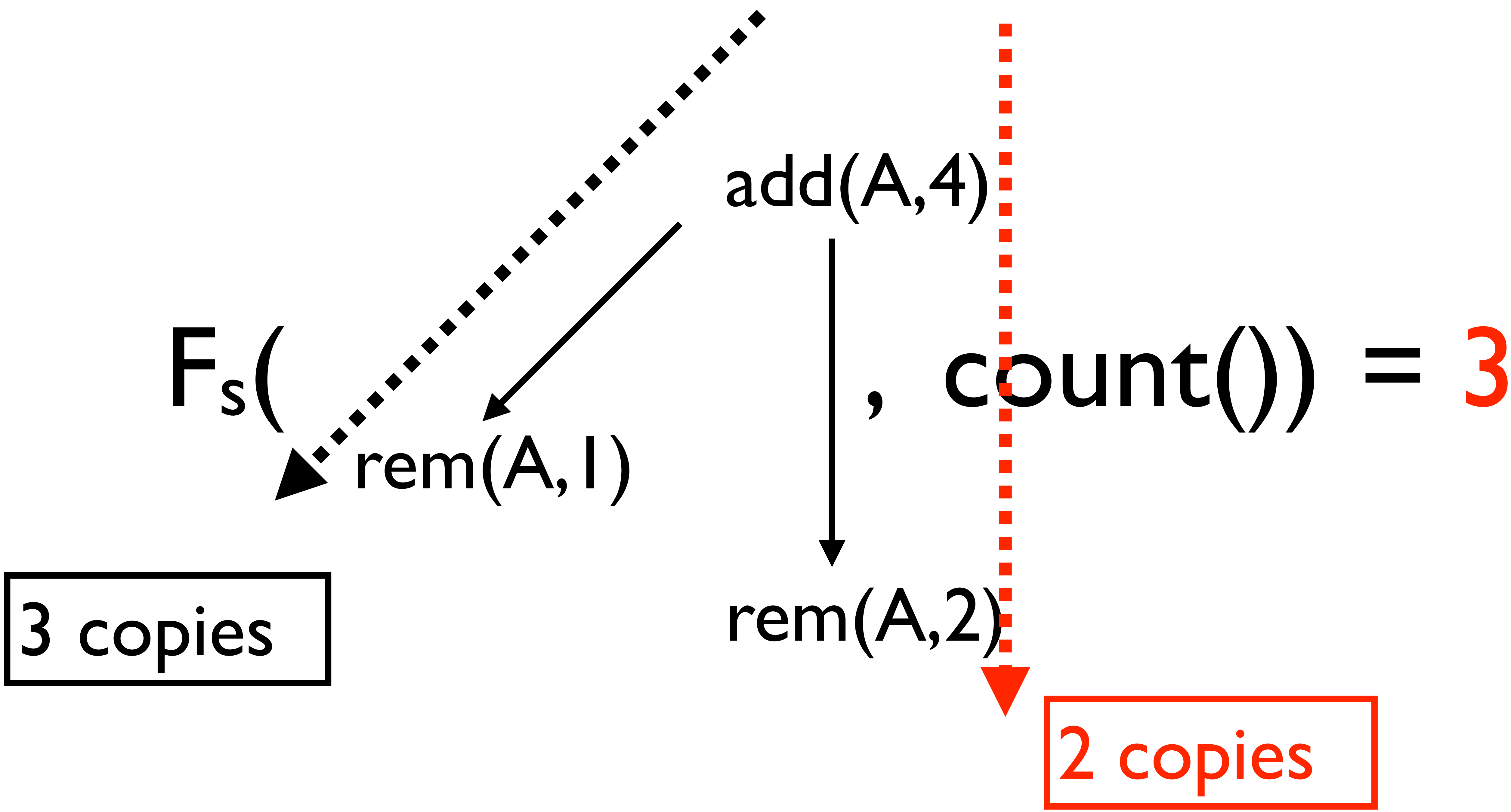

Semantics

- Developed F-based semantics of distributed programs with weak transactions.
- It is equivalent to a more standard semantics.
- The details are in our tech report. Email me if you are interested.

Take-home message

Developing a good programming model for weakly consistent distributed stores is a big challenge.